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Improved Pade-Pole Clustering Approach using Genetic Algorithm for Model Order Reduction

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ABSTRACT

Reducing the order of higher order systems by mixed approach is Improved Pade-Pole clustering based method to derive a reduced order approximation for a stable continuous time system is presented. In this method, the denominator polynomial of the reduced order model is derive by improved pole-clustering approach and the numerator polynomial are obtain through Padé approximation technique and by parameter optimization by minimizing the mean square error between the time responses of the original and reduced system element through genetic algorithm. The reduced order model so obtained by improved clustering algorithm guaranteed the stability in the reduced model.

Keywords

Padé approximation, Improved Pole clustering, Dominant pole, IDM, Mean Square Error, Genetic Algorithm.

1. INTRODUCTION

MOR originate to stand for producing compact system models from all sorts of physical systems modelling tools for the engineering application. There are lots of methods proposed in the literature [1-9] replicate the importance of constructing a reduced order model for the higher order system. Every Physical system needs mathematical modelling. Some point of concern for conversion the higher order system to reduced order system are

Physical systems,

Reduced computational complexity,

Reduced hardware complexity,

Simplified controller design and cost effective solutions.

Reduction of model are often take attention in system modelling and design of systems. Here author presented a mixed approach for order reduction. Projected technique based on Improved Pade-Pole clustering approach is dominant pole based pole clustering and with classical dominance. Author combined this technique with Genetic Algorithm for better an optimized result. There are so many approaches for the stable reduced order models presented in literature in SISO system or MIMO with mixed method [10-17] and allot work in time domain as well as in frequency domain with Genetic Algorithm.

The reduced-order model retains the basic physical features (such as time constants) of the original system and the stability of the simplified model is guaranteed. Proposed mixed methods based on the improved Padé-poles clustering approximations. Proposed method describes the poles of the reduced system is use as the cluster centre of the pole clusters of the original system which obtained by Inverse distance measure (IDM) criteria [1]. The choice of the clusters are either taken arbitrarily based on the order of reduction or by the investigator.

In proposed method, the reduced order denominator polynomial has been obtained using a dominant pole based pole-clustering approach for reduced order model. The method uses the improved pole clustering approach by deciding the value of the ratio of the residue to real parts of poles, taken in descending order and its corresponding reduced order model was obtained through a simple mathematical procedure. In classical approach the modes with the largest time constants i.e. slow modes or the poles nearest to imaginary axis are usually considered dominant, slow modes may not be dominant In proposed approach there two way to find numerator polynomial are obtained through Pade approximation and by Genetic algorithm. Error comparison done by Mean Square Error (MSE) for further effectiveness of system. GA calculate numerator polynomial by minimize the objective function which is MSE between the higher order system and reduced order system. The model so obtained preserves the stability. In spite of having several reduction methods, no approach always provides the satisfactory results for all systems.

2. DESCRIPTION OF PROPOSED METHOD

Consider a linear SISO time invariant system of n^{th} order. Higher order transfer function be in the form

$$G(s) = \frac{N(s)}{D(s)} = \frac{\sum_{j=0}^{m} q_j s^j}{\sum_{j=0}^{n} p_j s^j} \quad (1)$$

Where m ≤ n

Corresponding desired reduced order model of r^{th} order should be given by

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{\sum_{i=0}^{l} a_i s^i}{\sum_{i=0}^{r} y_i s^i} \quad (2)$$

Where $l \le$ r

New denominator $D_r(s)$ for reduced order model is obtain through pole clustering technique

$$D_r(s) = (s - P_0)(s - P_1) \ldots\ldots (s - P_r) \quad (3)$$

New numerator $N_r(s)$ for reduced order model is obtain through Padé approximation technique as

$$\frac{N(s)}{D(s)} = \frac{N_r(s)}{D_r(s)} \;;\quad N_r(s) = D_r(s) * \frac{N(s)}{D(s)} \quad (4)$$

Here important characteristics of original system retain by obtained reduced order model through this mixed method of model order reduction.

For computation of reduced order model consists two steps are as follows:

2.1 Denominator Polynomial Computation

Computation of denominator polynomial by improved pole clustering approach based on dominant pole and also by classical approach as shown below:

Dominant pole based pole-clustering approach the given system in (1) can be shown as

$$G(s) = \sum_{i=1}^{n} \frac{R_i}{s-\lambda_i} \quad (5)$$

By the given equation corresponding to every pole λ_i is the ratio of residue to pole intended as

$$\text{Ratio of residue to poles} = \frac{|R_i|}{|Re(\lambda_i)|}$$

The poles λ_i arrange in descending value of ratio of residue to poles and then n-poles $\lambda_1\ \lambda_2\ \lambda_{3.....}\lambda_n$ arranged in group as most dominant pole for r-clusters such that the poles are equal or in maximum number of clusters. Number of cluster and cluster center from each cluster is depend upon the order of the reduced system.

Projected Algorithm:

Let k number of poles available in one cluster group as: P_1, P_2, P_3 ... P_k cluster center obtained by inverse distance measure. The arrangement of poles are as follows $|P_1| < |P_2| < |P_3| \ldots\ldots < |P_k|$. Forming the cluster and find Cluster center for the reduced order system is obtained through the following procedure.

Step 3.1 - Let the k number of poles available be arrange as $|P_1| < |P_2| < |P_3| \ldots\ldots < |P_k|$,

Step 3.2 - Set M = 1,

Step 3.3 - Calculate for the pole cluster as

$$C_M = \left[\left(-\frac{1}{|P_1|} + \sum_{i=2}^{k} -1/|P_i - P_1|\right) \div k\right]^{-1},$$

Step 3.4 - Check if M = k, then the final cluster center is $C_C = C_M$ and terminate the process else go for next step.

Step 3.5 - Again set M = M+1,

Step 3.6 - The improved cluster center from

$$C_M = -\sqrt{P_1 * C_{M-1}}$$

Step 3.7 - Check M = k. If no then go to step 3.5 otherwise for next step,

Step 3.8 - The final cluster center is $C_C = C_M$

Here if the system to be reduced have pole lie on imaginary axis that should be retain as the cluster center as a single pole and other remaining are clustered in another clusters based on algorithm. We have following three cases for new denominator formation as

Case (1) - If all denominator poles or cluster center obtained are real. The denominator polynomial of reduced order in the form as:

$$D_r(s) = (s + C_{c1})(s + C_{c2}) \ldots.\ (s + C_{cr}) \quad (6)$$

The improved cluster values given as $C_{c1} C_{c2} \ldots\ldots C_{cr}$ and r is the order of reduced system.

Case (2) - If all poles or cluster center obtained are complex. The denominator polynomial of reduced order in the form as:

Let k complex conjugate poles in single cluster group be $(P_1 \pm jQ_1, P_2 \pm jQ_2, \ldots\ldots P_k \pm jQ_k)$. Where $|P_1| < |P_2| < |P_3| \ldots\ldots < |P_k|$ are obtain through same algorithm is proposed above, it follows separately for real and complex poles. Then the improved cluster center is as

$$\beta_i = P_i \pm jQ_i$$

Corresponding reduced order polynomial is

$$D_r(s) = (s + |\beta_1|)(s + |\beta_2|) \ldots.\ (s + |\beta_i|) \quad (7)$$

Case (3) - If some cluster center are real and remaining are in the complex form. Then applying the algorithm separately for real part and then for complex terms. To get cluster center for reduced system combine both of them to find denominator polynomial.

Classic dominance based pole clustering approach:

In classic dominance based approach pole cluster formation is made by most dominant pole first, and most dominant pole is decided through the modes with the largest time constants or nearest pole to the origin. And further cluster center is obtained through the same algorithm proposed.

2.2 Numerator Polynomial Computation

Numerator polynomial is calculate by Pade approximation equating the original higher order system transfer function with generated reduced system transfer function and another way is optimization through Genetic Algorithm by minimizing the mean square error between the time responses of the original and reduced system element. The denominator polynomial of reduced system obtained from first step is use here to get the unknown coefficient of reduced system. As given

$$\frac{N(s)}{D(s)} = \frac{N_r(s)}{D_r(s)}$$

$$\frac{q_0+q_1s+q_2s^2+\cdots+q_{m-1}s^{m-1}+q_ms^m}{p_0+p_1s+p_2s^2+\cdots+p_{n-1}s^{n-1}+p_ns^n} = \frac{a_0+a_1s+a_2s^2+\cdots+a_{l-1}s^{l-1}+a_ls^l}{y_0+y_1s+y_2s^2+\cdots+y_{r-1}y^{r-1}+y_rs^r} \quad (8)$$

By cross multiplying the equation and comparing the coefficient of same power of 's' and we get the new numerator coefficient as

$$q_0y_0 = p_0a_0$$

$$q_0y_1 + q_1y_0 = p_0a_1 + p_1a_0$$

$$q_0y_2 + q_1y_1 + q_2y_0 = p_0a_2 + p_1a_1 + p_2a_0$$

$$\vdots$$

$$q_my_r = p_na_l$$

The unknown coefficient $a_0, a_1, a_3, \ldots\ldots a_l$ can get easily. The numerator polynomial of reduced order system is shown in form of

$$N_r(s) = y_0 + y_1 s + y_2 s^2 + \;.... \qquad y_{r-1}s^{r-1} + y_r s^r \quad (9)$$

Mean square error (MSE)

MSE is determine for the proposed approach as

$$\text{MSE} = \frac{1}{N}\sum_{i=1}^{N}(y_0(t_i) - y_r(t_i))^2 \quad (10)$$

Here $y_0(t_i)$ is the step response of original higher order system and $y_r(t_i)$ is step response of reduced order model and N is the number of elements in the output vectors.

Genetic algorithm

Genetic Algorithms are the heuristic search and optimization techniques that mimic the process of natural evolution. GAs are based on principles inspired from the genetic and evolution mechanisms observed in natural systems. Their basic principle is the maintenance of a population of solutions to the problem that evolves in time. They are based on the triangle of genetic reproduction, evaluation and selection.

GA consist few steps are Initialization, Evaluation, Selection, Crossover, Mutation, Replacement, Termination.

Through GA find new numerator by optimization of MSE between transient response of original and reduced system. GA estimation process is performed based on the calculation of the fitness function defined as

$$\text{Fitness} = \frac{1}{1 + MSE}$$

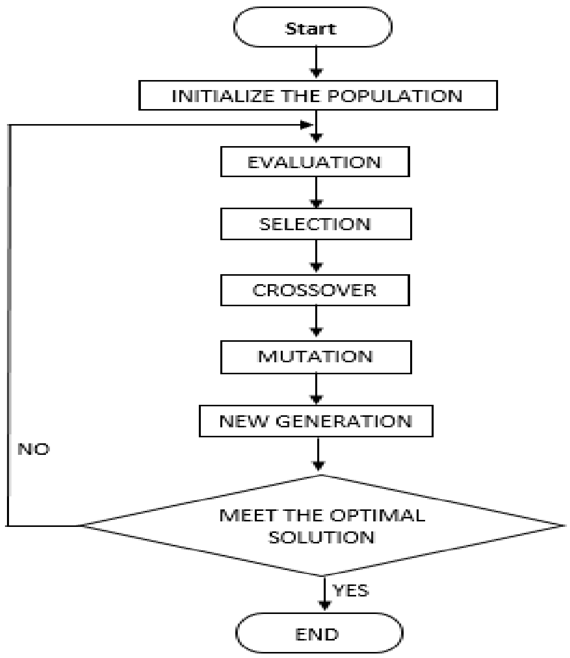

Fig 1- Flowchart of genetic algorithm

The parameters for the Genetic Algorithm

Number of parameter to be optimize = 3

Number of generations=100

Population size = 50

Selection = 0.5

Mutation rate = 0.02

Termination method = Maximum generation

3. NUMERICAL EXAMPLE AND SIMULATION RESULT

Original system

$$G(s) = \frac{N(s)}{D(s)}$$

$$N(s) = 40320 + 185760s + 222088s^2 + 122664s^3 + 36380s^4 + 5982s^5 + 514s^6 + 18s^7$$

$$D(s) = 40320 + 109584s + 118124s^2 + 67284s^3 + 22449s^4 + 4536s^5 + 546s^6 + 36s^7 + s^8$$

Reduced system transfer function by DPPCA

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{10.75\, s^2 + 26.63\, s + 7.49}{s^3 + 6.312\, s^2 + 12.48\, s + 7.49}$$

After Optimization by Genetic Algorithm best solution for system

Numerator parameter

12.3013 25.126 7.66283

System transfer function after GA (shows in fig.2)

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{12.30\, s^2 + 25.126\, s + 7.66283}{s^3 + 6.312\, s^2 + 12.48\, s + 7.49}$$

Reduced system transfer function by classical dominance

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{13.64\, s^2 + 40.1\, s + 11.45}{s^3 + 8.019\, s^2 + 18.47\, s + 11.45}$$

After Optimization by Genetic Algorithm best solution for system

Numerator parameter

14.5603 39.583 11.4483

System transfer function after GA (shows in fig.3)

$$G_r(s) = \frac{N_r(s)}{D_r(s)} = \frac{14.5603\, s^2 + 39.583\, s + 11.4483}{s^3 + 8.019\, s^2 + 18.47\, s + 11.45}$$

Table 1. Comparison of MSE value

S. No.	Approach	MSE	MSE after optimization
1.	DPPCA	0.0066086	0.0032678
2.	Classical dominance	0.0014263	0.00089461

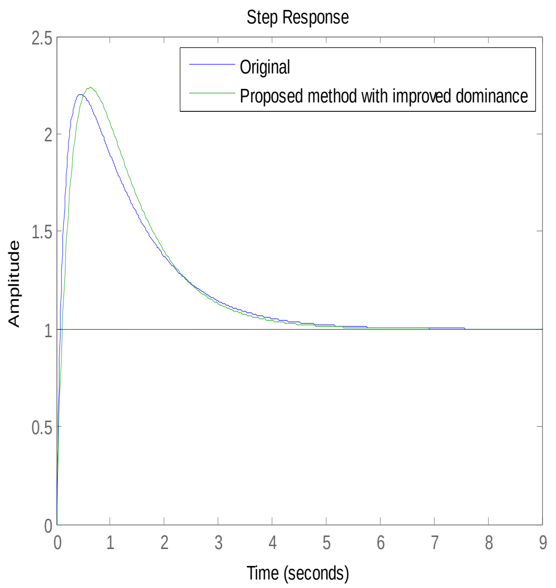

Fig.2. Step response of original and reduced system with improved dominance

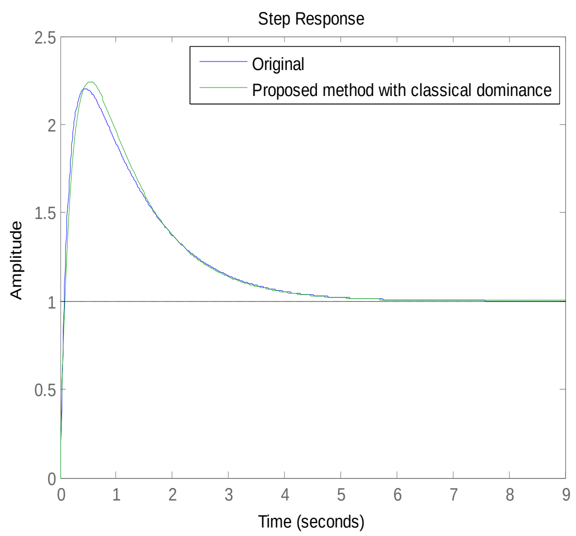

Fig.3. Step response of original and reduced system with classical dominance

4. CONCLUSION

Proposed mixed method for order reduction of higher order system model using improved Pade-Pole clustering approach applying with the DPPCA and classical dominance approaches. Denominator polynomial is obtain through pole clustering approach and numerator polynomial through Pade approximation for the reduced order model and by Genetic Algorithm via minimizing the mean square error between time response of original and reduced system transfer function which is optimization function for GA. Comparison of MSE shown in table. GA gives better and more optimized result with improved Pade-Pole clustering approach. The comparison of MSE shown in table with proposed techniques before optimization GA and after optimization. Characteristics and stability of original system are retained in reduced order system model.

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