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# Characteristics of a Fuzzy Project Network using Statistical Data

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## ABSTRACT
In the present paper, Characteristics of fuzzy project network using statistical data are discussed in detail in order to calculate the fuzzy critical path, fuzzy earliest times, fuzzy latest times and fuzzy total float. Fuzzy number as fuzzy activity time is constructed using interval estimate by calculating mean, variance and standard error. A new ranking function is used to discriminate the fuzzy numbers used as activity times in the fuzzy project network and used it in distance measure. The appropriateness and contribution of characteristics of fuzzy project network to ship building is discussed and calculated as an application to real life problem using statistical parameters.

## Keywords
Critical path, project network, fuzzy numbers, metric distance

## 1. INTRODUCTION
A project can be exemplified by a directed acyclic graph (project network) where the nodes stand for activities and the arcs for precedence relations when resource limitations are taken into consideration. Critical Path Method (CPM) provides the characteristics of project network, marginal project duration and identifies the critical path(s) when the activity times in a project are deterministic. Moreover, there are number of cases where the activities are not deterministic, but random assessments, Program Evaluation and Review Technique (PERT) that is built on the theory of probability can be used. Nonetheless, in real world applications, some activity times ought to be predicted subjectively such as utilizing human judgement, the wisdom of the decision maker, professional knowledge, experience, that should be replacing stochastic assumptions to determine activity times. A substitute method to deal with vague data is to put into use the concept of fuzziness, whereby the inexact activity times can be signified by fuzzy sets.

A fuzzy project network (FPN) was developed by [1] to forecast the activity durations and applied fuzzy algebraic operations in order to calculate the characteristics and critical path with project completion time. In [2], the composite and comparison methods of analyzing fuzzy numbers in

a proficient scheme for solving problems pertaining to project scheduling in a FPN are discussed. The fuzzy Delphi method [3] is used to establish the fuzzy activity time estimates in an FPN.In an FPN, [4] expounded a fuzzy project scheduling decision support system and it is used to earmark resources amid dependent activities in a software project scheduling environment. In [5] , a resource-held back project scheduling method that deals with three performance objectives; (1) expected project completion time; (2) utilization of resources; and (3) interruption of resources is introduced. Chen et al. [6] assimilated time-window constraint and time schedule constraint into the traditional activity FPN and also worked out a linear time algorithm for finding the critical path in an activity network with the time-constraints. In [7] proposed a procedure for ranking fuzzy numbers without the prerequisite for any postulations and used positive as well as negative values to define ordering which, then is applied, to fuzzy critical path method. They then did away with the fuzziness by putting into use the signed distance ranking for those fuzzy numbers to co-construct the activity network in the fuzzy sense, after which the fuzzy critical path can be acquired without difficulty. In [8] , the notion of criticality in the network with fuzzy activity times is introduced and two procedures of calculation of the path degree criticality are discussed. In [9] depicted a new line of method to a fuzzy critical path method for activity network created on statistical buoyancy interval estimates and a ranking method for level (1-α) fuzzy numbers. His focus was to introduce an approach that combined fuzzy set theory with statistics that incorporates the signed distance ranking of level (1-α) fuzzy numbers derived from (1-α)*100%. In [10], brought forward a new methodology to fuzzy critical path method that is based on statistical confidence interval estimates. An effort to combine statistics and fuzzy mathematics incorporating the signed distance ranking level (1-α) of fuzzy numbers was also focused. In [11] initiated a fuzzy critical path method constructed on statistical confidence-interval estimates and a signed distance ranking for (1-α)fuzzy number levels. In [12] encompassed some results for interval numbers to the fuzzy case for determining the possibility distributions depicting state of the art starting times for activities. In [13] expounded a way of modeling the well-defined collaboration process built on workflow technology, probability theory and fuzzy set theory. In [14] handed out an uncomplicated approach to solve the critical path method problem with fuzzy activity times which are fuzzy numbers on the foundation of the linear programming formulation and the fuzzy number ranking method that are more pragmatic than crisp ones. In [15], the algorithms for calculating the interval value of the state-of-the-art starting times and maximal floats of activities in the network with interval activity and time lag durations are familiarized. In [16], a fuzzy approach constructed on statistical confidence interval estimates and a distance ranking method for (1-α) fuzzy number levels is discussed.

In Section 2, of the paper some definitions on fuzziness, arithmetic operations between the fuzzy numbers are dealt with the definition of metric distance and signed distance ranking system for fuzzy numbers are reviewed. Existing Methods for characteristics of fuzzy project network(FPN) using distance measure of fuzzy activity time as fuzzy number like characteristics of FPN using metric distance and FCPM using sign distance ranking are presented in Section 3. Section 4 plays an vital role such as fuzzy number construction using statistical data like mean ,variance, and confidence interval. In this section, fuzzy ranking using centroid ofcentroids and evolution of $t^*$ are focused. Finally, the characteristics of FPN are discussed in detail. In section 5, an application to real life problem using statistical parameters ship building FPN for five projects is considered and characteristics of FPN are calculated in detail.

## 2. BACKGROUND INFORMATION

In this section, fuzzy preliminaries, metric distance ranking of fuzzy numbers and sign distance ranking of fuzzy numbers are reviewed.

### 2.1 Fuzzy Preliminaries

For a fuzzy critical path method, all relevant definition of fuzzy sets are given below

#### *2.1.1 Fuzzy Point:*

Let $\tilde{A}$ be a fuzzy set[17] on R=$\left(-\infty,\infty\right)$.It is called a fuzzy membership function is given below

$$\mu_{\tilde{A}} = \begin{cases} 1, & x = a \\ 0, & x \neq a \end{cases}$$

#### *2.1.2 Level $\alpha$ Fuzzy Interval:*

Let $\left[p_\alpha, q_\alpha\right]$ be a fuzzy set on R=$\left(-\infty,\infty\right)$.It is called a level $\alpha$ fuzzy interval, $0 \leq \alpha \leq 1$,if its member ship function is

$$\mu_{[p_\alpha, q_\alpha]}\left(x\right) = \begin{cases} \alpha, & p \leq x \leq q \\ 0, & \text{otherwise} \end{cases}$$

Let the family of all level $\lambda$ fuzzy numbers be denoted by

$$F_N\left(\lambda\right) = \left\{\left(p, q, r; \lambda\right) / \forall p < q < r,\, p, q, r \in R\right\}$$

#### *2.1.3 Arithmetic Operations of Trapezoidal Fuzzy Numbers:*

Let $\tilde{A}$ and $\tilde{B}$ be two trapezoidal fuzzy numbers where $\tilde{A} = \left(p_1, q_1, q_1', r_1\right)$ and $\tilde{B} = (p_2, q_2, q_2', r_2)$

Fuzzy numbers addition:

$$\tilde{A} \oplus \tilde{B} = \left(p_1 + p_2, q_1 + q_2, q_1' + q_2', r_1 + r_2\right)$$

Fuzzy numbers subtraction :

$$\tilde{A} \ \Theta \ \tilde{B} = (p_1 - r_2, q_1 - q_2', q_1' - q_2, r_1 - p_2)$$

### 2.2 Metric Distance Ranking of Fuzzy Numbers

Chen and Cheng [18] introduced a metric distance method to rank fuzzy numbers. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers defined as follows :

$$\zeta_{\tilde{A}}(x) = \begin{cases} \zeta^L_{\tilde{A}}(x), & x < \mu_{\tilde{A}} \\ \zeta^R_{\tilde{A}}(x), & x \geq \mu_{\tilde{A}} \end{cases}$$

$$\zeta_{\tilde{\mathbf{B}}}(x) = \begin{cases} \zeta^L_{\tilde{\mathbf{B}}}(x), & x < \mu_{\tilde{\mathbf{B}}} \\ \zeta^R_{\tilde{\mathbf{B}}}(x), & x \geq \mu_{\tilde{\mathbf{B}}} \end{cases}$$

Where $\mu_{\tilde{A}}$ and $\mu_{\tilde{B}}$ are the mean of $\tilde{A}$ and $\tilde{B}$.

The metric distance between $\tilde{A}$ and $\tilde{B}$ can be calculated as

$$MDist(\tilde{A}, \tilde{B}) =$$

$$\left[\int_0^1 \left(\xi^L_{\tilde{A}}(y) - \xi^L_{\tilde{B}}(y)\right)^2 dy + \int_0^1 \left(\xi^R_{\tilde{A}}(y) - \xi^R_{\tilde{B}}(y)\right)^2 dy\right]^{\frac{1}{2}}$$

where $\xi^L_{\tilde{A}}$, $\xi^R_{\tilde{A}}, \xi^L_{\tilde{B}}$ and $\xi^R_{\tilde{B}}$ are the inverse functions of $\xi^L_{\tilde{A}}$, $\xi^R_{\tilde{A}}, \xi^L_{\tilde{B}}$ and $\xi^R_{\tilde{B}}$, respectively. In order to rank fuzzy numbers, in [18] introduces the fuzzy number $\tilde{B} = \tilde{0}$ (where $\tilde{0}$ = (0,0,0) for triangular and $\tilde{0}$ = (0,0,0,0) for trapezoidal) then the metric distance between $\tilde{A}$ and $\tilde{0}$ is calculated as follows :

$$MDist(\tilde{A}, \tilde{0}) = \left[\int_0^1 \left(\xi^L_{\tilde{A}}(y)\right)^2 dy + \int_0^1 \left(\xi^R_{\tilde{A}}(y)\right)^2 dy\right]^{\frac{1}{2}}$$

The larger the value of $MDist(\tilde{A}, \tilde{0})$ the better the ranking of $\tilde{A}$.

According to Chen and Cheng [18], a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ can be approximated as a symmetry fuzzy number S[$\mu$,$\sigma$], $\mu$ denotes the mean of $\tilde{A}$, $\sigma$ denotes the standard deviation of $\tilde{A}$, and the membership function of $\tilde{A}$ is defined as follows :

$$\zeta_{\tilde{A}}(x) = \begin{cases} \dfrac{x - (\mu - \sigma)}{\sigma}, & if \ \mu - \sigma \leq x \leq \mu \\ \dfrac{(\mu + \sigma) - x}{\sigma}, & if \ \mu \leq x \leq \mu + \sigma \end{cases}$$

Where $\mu$ and $\sigma$ are calculated as follows:

$$\sigma = \frac{2(a_4 - a_1) + a_3 - a_2}{4}, \quad \mu = \frac{a_1 + a_2 + a_3 + a_4}{4}.$$

If $a_2 = a_3$ , then $\tilde{A}$ becomes a triangular fuzzy number, where $\tilde{A} = (a_1, a_2, a_4)$ and $\mu$ and $\sigma$ can be calculated as follows :

$$\sigma = \frac{a_4 - a_1}{2}, \quad \mu = \frac{a_1 + 2a_2 + a_4}{4}$$

The inverse functions $\xi^L_{\tilde{A}}$ and $\xi^R_{\tilde{A}}$ of $\zeta^L_{\tilde{A}}$ and $\zeta^R_{\tilde{A}}$ respectively, are shown as follows

$$\xi^L_{\tilde{A}}(y) = \left(\mu - \sigma\right) + \sigma \times y$$

, $$\xi_{\tilde{A}}^{R}(y) = (\mu + \sigma) - \sigma \times y$$

As per our trapezoidal fuzzy number, μ and σ can be written as

$$\sigma = \frac{-a_1 - a_2 + a_3 - 2a_4}{4},$$

$$\mu = \frac{2(a_2 + a_3) - (a_1 + a_4)}{4}.$$

## 2.3 Sign Distance Ranking of Fuzzy Number

The signed –distance from $a$ to the origin 0 is defined by $Sgnd^{*}(a) = a$, $a$ is a real number. Let $\tilde{A} = (a, b, c) \in F_N$ .Then $Sgnd^{*}(\tilde{A}, \tilde{0}_1)$ is the sign distance of $\tilde{A}$, which is measured from $\tilde{0}_1$ =(0,0,0)(y-axis) as defined by

$$sgnd^{*}(\tilde{A}, \tilde{0}_1) = \int_0^1 sgnd\left(\left(\tilde{A}_L(\alpha)_\alpha, A_R(\alpha)_\alpha\right), \tilde{0}_1\right) = \frac{1}{4}(a + 2b + c) \quad (1)$$

Let $\tilde{A} = (a, b, c, d)$ be a trapezoidal fuzzy number then $Sgnd^{*}(\tilde{A}, \tilde{0}_1)$ is the sign distance of $\tilde{A}$ ,which is measured from $\tilde{0}_1$ =(0,0,0,0) (y-axis) as defined by

$$sgnd^{*}(\tilde{A}, \tilde{0}_1) = \int_0^1 d\left(\left(A_L(\alpha)_\alpha, A_R(\alpha)_\alpha\right), \tilde{0}_1\right) = \frac{1}{4}(a + b + c + d) \quad (2)$$

The Ranking for $\tilde{A}, \tilde{B} \in F_N$ is defined as

$$\tilde{A} < \tilde{B} \Leftrightarrow sgnd^{*}(\tilde{A}, \tilde{0}_1) < sgnd^{*}(\tilde{B}, \tilde{0}_1)$$
$$\tilde{A} \approx \tilde{B} \Leftrightarrow sgnd^{*}(\tilde{A}, \tilde{0}_1) = sgnd^{*}(\tilde{B}, \tilde{0}_1) \quad (3)$$

Property: $$sgnd^{*}(\tilde{A} \oplus \tilde{B}, \tilde{0}) = sgnd^{*}(\tilde{A}, \tilde{0}) + sgnd^{*}(\tilde{B}, \tilde{0}_1) \quad (4)$$

$$sgnd^{*}(\tilde{A} \Theta \tilde{B}, \tilde{0}) = sgnd^{*}(\tilde{A}, \tilde{0}) - sgnd^{*}(\tilde{B}, \tilde{0}_1) \quad (5)$$

# 3. EXISTING METHODS FOR CHARACTERISTICS OF FUZZY PROJECT NETWORK (FPN) USING DISTANCE MEASURE OF FUZZY ACTIVITY TIME AS FUZZY NUMBER

Two methods [19,20]for characteristics of fuzzy project network using distance measure of fuzzy activity time as fuzzy number are discussed.

## 3.1 Characteristics of FPN using Metric Distance

The operation time for each activity in the FPN is characterized as a positive fuzzy number. In accordance with CPM, the forward pass yields the fuzzy earliest-start and earliest-finish times:

$$\tilde{E}_i^{\,s} = \max_{j \in P(i)} \left(\tilde{E}_j^{\,s} \oplus \tilde{t}_j\right)$$

$$\tilde{E}_i^{\,f} = \tilde{E}_j^{\,s} \oplus \tilde{t}_i$$

Where $\tilde{E}_i^{\,s}$ is the fuzzy earliest –start time with $\tilde{E}_{\tilde{A}}^{\,s} = (0, 0, 0)$ at the initial node $i = \tilde{\text{A}}$ ,

$\tilde{E}_i^{\,f}$ is the fuzzy earliest finish time with $\tilde{E}_z^{\,f}$ equal to the completion time $\tilde{T}$ at the ending node $i = Z$ , $P(i)$ is the set of predecessors for activity $i$ and $\tilde{t}_i$ is the operation time for activity $i$. The backward pass is performed to calculate the fuzzy latest-start and latest finish times:

$$\tilde{L}_i^{\,f} = \min_{j \in S(i)} \left(\tilde{L}_j^{\,f} \Theta \tilde{t}_j\right)$$

$$\tilde{L}_i^{\,s} = \tilde{L}_i^{\,f} \oplus \tilde{t}_i$$

where $\tilde{L}_i^{\,f}$ is the fuzzy latest-finish time with $\tilde{L}_z^{\,f} = \tilde{T}$ at the end node $i = Z$ is the fuzzy latest start time and $S(i)$ is the set of successors for activity $i$ . Here, Maximum and Minim mum of fuzzy numbers are identified by metric distance ranking using section 2.2.

The fuzzy total float is either $\tilde{T}_i^{\,F} = \tilde{L}_i^{\,s} \Theta \tilde{E}_i^{\,s}$ Or $\tilde{T}_i^{\,F} = \tilde{L}_i^{\,f} \Theta \tilde{E}_i^{\,f}$ .All the possible paths are found the total slack fuzzy time of each path is calculated and ranked using metric distance ranking in section 2.2.

## 3.2 FCPM using Sign Distance

An activity on node network $N = (V, A, T)$, where $V = \{v_1, v_2, ..., v_n\}$ . Let $t_{v_i v_j}$ be the activity time for each activity$(v_i, v_j)$ . Let $D_j = \{v_i / v_i \in V \text{ and} (v_i, v_j) \in A\}$ be the set of events obtained from event $v_j \in V$ such that $(v_i, v_j) \in A$ and $v_i < v_j$ .The earliest event time for event $v_i$ as

$$t_{Ev_j} = \max_{v_i \in D_j} \left[t_{Ev_j} + t_{v_i v_j}\right]$$

And $t_{Ev_j} = t_{Lv_j} = 0$ similarly, $H_i = \{v_j / v_j \in V \text{ and} (v_i, v_j) \in A\}$ be the set of events obtained from $v_i \in V$ such that $(v_i, v_j) \in A$ and $v_i < v_j$ .The latest event time for event $v_j$ as $t_{Lv_j} = \min_{v_i \in H_i} \left[ t_{Lv_j} - t_{v_i v_j} \right]$

and $t_{Lv_j} = t_{Ev_j}$

This crisp assumption may cause several difficulties in practice. Therefore, we consider $t_{v_i, v_j}$ is the only an estimate and is imprecise. Thus $t_{v_i, v_j}$ is made fuzzy by using the following triangular fuzzy number (Fig.1).

$$\tilde{t}_{v_i v_j} = \left( t_{v_i v_j} - \Delta_{v_i v_j 1}, t_{v_i v_j}, t_{v_i v_j} + \Delta_{v_i v_j 2} \right),$$
$$0 < \Delta_{v_i v_j 1} < t_{v_i v_j}, 0 < \Delta_{v_i v_j 2}.$$

$t^*_{v_i, v_j}$ to be the estimate of the activity time for activity $\left( v_i, v_j \right)$ in the Fuzzy sense i.e.,

$$t^*_{v_i v_j} = sgnd^* \left( \tilde{t}_{v_i v_j}, \tilde{0}_1 \right)$$
$$= t_{v_i v_j} + \frac{1}{4} \left( \Delta_{v_i v_j 2} - \Delta_{v_i v_j 1} \right)$$

Similarly to the above $t_{v_i v_j}$ which corresponds to the interval $\left[ t_{Ev_i} - \Delta_{Ev_i 1}, t_{Ev_i} + \Delta_{Ev_i 2} \right]$ the fuzzy number of the earliest time and latest times are $t_{Ev_i}$ as

$$\tilde{t}_{Ev_i} = \left( t_{Ev_i} - \Delta_{Ev_i 1}, t_{Ev_i}, t_{Ev_i} + \Delta_{Ev_i 2} \right),$$

where $0 < \Delta_{Ev_i 1} < t_{Ev_j}, 0 < \Delta_{Ev_i 2}$ are defined $t^*_{Ev_j}$ to be an estimate of the earliest event time for event $v_i$ in the fuzzy sense i.e.,

$$t^*_{Ev_i} = sgnd^* \left( \tilde{t}_{Ev_i}, \tilde{0}_1 \right) = t_{Ev_i} + \frac{1}{4} \left( \Delta_{Ev_i 2} - \Delta_{Ev_i 1} \right)$$

Afterwards the following equation $t^*_{Ev_j} = t^*_{Ev_i} + t^*_{v_i v_j}$ is obtained.

Similarly, the fuzzy umber of $t_{Lv_i}$ is defined as

$$\tilde{t}_{Lv_j} = \left( t_{Lv_j} - \Delta_{Lv_j 1}, t_{Lv_j}, t_{Lv_j} + \Delta_{Lv_j 2} \right),$$

where $0 < \Delta_{Lv_j 1} < t_{Lv_j}, 0 < \Delta_{Lv_j 2}$.

$t^*_{Lv_j}$ is defined as an estimate of the earliest event time for event $v_j$ in the fuzzy sense, i.e.,

$$t^*_{Lv_j} = sgnd^* \left( \tilde{t}_{Lv_j}, \tilde{0}_1 \right) = t_{Lv_j} + \frac{1}{4} \left( \Delta_{Lv_j 2} - \Delta_{Lv_j 1} \right)$$

$$t^*_{Lv_j} = t^*_{Lv_i} - t^*_{v_i v_j}$$

The Fuzzy number of the total available time $t_{v_i v_j}$ is defined by

$$\tilde{T}_{v_i v_j} = \left( T_{v_i v_j} - \Delta'_{v_i v_j}, T_{v_i v_j}, T_{v_i v_j} + \Delta'_{v_i v_j} \right),$$
$$0 < \Delta'_{v_i v_j 1} < T_{v_i v_j}, 0 < \Delta'_{v_i v_j 2}$$

$$T^*_{v_i v_j} = sgnd^* \left( \tilde{T}_{v_i v_j}, \tilde{0}_1 \right) = T_{v_i v_j} + \frac{1}{4} \left( \Delta'_{v_i v_j 2} - \Delta'_{v_i v_j 1} \right)$$

$$T^*_{v_i v_j} = t^*_{Lv_j} - t^*_{Ev_i}$$

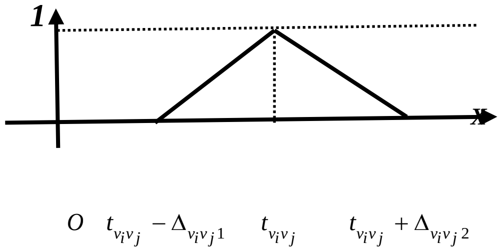


**Fig.1 A Triangular fuzzy number $\tilde{t}_{v_i v_j}$**

# 4. CHARACTERISTICS OF FPN USING STATISTICAL INFERENCE

The notations that are used throughout fuzzy critical path analysis and construction of fuzzy number by confidence interval are discussed in this section. This confidence interval built by using statistical data , a new ranking method and characteristics in a FPN are discussed to find the critical path of a project network under the fuzzy environment.

## 4.1 Construction of Fuzzy Number using Statistical Data

A statistical point estimate can be used in an unknown situation, there in inaccuracy problems involving the possibility of obtaining a greater deviation from the true value. Therefore, the confidence interval estimate with statistical view point is used.

### *4.1.1 Mean and Variance of the Sampling Distribution*

Population refers to the overall fuzzy project networks related to a field. Let $t_{ij}$ be the required time for a population and $t_{ijq}$ be the required time for the past $n_{ij}$, where $n_{ij}$ is the Statistical data of FPNs, $q = 1, 2, ..., n_{ij}$ for each activity $a_{ij}$ .The Point estimate for the time required for a population is $\bar{t}_{ij}$.

$$\bar{t}_{ij} = \frac{1}{n_{ij}} \sum_{q=1}^{n_{ij}} t_{ijq} \qquad (6)$$

The variance of the sampling distribution beyond $n_{ij}$ statistical data is $s_{ij}^2$.

$$s_{ij}^{\ 2} = \frac{1}{n_{ij}-1}\sum_{i=1}^{n}\left(t_{ijk} - \bar{t}_{ij}\right)^2 \qquad (7)$$

### *4.1.2 Confidence Interval and Fuzzy Number*

Let the samples be contemplated where an evaluated range of values can be calculated from the set of sample space, that is prone to incorporate an obscure population parameter. Let $0 < \alpha_{ijp} < 1$ , $p = 1, 2$ , $\alpha_{ij1} + \alpha_{ij2} = \alpha$ , $0 < \alpha < 1$ be the level of significance for each activity $a_{ij}$ in FPN. The $(1-\alpha)\times 100\%$ confidence interval time required for the population $t_{ij}$ is

$$I_{ij} = \left[\bar{t}_{ij} - t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij1}\right), \bar{t}_{ij} + t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij2}\right)\right] \qquad (8)$$

The $(1-\alpha)$ level Fuzzy number for the time required to each activity $t_{ij}$ is denoted by $\tilde{t}_{ij}$

$$\tilde{t}_{ij} = \left(\bar{t}_{ij} - t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij1}\right), \bar{t}_{ij}, \bar{t}_{ij} + t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij2}\right)\right) \qquad (9)$$

## 4.2 A New Ranking Function for Constructed Fuzzy Numbers using Centroid of Centroids

In this section, a new ranking function using centroid of centroid is introduced and this ranking function is applied to the constructed fuzzy number using statistical data. The distance $D\left(\tilde{t}_{ij}\right)$ from origin to the centroid is calculated and it is denoted by $t_{ij}^{*}$.

### *4.2.1 A New Ranking Function for Generalized Fuzzy Number*

Consider a generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d;\ w)$ (Fig.2). The Centroids of the three triangles in Fig.2 are

$G_1 = \left(\dfrac{a+b+c}{3}, \dfrac{w}{3}\right)$, $G_2 = \left(\dfrac{2c+b}{3}, \dfrac{2w}{3}\right)$ and $G_3 = \left(\dfrac{2c+d}{3}, \dfrac{w}{3}\right)$. Equation of the line $\overleftrightarrow{G_1G_3}$ is $y = \dfrac{w}{3}$ and $G_2$ does not lie on the line $\overleftrightarrow{G_1G_3}$. Therefore, $G_1, G_2$ and $G_3$ are non-collinear and they form a triangle. The centroid $G_{\tilde{A}}\left(\bar{x}_0, \bar{y}_0\right)$ of the triangle with vertices $G_1, G_2$ and $G_3$ of the generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d;\ w)$ as

$$G_{\tilde{A}}\left(\bar{x}_0, \bar{y}_0\right) = \left(\frac{a+2b+5c+d}{9}, \frac{4w}{9}\right)$$

As a special case, for triangular fuzzy number $\tilde{A} = (a, b, c, d;\ w)$ i.e., $c = b$ the centroid of Centroids is given by

$$G_{\tilde{A}}\left(\bar{x}_0, \bar{y}_0\right) = \left(\frac{a+7b+d}{9}, \frac{4w}{9}\right)$$

$$\text{Rank of } \tilde{A} = R(\tilde{A}) = \sqrt{x_0^2 + y_0^2}$$

$$= \frac{1}{9}\sqrt{(a+7b+c)^2 + 16w^2}$$

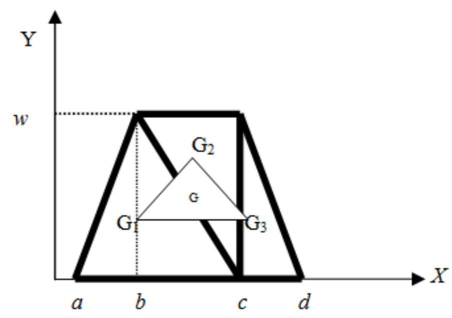


**Fig.2 A Trapezoidal Fuzzy Number**

### *4.2.2 A Ranking Function for Constructed Fuzzy Number*

The centroid of the centroid of a trapezoidal fuzzy number (Fig.2) becomes

$$G_{\tilde{t}_{ij}}\left(\bar{x}, \bar{y}\right) = \frac{1}{9}\left(\left(9\bar{t}_{ij} + t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij2} - \alpha_{ij1}\right)\right), 4\right) \qquad (10)$$

The ranking function is the distance $D\left(\tilde{t}_{ij}\right)$ from origin (0,0,0) to the centroid $G_{\tilde{t}_{ij}}\left(\bar{x}, \bar{y}\right)$. This $D\left(\tilde{t}_{ij}\right)$ can be represented as $t_{ij}^{*}$.

$$t_{ij}^{*} = D\left(\tilde{t}_{ij}\right) = \frac{1}{9}\sqrt{\left[9\bar{t}_{ij} + t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij2} - \alpha_{ij1}\right)\right]^2 + 16} \qquad (11)$$

The Eq. 6 becomes

$$t_{ij}^{*} = D\left(\tilde{t}_{ij}\right) = \frac{1}{9}\sqrt{\left[9\bar{t}_{ij} + z_{ij}\left(\alpha_{ij2}\right) - z_{ij}\left(\alpha_{ij1}\right)\right]^2 + 16} \qquad (12)$$

Where $z_{ij}\left(\alpha_{ij1}\right)=t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\alpha_{ij1}$ and

$z_{ij}\left(\alpha_{ij2}\right)=t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\alpha_{ij2}$

## 4.3 Characteristics of a Crisp and FPN

The method of finding characteristics of an FPN is explained using statistical data.

### 4.3.1 *Comparison of Average and Ranking Function Fuzzy Times with Statistical Data*

In this section some of the observations and comparison are discussed. First it is observed observe that the method to determine the values of $z_{ij}\left(\alpha_{ij1}\right)$ and $z_{ij}\left(\alpha_{ij2}\right)$ by using the standard deviation $s_{ij1}{}^{2}, s_{ij2}{}^{2}$ which can be obtained by separating the two sets. Next Comparison of $C\left(\bar{t}_{ij}\right)$ (centroid of $\bar{t}_{ij}$), ranking function $t_{ij}^{*}$,average statistical data $\bar{t}_{ij}$ and middle of the interval $M$ Finally it is observed that the $t_{ij}^{*}$ as an estimate of $t_{ij}$ in the fuzzy sense. The values of $z_{ij}\left(\alpha_{ij1}\right)$ and $z_{ij}\left(\alpha_{ij2}\right)$ are determined by using statistical data $t_{ijq}$ $q=1,2,...,n_{ij}$ for each activity $a_{ij}$. Suppose that the set $A_{ij}=\left\{t_{ijk} \text{ / forall } \bar{t}_{ij}>t_{ijk}\right\}$ and $B_{ij}=\left\{t_{ijp} \text{ / forall } \bar{t}_{ij}\le t_{ijp}\right\}$ the number of elements in $A_{ij}, B_{ij}$ are $m_{ij}$ and $r_{ij}$ respectively.

Let $s_{ij1}{}^{2}=\frac{1}{m_{ij}-1}\sum_{i=1}^{n}\left(t_{ijk}-\bar{t}_{ij}\right)^{2}$ and

$s_{ij2}{}^{2}=\frac{1}{r_{ij}-1}\sum_{i=1}^{n}\left(t_{ijp}-\bar{t}_{ij}\right)^{2}$

The determination of the values for $z_{ij}\left(\alpha_{ij1}\right)$ and $z_{ij}\left(\alpha_{ij2}\right)$ should include the following cases.

**Case-1** $s_{ij1}{}^{2}<s_{ij2}{}^{2}$

If $s_{ij1}{}^{2}<s_{ij2}{}^{2}$ then $\bar{t}_{ij}$ misrepresents the left side of the interval, The distance between $\bar{t}_{ij}$ and the left end point of the interval are less than that of $\bar{t}_{ij}$ ,and the right end of the interval. Noticeably, the decision-maker should choose the appropriate value for $\alpha_{ij1}$ and $\alpha_{ij2}$ to satisfy $0<\alpha_{ij2}<\alpha_{ij1}<1$, $\bar{t}_{ij}-z_{ij}\left(\alpha_{ij1}\right)>0$ and $\alpha_{ij1}+\alpha_{ij2}=\alpha$ for securing the $\left(1-\alpha\right)\times 100\%$ confidence interval $\left[\bar{t}_{ij}-z_{ij}\left(\alpha_{ij1}\right),\bar{t}_{ij}+z_{ij}\left(\alpha_{ij2}\right)\right]$.

Hence $z_{ij}\left(\alpha_{ij1}\right)<z_{ij}\left(\alpha_{ij2}\right)$

**Case-2** $s_{ij1}{}^{2}\ge s_{ij2}{}^{2}$

If $s_{ij1}{}^{2}\ge s_{ij2}{}^{2}$ then $\overline{t_{ij}}$ distort to the left side of the interval. The distance between the $\bar{t}_{ij}$ and the left endpoint of the interval are greater than that of $\bar{t}_{ij}$, and the right end of the interval. Evidently, the decision-maker should opt for the pertinents value for $\alpha_{ij1}$ and $\alpha_{ij2}$ to satisfy $0<\alpha_{ij1}<\alpha_{ij2}<1$ , $\bar{t}_{ij}-z_{ij}\left(\alpha_{ij1}\right)>0$ and $\alpha_{ij1}+\alpha_{ij2}=\alpha$ for obtaining the $\left(1-\alpha\right)\times 100\%$ confidence interval

$\left[\bar{t}_{ij}-z_{ij}\left(\alpha_{ij1}\right),\bar{t}_{ij}+z_{ij}\left(\alpha_{ij2}\right)\right]$.

Hence $z_{ij}\left(\alpha_{ij1}\right)>z_{ij}\left(\alpha_{ij2}\right)$

**Case-3** $s_{ij1}{}^{2}=s_{ij2}{}^{2}$

The decision maker should choose the appropriate value for $\alpha_{ij1}$ and $\alpha_{ij2}$, to satisfy

$0<\alpha_{ij1}=\alpha_{ij2}<1$, $\bar{t}_{ij}-z_{ij}\left(\alpha_{ij1}\right)>0$ and $\alpha_{ij1}=\alpha_{ij2}=\frac{\alpha}{2}$ ,for obtaining

$z_{ij}\left(\alpha_{ij1}\right)=z_{ij}\left(\alpha_{ij2}\right)$

Obviously $\bar{t}_{ij}$ is the middle of the interval.

## Comparison of $C\left(\bar{t}_{ij}\right)$(Centroid of $\bar{t}_{ij}$), $t_{ij}^{*}$, $\bar{t}_{ij}$ and Middle of the Interval $M$ :

The centroid of the level $\left(1-\alpha\right)$ fuzzy number is

$$C\left(\bar{t}_{ij}\right)=\frac{3\bar{t}_{ij}+\left(z_{ij}\left(\alpha_{ij2}\right)-z_{ij}\left(\alpha_{ij1}\right)\right)}{3}$$

The mean of the two end points $\left(1-\alpha\right)\times 100\%$ confidence interval

$\left[\bar{t}_{ij}-z_{ij}\left(\alpha_{ij1}\right),\bar{t}_{ij}+z_{ij}\left(\alpha_{ij2}\right)\right]$ is

$$M=\frac{2\bar{t}_{ij}-z_{ij}\left(\alpha_{ij1}\right)+z_{ij}\left(\alpha_{ij2}\right)}{2}.$$

The comparison of $C\left(\bar{t}_{ij}\right)$, $t_{ij}^{*}$, $\bar{t}_{ij}$ and M should include the following cases.

**Case 1**: $s_{ij1}{}^{2}<s_{ij2}{}^{2}$

$\bar{t}_{ij} - z_{ij}\left(\alpha_{ij1}\right) < \bar{t}_{ij} < t^*_{ij} < C\left(\bar{t}_{ij}\right) < M < \bar{t}_{ij} + z_{ij}\left(\alpha_{ij1}\right)$ The statistical data is skewed to the area greater than $\bar{t}_{ij}$. Therefore

$$t^*_{ij} = \frac{1}{9}\sqrt{\left[9\bar{t}_{ij} + t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij2} - \alpha_{ij1}\right)\right]^2 + 16} > \bar{t}_{ij}$$

**Case 2:** $s_{ij1}^{\ 2} \geq s_{ij2}^{\ 2}$

$\bar{t}_{ij} - z_{ij}\left(\alpha_{ij1}\right) < M < C\left(\bar{t}_{ij}\right) < t^*_{ij} < \bar{t}_{ij} < \bar{t}_{ij} + z_{ij}\left(\alpha_{ij1}\right)$ The statistical data is skewed to the area less than $\bar{t}_{ij}$ Therefore

$$t^*_{ij} = \frac{1}{9}\sqrt{\left[9\bar{t}_{ij} + t_{n_{ij}-1}\frac{s_{ij}}{\sqrt{n_{ij}}}\left(\alpha_{ij2} - \alpha_{ij1}\right)\right]^2 + 16} < \bar{t}_{ij}$$

**Case 3:** $s_{ij1}^{\ 2} = s_{ij2}^{\ 2}$

$\bar{t}_{ij} - z_{ij}\left(\alpha_{ij1}\right) < M = C\left(\bar{t}_{ij}\right) = t^*_{ij} = \bar{t}_{ij} < \bar{t}_{ij} + z_{ij}\left(\alpha_{ij1}\right)$ $\bar{t}_{ij}$ is the middle of the interval. Hence $\bar{t}_{ij} = t^*_{ij}$ .Finally $t^*_{ij}$ is better than $\bar{t}_{ij}$ .

### 4.3.2 Calculation of Fuzzy Earliest Times

The time variables are vague and should be regarded as fuzzy number. So the range of an estimate the interval $\left[E_i - \phi_{E_i1}, E_i + \phi_{E_i2}\right]$, $0 < \phi_{E_i1} < E_i$ and $0 < \phi_{E_i2}$ ,where $E_i$ is known. This interval will be changed in to fuzzy number with respect to the level $\left(1-\alpha\right)$ fuzzy number is defined as

$$\tilde{E}_i = \left(E_i - \phi_{E_i1}, E_i, E_i + \phi_{E_i2}; 1-\alpha\right), 0 < \phi_{E_i1} < E_i$$

and $0 < \phi_{E_i2}$ (13)

The estimate of $E_i$ in the fuzzy sense as

$$E^*_i = D\left(\tilde{E}_i\right) = \frac{1}{9}\sqrt{\left[9E_i + \phi_{E_i2} - \phi_{E_i1}\right]^2 + 16} > 0 \qquad (14)$$

The centroid of the fuzzy number is

$$C\left(\tilde{E}_i\right) = \frac{3E_i + \left(\phi_{E_i2} - \phi_{E_i1}\right)}{3}$$

$C\left(\tilde{E}_i\right)$ also irrelevant to $\left(1-\alpha\right)$,because the middle of the interval $\left[E_i - \phi_{E_i1}, E_i + \phi_{E_i2}\right]$ is

$$M_{E_i} = \frac{2E_i + \left(\phi_{E_i2} - \phi_{E_i1}\right)}{2}$$

Thus $E^*_i$ is closer to $E_i$ than $C\left(\tilde{E}_i\right)$ and $C\left(\tilde{E}_i\right)$ is closer to $M_{E_i}$ than $E^*_i$

From Eq.1, Eq.2 and Eq.3, it can be written

$$D\left(\tilde{E}_i\right) + D\left(\tilde{t}_{ij}\right) \leq D\left(\tilde{E}_j\right)$$

Where at least one equal sign holds.From (12) and (14) to get

$$E^*_i + t^*_{ij} \leq E^*_j$$

Where at least one equal sign holds. Thus

$$E^*_j = \max_{i \in D_j}\left(E^*_i + t^*_{ij}\right) \qquad (15)$$

### 4.3.3 Calculation of Latest Times

Consider the second variable $L_j$ , the time variables are vague and should be regarded as fuzzy number. So the range of an estimate for is the interval $\left[L_j - \phi_{L_j1}, L_j + \phi_{L_j2}\right]$, $0 < \phi_{L_j1} < L_j$ and $0 < \phi_{L_j2}$ . This interval will be changed in to fuzzy number with respect to the level $\left(1-\alpha\right)$ fuzzy number is defined as

$$\tilde{L}_j = \left(L_j - \phi_{L_j1}, L_j, L_j + \phi_{L_j2}; 1-\alpha\right),$$

$0 < \phi_{L_j1} < L_j$ and $0 < \phi_{L_j2}$ (16)

The estimate of $L_j$ in the fuzzy sense as

$$L^*_j = D\left(\tilde{L}_j\right) = \frac{1}{9}\sqrt{\left[9L_j + \phi_{L_i2} - \phi_{L_i1}\right]^2 + 16} > 0 \qquad (17)$$

The centroid of the fuzzy number is

$$C\left(\tilde{L}_j\right) = \frac{3L_j + \left(\phi_{L_i2} - \phi_{L_i1}\right)}{3}$$

$C\left(\tilde{L}_j\right)$ also irrelevant to $\left(1-\alpha\right)$,because the middle of the interval $\left[L_j - \phi_{L_j1}, L_j + \phi_{L_j2}\right]$

$$M_{L_j} = \frac{2L_j + \left(\phi_{L_i2} - \phi_{L_i1}\right)}{2}$$

Thus $L^*_j$ is closer to $L_j$ then $C\left(\tilde{L}_j\right)$ and $C\left(\tilde{L}_j\right)$ is closer to $M_{L_j}$ than $L^*_j$ from section 3.1

$$L_i = L_j - t_{ij}$$

It is clear that at least one equal sign holds. The same method is used to fuzzify both sides of the equation, thus obtaining fuzzy inequality

$$\tilde{L}_i = \tilde{L}_j - \tilde{t}_{ij}$$

In addition that the following equation holds

$$L_i^* = \min_{j \in H_i} \left( L_j^* - t_{ij}^* \right) \quad (18)$$

### 4.3.4 4.3.4 Proof of $E_n^* = L_n^*$

Consider the equation in section 3.1, $E_1 = 0 = L_1$ after fuzzifying the equation will be $\tilde{E}_1 = 0 = \tilde{L}_1$ and $D\left(\tilde{E}_i\right) = D\left(\tilde{t}_{ij}\right) = D(0)$.Thus $E_1^* = 0$ and $L_1^* = 0$ and the additional conditions are $\phi_{E_i 1} = \phi_{E_i 2} = \phi_{L_i 1} = \phi_{L_i 2} = 0$

Also consider the equation $E_n = L_n$ in in section 3.1,after fuzzifying this equation to get the following equation

$$\tilde{E}_n = \left( E_n - \phi_{E_n 1}, E_n, E_n + \phi_{E_n 2}; 1-\alpha \right)$$
$$= \left( L_n - \phi_{L_n 1}, L_n, L_n + \phi_{L_n 2}; 1-\alpha \right)$$
$$= \tilde{L}_n$$

Apply Eq.11

$$\frac{1}{9}\sqrt{\left[9E_n + \phi_{E_n 2} - \phi_{E_n 1}\right]^2 + 16}$$
$$= \frac{1}{9}\sqrt{\left[9L_n + \phi_{L_n 2} - \phi_{L_n 1}\right]^2 + 16}$$

Whenever $\phi_{E_n 2} - \phi_{E_n 1} = \phi_{L_n 2} - \phi_{L_n 1}$ thus

$$E_n^* = L_n^*$$

### 4.3.5 Calculation of Total float $T_{ij}^*$

Consider the final variable $T_{ij}$, the time variables are vague and should be regarded as fuzzy number. So the range of an estimate for is the interval $\left[ T_{ij} - \delta_{ij1}, T_{ij} + \delta_{ij2} \right]$, $0 < \delta_{ij1} < T_{ij}$ and $0 < \delta_{ij2}$. This interval will be change in to fuzzy number with respect to the level $(1-\alpha)$ fuzzy number is defined as

$$\tilde{T}_{ij} = \left( T_{ij} - \delta_{ij1}, T_{ij}, T_{ij} + \delta_{ij2}; 1-\alpha \right),$$

$0 < \delta_{ij1} < T_{ij}$ and $0 < \delta_{ij2}$

The estimate of $T_{ij}$ in the fuzzy sense as

$$T_{ij}^* = D\left(\tilde{T}_{ij}\right) = \frac{1}{9}\sqrt{\left[9T_{ij} + \delta_{ij2} - \delta_{ij1}\right]^2 + 16} > 0$$

The centroid of the fuzzy number is

$$C\left(\tilde{T}_{ij}\right) = \frac{3T_{ij} + \delta_{ij2} - \delta_{ij1}}{3}$$

$C\left(T_{ij}\right)$ also irrelevant to $(1-\alpha)$,because the middle of the interval $\left[ T_{ij} - \delta_{ij1}, T_{ij} + \delta_{ij2} \right]$ is

$$M_{\tilde{T}_{ij}} = \frac{2T_{ij} + \left(\delta_{ij2} - \delta_{ij1}\right)}{2}$$

Thus $T_{ij}^*$ is closer to $T_{ij}$ than $C\left(T_{ij}\right)$ and $C\left(T_{ij}\right)$ is closer to $M_{\tilde{T}_{ij}}$ then $T_{ij}^*$.

From the section 3.2,

$$T_{ij} = L_j - E_i$$

It is clear that at least one equal sign holds. The same method is used to fuzzily both sides of the equation, thus obtaining fuzzy inequality

$$D\left(\tilde{T}_{ij}\right) = D\left(\tilde{L}_j\right) - D\left(\tilde{E}_i\right)$$

In addition that the following equation holds

$$T_{ij}^* = L_j^* - E_i^* \quad (19)$$

## Determination time of activity (i,j):

$(Es)_{ij}^* = E_i^*$, $(Ef)_{ij}^* = (ES)_{ij}^* + t_{ij}^*$, $(Lf)_{ij}^* = L_j^*$,

$(Ls)_{ij}^* = (Lf)_{ij}^* - t_{ij}^*$, $TF_{ij}^* = (Ls)_{ij}^* - (Es)_{ij}^*$.

## 5. NUMERICAL EXAMPLE

Project networks of 5 ship bulding organizations in India are collected and the activity involved in the projectes are presented in the Table I. A network diagram related to Table I is constructed and represented in Fig.3. Activity times for FPN are presented in Table II.

The five ship building companies in India are

Hindustan Shipyard Limited (HSL) (Andhra Pradesh)

Garden Research Work shop(GRW)(Kolkata)

Mazogon Docks(MD) ( Mumbai)

Cochin shipyard (CS) (Cochin)

ABN shipyard (ABN) (Gujarath)

**Table I : Activities involved in Ship building process**

| Activity | Description | Immediate Predecessor(s) |
|---|---|---|
| 1 | Bill of materials(drawing office) | --- |
| 2 | Purchasing | 1 |
| 3 | Storing of materials | 2 |
| 4 | Hull shop | 3 |
| 5 | Contractors to do some panels | 3 |
| 6 | Prefabrication | 4 |
| 7 | Welding | 6 |
| 8 | Erection | 3,5,7 |
| 9 | Assembling | 8 |

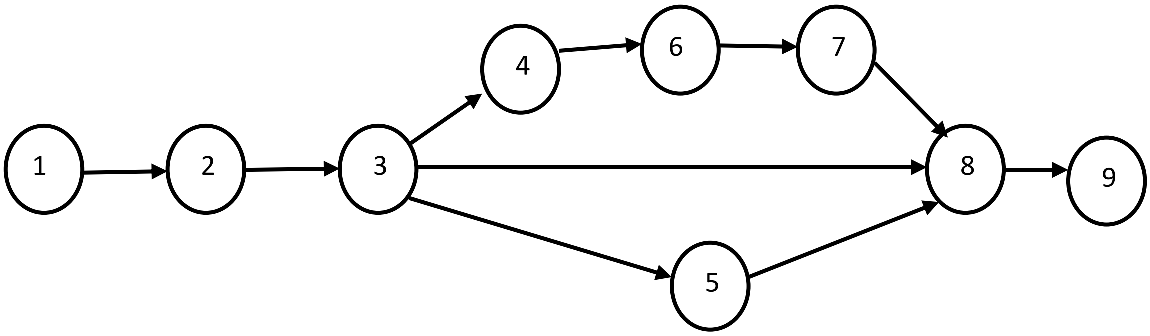


**Fig .3 FPN for Ship Building**

**Table II : Activity times in Ship Building Process for various organizations**

| **Activity** $(i, j)$ | HSL **(weeks)** | GRW **(weeks)** | MD **(weeks)** | CS **(weeks)** | ABN **(weeks)** |
|---|---|---|---|---|---|
| $(1,2)$ | 2 | 2 | 4 | 3 | 2 |
| $(2,3)$ | 16 | 14 | 12 | 18 | 16 |
| $(3,4)$ | 12 | 14 | 16 | 15 | 14 |
| $(3,5)$ | 24 | 20 | 26 | 24 | 23 |
| $(3,8)$ | 24 | 20 | 21 | 22 | 28 |
| $(4,6)$ | 4 | 6 | 2 | 4 | 3 |
| $(5,8)$ | 8 | 7 | 6 | 9 | 10 |

| | | | | | |
|---|---|---|---|---|---|
| $(6,7)$ | 8 | 9 | 11 | 10 | 7 |
| $(7,8)$ | 12 | 10 | 13 | 12 | 10 |
| $(8,9)$ | 24 | 22 | 30 | 28 | 22 |

Using Table II and the sections 4.1.1, 4.1.2, 4.2.2, Table III constructed.

**Table III: Confidence interval for each activity**

| Activity | $\bar{t}_{ij}$ | $s_{ij}$ | $z_{ij}(\alpha_{ij1})$ | $z_{ij}(\alpha_{ij2})$ | $\left[\bar{t}_{ij} - z_{ij}(\alpha_{ij1}), \bar{t}_{ij} + z_{ij}(\alpha_{ij2})\right]$ |
|---|---|---|---|---|---|
| $(1,2)$ | 2.6 | 0.89 | 0.93 | 1.49 | [1.66,4.09] |
| $(2,3)$ | 15.2 | 2.23 | 2.37 | 3.82 | [12.82,19.02] |
| $(3,4)$ | 14.2 | 1.48 | 1.54 | 2.48 | [12.65,16.68] |
| $(3,5)$ | 23.4 | 2.19 | 2.28 | 3.66 | [21.11,27.06] |
| $(3,8)$ | 23 | 3.16 | 3.29 | 5.29 | [19.70,28.29] |
| $(4,6)$ | 3.8 | 1.48 | 1.54 | 2.48 | [2.25,6.28] |
| $(5,8)$ | 8 | 1.58 | 1.64 | 2.64 | [6.35,10.64] |
| $(6,7)$ | 9 | 1.58 | 1.64 | 2.64 | [7.350,11.64] |
| $(7,8)$ | 11.4 | 1.34 | 1.39 | 2.24 | [10.00,13.64] |
| $(8,9)$ | 25.2 | 3.63 | 3.79 | 6.08 | [21.40,31.28] |

Using Table III and the sections 4.1.2, 4.2.2, 4.3.5, Table IV constructed

**Table IV : Triangular fuzzy number and time characteristics**

| Activity | $\tilde{t}_{ij}$ | $t^*_{ij} = d\left(\tilde{t}_{ij}, \tilde{0}\right)$ | $ES^*_{ij}$ | $EF^*_{ij}$ | $LS^*_{ij}$ | $LF^*_{ij}$ | $TF^*_{ij}$ |
|---|---|---|---|---|---|---|---|
| $(1,2)$ | $(1.66, 2.6, 4.09)$ | 2.69 | 0 | 2.69 | 0 | 2.69 | 0 |
| $(2,3)$ | $(12.82, 15.2, 19.02)$ | 15.36 | 2.69 | 18.06 | 2.69 | 18.06 | 0 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $(3,4)$ | $(12.65,14.2,16.68)$ | 14.31 | 18.06 | 32.37 | 18.06 | 32.37 | 0 |
| $(3,5)$ | $(21.11,23.4,27.06)$ | 23.55 | 18.06 | 41.62 | 25.25 | 48.80 | 7.19 |
| $(3,8)$ | $(19.70,23,28.29)$ | 23.22 | 18.06 | 41.29 | 33.70 | 56.93 | 15.63 |
| $(4,6)$ | $(2.25,3.8,6.28)$ | 3.92 | 32.37 | 36.30 | 32.37 | 36.30 | 0 |
| $(5,8)$ | $(6.35,8,10.64)$ | 8.12 | 41.62 | 49.74 | 48.80 | 56.93 | 7.18 |
| $(6,7)$ | $(7.35,9,11.64)$ | 9.12 | 36.30 | 45.42 | 36.30 | 45.42 | 0 |
| $(7,8)$ | $(10.0,11.4,13.64)$ | 11.50 | 45.42 | 56.93 | 45.42 | 56.93 | 0 |
| $(8,9)$ | $(21.41,25.2,31.28)$ | 25.45 | 56.93 | 82.39 | 56.93 | 82.39 | 0 |

**Critical path:**
From the Table IV,the critical path of the FPN is 1-2-3-4-6-7-8-9

## 6. ADVANTAGES USING STATISTICAL DATA

The focus of this study was to introduce an approach that combined fuzzy set theory with statistics that includes a new fuzzy number. We would like to point out the following significant results obtained in this paper for characteristics of fuzzy project network.

(i) The activity fuzzy project network is same as the fuzzy project network based on confidence interval estimate.

(ii) It was shown that some relations exist between the notion of fuzzy criticality and the notion of criticality in the fuzzy project network with newly constructed fuzzy numbers.

(iii) It can be noted that this approach is used to solve practical problems in unknown or vague situations, without any assumptions, using fuzzy numbers.

(iv) Calculating the past statistical data average, we observed that there is inaccuracy for the possibility of obtaining a greater deviation from the true value.

## 7. CONCLUSIONS

This paper has presented characteristics of fuzzy project network using statistical information related to the project by calculating mean and variance. Fuzzy number has been constructed using interval estimate. A new ranking function has been developed to discriminate various fuzzy numbers. The fuzzy number combined with ranking function, a new formula for calculating characteristics of fuzzy network using $t$ distribution has been developed.

This method has been applied on five ship building projects collected from various places in India. Throughout the paper, it has been observed that: (i) A new fuzzy number is constructed to calculate characteristics of a fuzzy project network using statistical data.

(ii) A new ranking function is developed on the newly constructed fuzzy number which gives more accurate results in discriminating fuzzy numbers.

(iii) The appropriateness and contribution of characteristics of fuzzy project network to ship building is discussed using statistical parameters.

This method gives fuzzy project network researchers new tools and ideas on how to approach fuzzy project network problems using statistical data.

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