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A Virtual Reality Environment for 5-DOF Robot Manipulator based on XNA Framework

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ABSTRACT
This work presents a complete solution for the Forward and Inverse Kinematics for a 5DOF robot manipulator and use XNA game studio as a simulator. The work include mathematical modeling of the robot Kinematics which it used in a 3D virtual environment. This text show that it can be use XNA GS as an excellent tool to simulate robots behavior.

Keywords
inverse kinematics, Lab volt 5150, XNA, Simulation, Virtual reality.

1. INTRODUCTION
Many researches dealt with the analysis for inverse Kinematics of a 5-DOF manipulator considered that it should deal with the problem as an analysis for 6-DOF manipulator losing one degree of freedom [1], [2] and deficiency of degrees of freedom makes the control of manipulators with less than 6-DOF very difficult when following a given trajectory[3].

Some algebraic solutions for inverse Kinematics of a 5-DOF manipulator assume that summation of joint angles 2,3 and 4 (θ_234) is a constant value, so it didn't solve all the joint angles. [4] introduced analysis for Forward Kinematics of 5-Dof and didn't deal with inverse Kinematics of the robot. C. S. G Lee, M. Ziegler [5] solve the problem of choosing a solution among multi solutions generated as a result of left and above arm, right and above arm, left and below arm, right and below arm and wrist flip configurations by assuming an indicators, this way need that joint angles already known. In this paper, a 5-DOF manipulator which is Lab-Volt 5150 series was modeled and analyzed without assuming any auxiliary frame. All the driven forward and inverse Kinematics shows a great deal of accuracy theoretically and experimentally, also the problem of multi solutions was solved by suggesting an algorithm that choose the best configuration for a point in space represented by a position vector and orientation matrix.

A virtual reality environment developed to simulate the movements of the robot arm using C#.Net language XNA framework. Virtual reality is a system which allows one or more users to move and react in a computer generated environment. The basic VR systems allow the user to visual information using computer screens [6]. Section 2 shows the modeling of the robot manipulator and D-H parameters associated with the model. Section 3.1 introduces the analysis of the forward Kinematics and use Lab-Volt 5150 as a case study. Section 3.2 introduced the problem of the singularity and it was proved that there are no singularity. Section 4 offer the inverse Kinematics analysis and set all the governing equations for all the joints angles. Section 5 give the theoretical and experimental results done. finally section 6 give the conclusions.

2. COORDINATE FRAME FOR LAB VOLT 5150
For this study, Lab-volt 5150 robot manipulator was used as a case study. It is an educational robot from the family of Lab Volt Automation. as shown in Fig (3),it is a 5 DOF manipulator driven by five stepper motors and has a griper as an end-effector.

Fig (1) Lab Volt 5150 manipulator

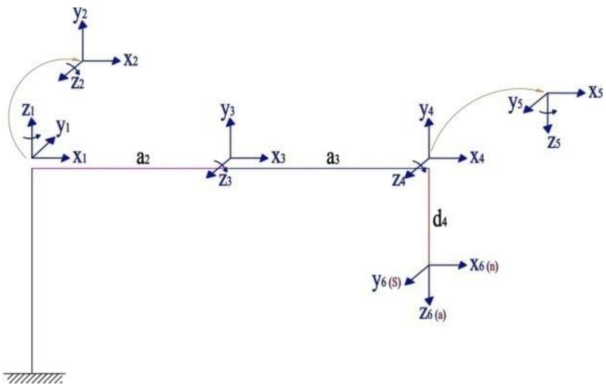


Fig (2) Coordinate frames for Lab Volt 5150 manipulator

Coordinate frames are assigned as shown in the Fig (4). They are established using the principles of the Denativ-Hartenberg (D-H) convention.

3. FORWARD KINEMATICS
Forward Kinematics is the multiplication of set of matrices. Each matrix represents the position and orientation of a link relative to the former one in a serial links, these matrices

called homogenous transformation matrices [7]. Specifically, given a set of joint angles, the forward kinematic problem is to compute the position and orientation of the tool frame relative to the base frame. Sometimes, it can be said the forward Kinematics changes the representation of manipulator position from a joint space Coordinate into a Cartesian space Coordinate.

Table (1) shows (D-H) parameters which are concluded from the frame assignment and used to generate Forward Kinematics

Table (1) D-H parameters for the manipulator

$Link$	$\propto_i$	a_i	d_i	θ_i
1	90	0	0	θ_1
2	0	a_2	0	θ_2
3	0	a_3	0	θ_3
4	90	0	0	θ_4
5	0	0	d_4	θ_5

where : $a3 = a2 = 190.25mm$, $d4 = 115mm$

transformation matrix for each link can be obtained by substituting these parameters in Table 1 into D.H matrix.

$$T_1=\begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .....(1)$$

$$T_2=\begin{bmatrix} C_2 & -S_2 & 0 & a_2C_2 \\ S_2 & C_2 & 0 & a_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ....(2)$$

$$T_3=\begin{bmatrix} C_3 & -S_3 & 0 & a_3C_3 \\ S_3 & C_3 & 0 & a_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ....(3)$$

$$T_4=\begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ....(4)$$

$$T_5=\begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} ....(5)$$

$$T = T_1 * T_2 * T_3 * T_4 * T_5 ....(6)$$

T represents transformation matrix which hold the position vector and orientation matrix of the end-effector with respect to the base frame

$$T=\begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} ....(7)$$

where $\{p_x, p_y, p_z\}$ represent the position vector and $\{\{n_x, n_y, n_z\}$,$\{o_x, o_y, o_z\}$,$\{a_x, a_y, a_z\}$ $\}$ represent the orientation of the center of the end-effector.It is clear that the position and orientation are a function of joint variables and (D-H) parameters as shown in equations (8) to (18).

$n_x = S_1S_5 + C_5C_1C_{234}$ ....(8)

$n_y = -C_1S_5 + C_5S_1C_{234}$ ....(9)

$n_z = C_5S_{234}$ ....(14)

$o_x = C_5S_1 - S_5C_1C_{234}$ ....(10)

$o_y = -S_5S_1C_{234} - C_1C_5$ ....(11)

$o_z = -S_5S_{234}$ ....(12)

$a_x = C_1S_{234}$ ....(13)

$a_y = S_1S_{234}$ ....(14)

$a_z = -C_{234}$ ....(15)

$P_x = C1(d_4S_{234} + a_2C_2 + a_3C_{23})$ ......(16)

$P_y = S1(d_4S_{234} + a_2C_2 + a_3C_{23})$ ......(17)

$P_z = a_2S_2 - d_4C_{234} + a_3S_{23}$ .....(18)

where $C_i = \cos(\theta_i)$,$S_i = \sin(\theta_i)$,

$S23 = \sin(\theta_2 + \theta_3)$, $C23 = \cos(\theta_2 + \theta_3)$

$a3 = a2 = 190.25mm$

$d4 = 115mm$

3.1 Singularity

The singular position might emerge when frame $x_4y_4z_4$ rotate 90°, which causes the links 2,3 and 4 to be on the same plane. this has to be checked for, before driving the inverse Kinematics. If $\theta_4 = 90$, then

$$T_4=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .....(19)$$

from (10) and substitute T_4 from (20)

$$(T_1)^{-1} * T = T_2 * T_3 * T_4 * T_5 . ...(21)$$

where $(T_1)^{-1} * T =$

$$\begin{bmatrix} n_xC_1 + n_yS_1 & o_xC_1 + o_yS_1 & a_xC_1 + a_yS_1 & P_xC_1 + P_yS_1 \\ n_z & o_z & a_z & P_Z \\ n_xS_1 - n_yC_1 & o_xS_1 - o_yC_1 & a_xS_1 - a_yC_1 & P_xS_1 - P_yC_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_2 * T_3 * T_4 * T_5 =$

$$\begin{bmatrix} -C_5S_{23} & S_5S_{23} & C_{23} & d_4C_{23} + a_2C_2 + a_3C_{23} \\ C_5C_{23} & -S_5C_{23} & S_{23} & d_4S_{23} + a_2S_2 + a_3S_{23} \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

by equating elements (3,4) from both sides of (21)

it is found that

$P_x S_1 - P_y C_1 = 0$.....(22)

from elements (3,1)

$S_5 = n_x S_1 - n_y C_1$....(23)

elements (1,4)

$d_4 C_{23} + a_2 C_2 + a_3 C_{23} = P_x C_1 + P_y S_1$

.......(24)

and from elements (2,4)

$d_4 S_{23} + a_2 S_2 + a_3 S_{23} = P_Z$ ....(25)

It clear from equations (22) to (25) that there are a solutions for joint angles θ_1 to θ_5 In other words, there exists no singular problem in Lab Volt 5150 in the case of $\theta_4 = 90$.

4. INVERSE KINEMATICS

Determining the joints variables of the manipulator given the transformation matrix that relate the position and orientation of the end-effector frame to the base frame.

referring to (10) , T_5^0 is given, the equation

$^{-1}(T_1^0) * T_5^0 = T_2^1 * T_3^2 * T_4^3 * T_5^4$ . ...(26)

where $(\mathrm{T}_1)^{-1} * T =$

$$\begin{bmatrix} n_x C_1 + n_y S_1 & o_x C_1 + o_y S_1 & a_x C_1 + a_y S_1 & P_x C_1 + P_y S_1 \\ n_z & o_z & a_z & P_Z \\ n_x S_1 - n_y C_1 & o_x S_1 - o_y C_1 & a_x S_1 - a_y C_1 & P_x S_1 - P_y C_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\mathrm{T}_2 * \mathrm{T}_3 * \mathrm{T}_4 * \mathrm{T}_5 =$

$$\begin{bmatrix} C_5 C_{234} & -S_5 C_{234} & S_{234} & a_2 C_2 + d_4 S_{234} + a_3 C_{23} \\ C_5 S_{234} & -S_5 S_{234} & -C_{234} & a_2 S_2 - d_4 C_{234} + a_3 S_{23} \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

will use to determining the joint angles for the robot

4.1 Solution for θ_1

By equating elements (3,4) from both sides of (26) will get

$P_x * S_1 - P_y * C_1 = 0$ ....(27)

then

$\theta_1 = atan2(P_y, P_x)$ ....(28)

the other solution will be

$\grave{\theta_1} = \theta_1 + 180$ ....(29)

4.2 Solution for θ_5

equating elements (3,1) from both sides give

$S_5 = n_x * S_1 - n_y * C_1$ ....(30)

and from equating elements (3,2)

$C_5 = o_x * S_1 - o_y * C_1$.......(31)

Then,

$\theta_5 = atan2(S_5, C_5)$ ....(32)

4.3 Solution for θ_{234}

where $\theta_{234} = \theta_2 + \theta_3 + \theta_4$

referring to (26) and equating elements (1,1) from both sides of the equation will give

$C_5 C_{234} = n_x * C_1 + n_y * S_1$ ....(33)

then, $C_{234} = n_x * C_1 / C_5 + n_y * S_1 / C_5$ .....(34)

and $S_{234} = \mp\sqrt{1 - {C_{234}}^2}$ ....(35)

then, $\theta_{234} = atan2(S_{234}, C_{234})$ ....(36)

4.4 Solution for θ_3

equating elements (1,4) from both sides of (26) will results

$P_x C_1 + P_y S_1 = a_2 C_2 + d_4 S_{234} + a_3 C_{23}$ ...(37)

and equating (2,4) results,

$P_z = a_2 S_2 - d_4 C_{234} + a_3 S_{23}$ ......(38)

by squaring and adding equations (37) and (38)

$C_3 = \frac{(P_x C_1 + P_y S_1 - d_4 S_{234})^2 + (P_z + d_4 C_{234})^2 - a_2^2 - a_3^2}{2 a_2 a_3}$ ...(39)

and $S_3 = \mp\sqrt{1 - {C_3}^2}$ ....(40)

then, $\theta_3 = atan2(S_3, C_3)$ ....(41)

4.5 Solution for θ_2

referring to (37) and (38) by rearranging and solve for θ_2 then,

$C_2 = \frac{(P_x C_1 + P_y S_1 - d_4 S_{234})(a_3 C_3 + a_2) + a_3 S_3 (P_z + d_4 C_{234})}{(a_3 C_3 + a_2)^2 + a_3^2 S_3^2}$ ....(42)

and $S_2 = \mp\sqrt{1 - {C_2}^2}$.......(43)

then , $\theta_2 = atan2(S_2, C_2)$ ......(44)

4.6 Solution for θ_4

simply by subtracting the values of $\theta_2$and θ_3 from θ_{234} , the value of θ_4 will be determined

$\theta_4 = \theta_{234} - \theta_2 - \theta_3$ ....(45)

Equations (28) to (45) shows that there are multi solutions for a single point in space represented in a homogenous transformation matrix. This text introduces an algorithm generates eight possible solutions and choose the feasible solution which is achieves minimum joint displacements , see Fig (3).

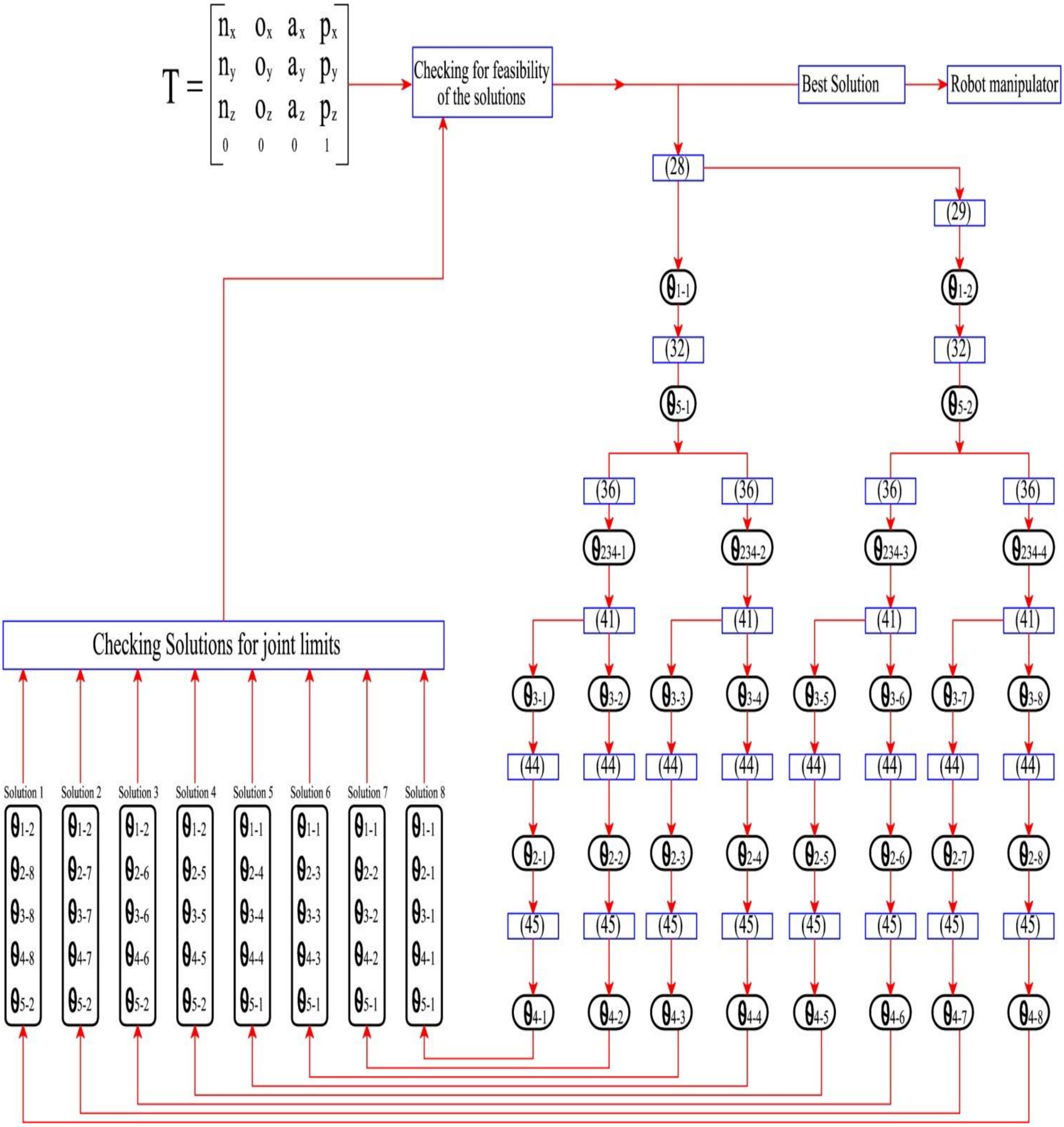


Fig (3) algorithm of choosing optimal solution among 8 possible solution for a point

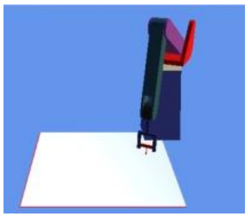
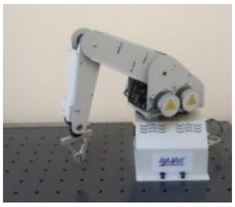
5. VIRTUAL REALITY (VR) AND SIMULATION

Virtual reality [6] is an attractive way to visualize the data in three dimensional environment generated using computer. VR make advantage of user interaction devices and the governing equations for a system which is need to be simulated. In robotics VR is very important because it shows the behavior of the robots in a duty, so users can avoid any damages may be occurs. In recent years Microsoft had announced a new technology for game developers and called it XNA which is a toolset with a managed runtime environment , it simplify the video game development and it was based on .NET framework. XNA support modeling objects as well as dealing with CAD files as an object, but CAD files must be exported to .fbx or xfile so as to be possible to be added to the virtual environment. This text use XNA to develop a computer-generated-environment to simulate a 5-DOF manipulator.

6. RESULTS AND DISCUSSION

The driven forward and inverse Kinematics equations have been investigated and show an accurate results in a real and virtual environment for different configurations for the manipulator. Four different arbitrary configurations was tested by setting the joint angles of the real and virtual model of Lab-Volt 5150 to the set of angles for each configuration in table (2) and the two models had a great matching in end-effector position.

Table (2) Comparison between real and virtual configuration

Case	Joint variables	Virtual end-effector position mm	Actual end-effector position mm	Error %	Real model	Virtual model
1	$\theta_1 = 45$ $\theta_1 = 24$ $\theta_1 = -128$ $\theta_1 = 104$ $\theta_1 = 0$	$P_x = 90.12$ $P_y = 90.12$ $P_z = 33.19$	$P_x = 90.1$ $P_y = 90.1$ $P_z = 33.18$	0.022 0.022 0.03		
2	$\theta_1 = 64.65$ $\theta_1 = 25.09$ $\theta_1 = -103.5$ $\theta_1 = 78.4$ $\theta_1 = 0$	$P_x = 90.02$ $P_y = 190.04$ $P_z = 34.72$	$P_x = 90.06$ $P_y = 190.1$ $P_z = 34.74$	0.044 0.031 0.057		
3	$\theta_1 = 45$ $\theta_1 = 19.12$ $\theta_1 = -81.12$ $\theta_1 = 62.01$ $\theta_1 = 0$	$P_x = 190.15$ $P_y = 190.15$ $P_z = 34.72$	$P_x = 190.14$ $P_y = 190.14$ $P_z = 34.72$	0.0052 0.0052 0		
4	$\theta_1 = 25.35$ $\theta_1 = 25.1$ $\theta_1 = -103.5$ $\theta_1 = 78.41$ $\theta_1 = 0$	$P_x = 190.04$ $P_y = 190.02$ $P_z = 34.72$	$P_x = 190.05$ $P_y = 190.04$ $P_z = 34.74$	0.0053 0.01 0.057		

7. CONCLUSIONS

A complete analytical solution to the forward and inverse Kinematics of the 5 DOF manipulator was driven in this paper and all the driven governing equations was proved that they are correct and accurate theoretically and experimentally. All the joint angles were delivered to the virtual model of Lab-Volt 5150 robot manipulator then these angles delivered to a real one and we compared the deviation in x, y, and z axis. We recommended the driven forward and inverse kinematics to use in other applications like path planning or collision avoidance. Also XNA framework is reliable and can be used to simulate very wide range of applications because it has a lot of built- in and customizable functions for math and graphics. We have focused on simulate the robot manipulator Kinematically, so including the Kinetic behavior is suitable for a future work.

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