Design and Analysis of Gimbal Thruster Configurations for 3-Axis Satellite Attitude Control

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ABSTRACT
The satellite thruster's configuration plays also an important role in providing the attitude control torques. In this paper, after discussing the gimbal thruster's structure and its benefits, several configurations based on 2, 3 and 4 gimbal thrusters are investigated in order to identify the most suitable orientation that consume less fuel and raise reliability. Then, a 3-axis attitude controller based on proportional-derivative control law is applied to satellite dynamics under these configurations. All the configurations are analyzed in terms of their torque workspace (controllable directions), attitude control performances and gimbal angles changes. The results show that the 4-thrusters configuration is more reliable and gimbal angles changes are smoother.

Keywords
Gimbal reaction thrusters; 3-axis attitude control; Torque workspace; Gimbal angles; Proportional-derivative controller

1. INTRODUCTION
Attitude control of a satellite has implemented by using various actuators. These actuators are Reaction and Momentum wheels, Control moment gyros, Reaction thrusters and Magnetic torquers. Reaction thrusters significantly used because they can product large torques which are required for attitude control.

The level of thruster torque around a satellite axis depends not only on its thrust level but also on the torque-arm length. Thus selection of a suitable thruster depends primarily on its location on the satellite, and also on its inclination to the satellite body axes. In some cases, different torque levels might be needed around the three principal body axes, so the location of the thrusters and their direction must be carefully studied before a final configuration for the propulsion system. The location and direction of the thrusters is also affected by the location of the optical sensors and solar panels, which must not be damaged by the thrusters exhaust.

Zuliana Ismail *et al* in [1] investigated several configurations based on three or four reaction wheels in order to identify the most suitable orientation that consumes a minimum power. The conventional PD-type (proportional-derivative) controller and PI-type (proportional-integral) controller are adopted for the satellite attitude and wheel angular momentum controls, respectively. But use of reaction thrusters has several advantages. Namely they can produce different torques in different intervals. Therefore, thrusters are more efficient rather than wheels in some usages.

Sidi in [2] illustrated two configurations of satellite thrusters with six and four thrusters. Considering six fixed thrusters, each thruster is capable of acting only in one direction. Another thruster must be activated around the same axis on the opposite direction, in order to achieve a torque around the same axis on the opposite sign. Use of additional thrusters may cause increase satellite mass and fuel consumption. In other case, with four thrusters, the possibility of achieving linear velocity augmentation in desired body directions is no longer available.

More researches have been achieved about actuators configuration scheme for attitude control system in recent years. These schemes have been applying in order to optimize fuel consumption [3]. For this purpose, using Gimbal thruster is offered. Gimbal system allows thrusters to rotate in the ideal direction to produce desired torques.

Anzel. B. M *et al* in [4], Daryl K *et al* in [5] and Glogowski M. J in [6], have presented models with gimbal thrusters configuration. In these models, different tasks are considered for thrusters. But these models have problems in attitude control. The model is proposed by [4] use 8 fixed thrusters, in addition to 2 gimbal thrusters. As a result, this may be increases fuel consumption in some large maneuvers. According to thrusters configuration of the model is described by [5], thrusters are not capable of producing torques at anti-earth direction. This model needs other actuators like reaction wheels for complete attitude control. The thrusters task in the model proposed by [6] is station keeping. Designer has used single gimbal thrusters in this model.

However, using additional thrusters increases accuracy, but also increases fuel consumption. Hence, it is necessary to obtain configuration method that 1st well done control tasks and 2nd decrease fuel consumption. On the other hand, reliability of attitude subsystem is one of the most considerable problems. For checking reliability, consider what the effect on product torques is in different directions while one or more thrusters or gimbal platforms get damaged. In spite of decreasing the number of thrusters, all the proposed configurations in this paper completely cover the torques workspace about the three axis of satellite compared to the previous models. Therefore, increase reliability of the configurations.

Also, because stepper motors are used in gimbal systems, it should avoid corrosion of stepper motor segments with desired control algorithm. The thrust level is considered to be constant and equal for all thrusters. As a result, different torques provide with gimbal angles changes only. Whatever gimbal angle changes have less amplitude, gimbal hardwares would have less corrosion. The approach is employed for dividing torques. Then the gimbal angles get smooth changes.

In this paper in section 2, gimbal thrusters and their benefits in comparison with fixed thrusters are discussed. In section 3, various configurations of gimbal thrusters and covering torques workspace are studied. In section 4, competency of configurations is analysed. In section 5, attitude dynamics and kinematics of satellite and pd control law are presented. In section 6, gimbal thruster control strategy is modeled and finally in section 7, 3-axis attitude control performance of configurations under pd controller are presented.

2. GIMBAL THRUSTERS

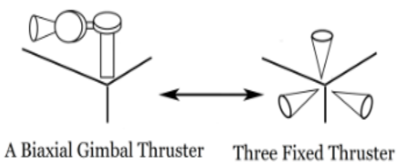
Gimbal is a rotary system about one axis and called single-axis gimbal. This system consists of a small-angle permanent magnet stepper motor coupled to a harmonic drive speed reducer, with a large, rotating flange output member. Nowadays several mechanisms are using for avoiding corrosion in gimbal systems.

When there is necessity to move a load on two orthogonal axes, two actuators can be combined to provide this capability. These biaxial gimbals are designed in modular fashion, making it easy to change units of either elevation or azimuth angles. One of the properties of biaxial gimbal system is covering all of the torque workspace around it by using change 360 degrees of azimuth and 180 degrees of elevation. Therefore, if thruster mounts on biaxial gimbal platform, it can produce enough force in desired direction and control the attitude and position of satellite. Figure 1 represents biaxial gimbal system.

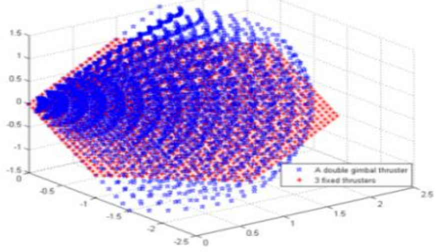
Fig 1: a. biaxial gimbal scheme [7], b. gimbal platform sample

Each gimbal mechanism includes a first gimbal for pivotal movement about an axis parallel to the pitch, that is, Y-axis and in a plane parallel to a plane defined by the roll (that is, X-axis) and yaw (that is, Z-axis) axes. Each gimbal mechanism also includes a second gimbal which is suitably connected to the first gimbal for pivotal movement about an axis parallel to the roll axis so as to have a component of thrust directed out of the plane defined by the pitch and yaw axes. The single gimbal thruster vector may be generated by two fixed thrusters and the biaxial gimbal thruster vector may be generated by three fixed thrusters as shown in Figure 2a [8]. For evaluation this matter, the torque workspace covering of a biaxial gimbal thruster is compared with three fixed thrusters in Figure 2b. The benefits and the disadvantages of gimbal and fixed thrusters are summarized in Table 1.

As can be seen, gimbal thrusters have several benefits in spite of having complex analysis. Also, gimbal thrusters configurations have more reliability toward fixed thruster configuration. Namely, if one of the gimbal thrusters gets damaged, then the other thrusters can compensate this deficiency. These benefits are the main reason for preferring to use gimbal instead of fixed thrusters.



(a)



(b)

Fig 2: a. Replace three fixed thrusters by a biaxial gimbal thruster, b. Compare controllable directions by the 2 types

Table 1. Comparison between fixed and gimbal thrusters

Benefits	Disadvantages	
- Required less thrusters - Less fuel consumption - Access more surface of satellite - Exhaust plumes do not impinge on the other satellite hardwares with desired configuration	- Higher electrical power consumption with increase stepper motors - Possibility gimbal mechanism parts wear with gimbal angles change - Complex analysis	Gimbal
- Simple analysis	- Required further thrusters - More fuel consumption - Occupy more surface of the satellite and create limitation on the other satellite hardwares	Fixed

3. CONFIGURATIONS OF GIMBAL THRUSTERS

This section focuses on the torque workspace (controllable directions) covering of various configurations with two, three and four gimbal thrusters for 3-axis attitude control of satellite. The gimbal mechanism using in these configurations is biaxial gimbal. In these configurations, X-axis is in satellite orbital movement direction and perpendicular west/east plane, Y-axis is perpendicular north/south plane and Z-axis is directed toward the earth center of mass according to figure 3.

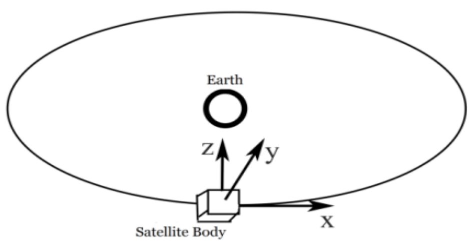


Fig 3: Satellite orientation in space

The proposed various gimbal thruster configurations will be considered as follows.

3.1 Configuration based on Two Thrusters

There are two gimbal thrusters TH1 and TH2 that are mounted on the corner of north and south planes and near the anti-earth plane of satellite (Figure 4).

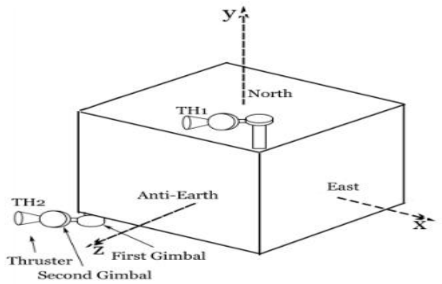


Fig 4: Location of 2-thrusters

Each gimbal thruster has a vertical gimbal with elevation angle (α) and a horizontal gimbal with azimuth angle (β) (Figure 5). These angles are controlled independently.

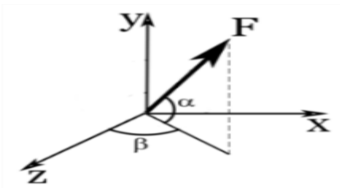


Fig 5: Rotation of gimbal angles (elevation & azimuth)

3.2 Configuration based on Three Thrusters

There are three gimbal thrusters TH1, TH2 and TH3 that are mounted on the north and south planes. Two of them put at the corner of north plane and one of them put at the center of south and near the anti-earth planes (Figure 6). The gimbal system is used as in the previous case.

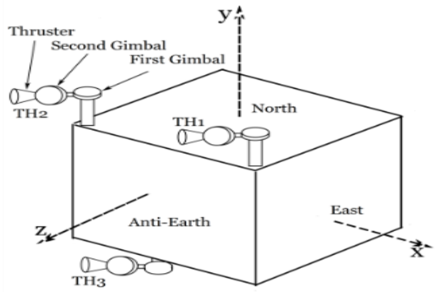


Fig 6: Location of 3-thrusters

3.3 Configuration based on Four Independent Thrusters

There are four gimbal thrusters TH1, TH2, TH3 and TH4 that are mounted on the corner of north and south planes and near the anti-earth plane. Also, the gimbal angles are controlled independently. Consequently, the various control torques are produced about the center of mass of satellite by this configuration (Figure 7).

The force components equations produced by thrusters at the all above configurations are as follows:

$$F_i = [F_{xi} \quad F_{yi} \quad F_{zi}]$$

$$\begin{cases} F_{xi} = f_i \cos\alpha_i \sin\beta_i \\ F_{yi} = f_i \sin\alpha_i \\ F_{zi} = f_i \cos\alpha_i \cos\beta_i \end{cases} \tag{1}$$

Where $i = 1, \cdots, N$ is subscript of thruster's number and f_i is thrust level of them.

If the torque arms of forces toward the center of mass are r_is, then total torque about the center of mass is as follows:

$$T_i = r_i \times F_i$$
$$T = \sum_{i=1}^{N} T_i \tag{2}$$

Where T_i is product torque of the i th thruster, T is the total torque about the center of mass and N is the number of thrusters.

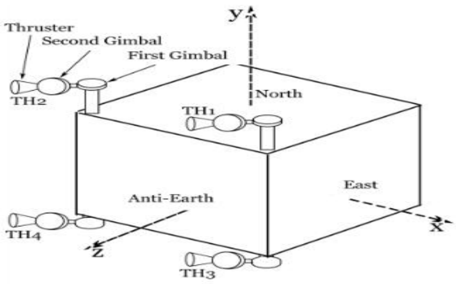


Fig 7: Location of 4-thrusters

3.4 Configuration based on Two Paired Thrusters

There are four thrusters TH1, TH2, TH3 and TH4 that are using in pairs instead of independent thrusters. This means that the thrusters are controlled dependently and pairwise. Therefore, the gimbal angles (α) and (β) have similar changes. Each of paired thrusters has a north thruster TH1 and TH2 and a south thruster TH3 and TH4 (Figure 7). Thus, the four thrusters are mounted pairwise on the north and south planes. For example, the control signal that is applied to TH1, also applied to TH3 and vice versa. Consequently, gimbal angles (α_1) and (α_2) or (β_1) and (β_2) have the same changes. This configuration is divided into two types:

1- Equal in magnitude and same in direction of the azimuth changes

2- Equal in magnitude and opposite in direction of the azimuth changes

The force components equations produced by thrusters are as follows:

$$F_i = [F_{xi} \quad F_{yi} \quad F_{zi}]$$

First paired thruster:

$$\begin{cases} F_{xi} = f_i \cos\alpha_1 \sin\beta_1 \\ F_{yi} = f_i \sin\alpha_1 \qquad i = 1,3 \\ F_{zi} = f_i \cos\alpha_1 \cos\beta_1 \end{cases} \tag{3}$$

Second paired thruster:

$$\begin{cases} F_{xi} = f_i \cos\alpha_2 \sin\beta_2 \\ F_{yi} = f_i \sin\alpha_2 \qquad i = 2,4 \\ F_{zi} = f_i \cos\alpha_2 \cos\beta_2 \end{cases}$$

And total torque about center of mass is:

$$T_i = r_i \times F_i$$
$$T = \sum_{i=1}^{4} T_i \tag{4}$$

In the second type, (β_1) and (β_2) change to ($-\beta_1$) and ($-\beta_2$) for south thrusters of each pair (Figure 8)

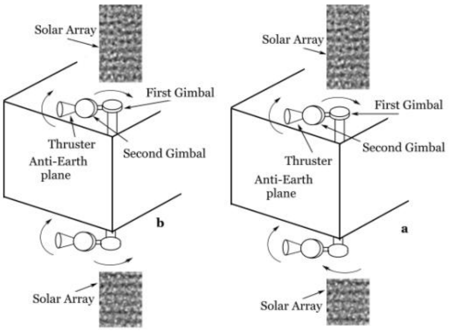


Fig 8: a. Same, b. Opposite direction in azimuth

In the all above configurations, the various torques are produced about 3 axis of satellite. The torque workspace covering of proposed configurations are represented on table 2. Simulations are implemented with MATLAB. As can be seen from simulation results, all above configurations cover the torque workspace about the center of mass. Thus, control torques are possible in all directions and so, 3-axis attitude control is feasible.

4. CONFIGURATION COMPETENCY

As seen in previous section, all proposed configurations were successful in workspace covering. Basically, knowledge of control torque workspace covering makes it possible to study competency of configurations. Criterion of competency is based on the reliability of configuration. Thus, for study about competency, consider one or more thrusters or gimbal mechanisms of thrusters get damaged. This means that the thruster cannot produce any torque about 3-axis of satellite. Simulation has been done in normal form. The thrust level of thrusters is considered to be unit. Also, the components of r (r_x, r_y and r_z) are considered to be unit.

As can be seen from table 3, if one of the thrusters gets damaged in configuration based on two thrusters, then a significant amount of workspace (controllable directions) loses. As a result, the 3-axis attitude control of satellite will be difficult with a remaining thruster. Therefore, this configuration has very low reliability in spite of less fuel consumption.

Table 2: Torque workspace of different configurations

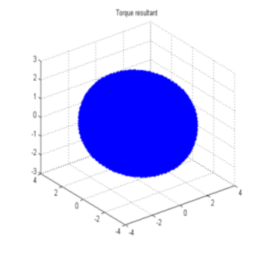
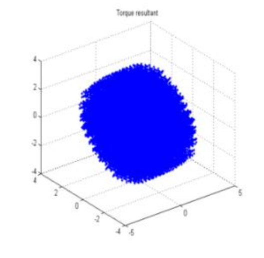
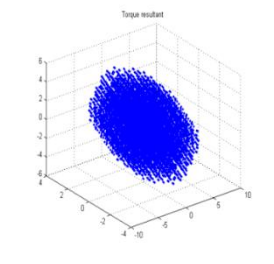
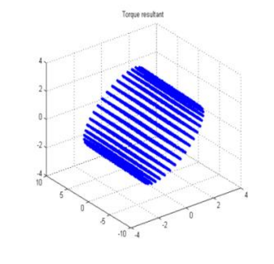
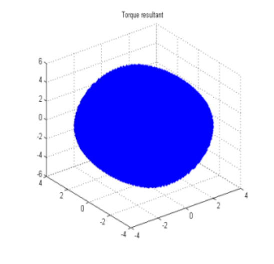
Independent control of each thruster		2 thruster’s
Independent control of each thruster		3 thruster’s
Independent control of each thruster		4 independent thruster’s
Dependent control of the north and south thrusters with same direction in azimuth		4 dependent thruster’s
Dependent control of the north and south thrusters with opposite direction in azimuth		4 dependent thruster’s

Table 3: Lose of torque workspace in 2-thrusters configuration

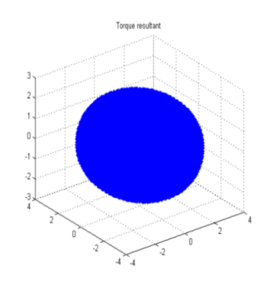
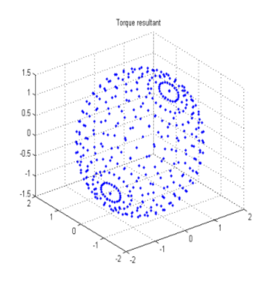
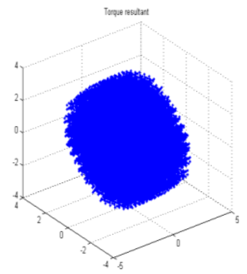
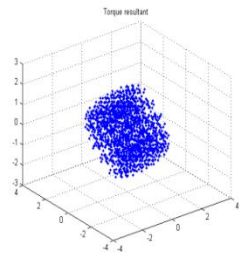
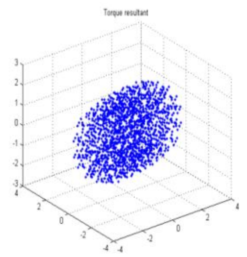
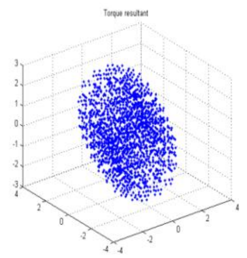
Angle Constraint	Product Torque	Configuratio n
Azimuth: ± 180 Elevation: ± 90 Independent control of thrusters		2 thruster’s
Azimuth: ± 180 Elevation: ± 90 Independent control of thrusters		2 thruster’s if TH_2 is damaged

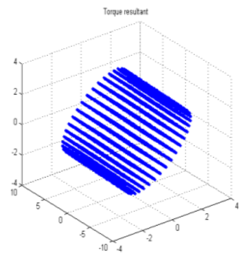
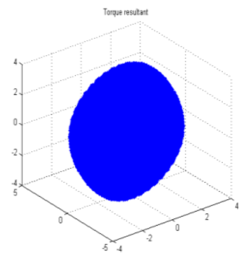
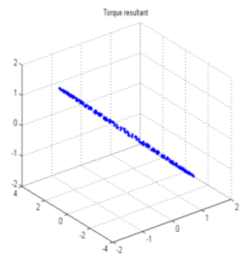
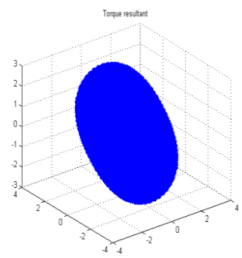
Table 4: Lose of torque workspace in 3-thrusters configuration

Angle Constraint	Product Torque	Config
Azimuth:±180 Elevation:±90 Independent control of thrusters		3 thruster's
Azimuth:±180 Elevation:±90 Independent control of thrusters		3 thruster's if TH_1 is damaged
Azimuth:±180 Elevation:±90 Independent control of thrusters		3 thruster's if TH_2 is damaged
Azimuth:±180 Elevation:±90 Independent control of thrusters		3 thruster's if TH_3 is damaged

As can be seen from table 4, if one of the north thrusters gets damaged in configuration based on three thrusters, then it has a few effects on torque workspace. But if the south thruster gets damaged, then it has a large effect on torque workspace.

As can be seen from table 5, if one of the thrusters gets damaged in configuration based on paired thrusters type 1, then a few amount of torque workspace loses. So, the other thrusters should produce additional torque to compensate absence of its pair thruster. But, when one of the paired thrusters that consist of a north thruster and a south thruster get damaged, the attitude control will be difficult.

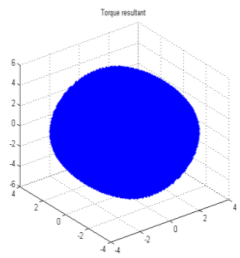
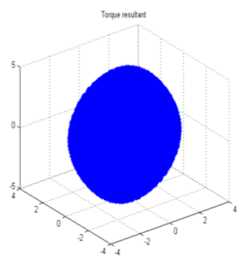
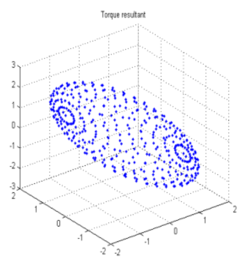
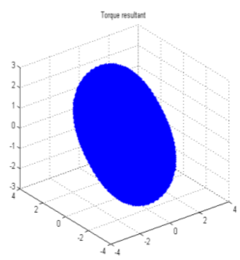
Table 5: Lose of torque workspace in 4 dependent thrusters configuration by same direction in azimuth

Angle Constraint	Product Torque	Config
Azimuth: ±180 Elevation:±90 Dependent control of the north and south thrusters with same direction in azimuth		4 dependent thruster's
Azimuth: ±180 Elevation:±90 Dependent control of the north and south thrusters with same direction in azimuth		4 dependent thruster's ifTH_4 is damaged
Azimuth: ±180 Elevation:±90 Dependent control of the north and south thrusters with same direction in azimuth		4 dependent thruster's if 2 eastern TH_1&TH_3 damaged
Azimuth: ±180 Elevation:±90 Dependent control of the north and south thrusters with same direction in azimuth		4 dependent thruster's if 2 southern TH_3&TH_4 damaged

As can be seen from table 6, this configuration consists of two paired thrusters type 2. The torque workspace covering is same as previous type. However, the magnitude of torque is less than previous type. It causes higher fuel consumption.

As can be seen from table 7, the thrusters are controlled independently. If one or two thrusters get damaged, attitude control has less changes and this absence compensate only with a few change in thrust level or gimbal angles of the other thrusters. In tables 3-7, whatever the torque workspace is become poor, the controllable directions have lost. Using gimbal thrusters have considerable effects on decrease fuel and power consumption.

Table 6: Lose of torque workspace in 4 dependent thrusters configuration by opposite direction in azimuth

Angle Constraint	Product Torque	Config
Azimuth:±180 Elevation:±90 Dependent control of the north and south thrusters with opposite direction in azimuth		4 dependent thruster's
Azimuth:±180 Elevation:±90 Dependent control of the north and south thrusters with opposite direction in azimuth		4 dependent thruster's if TH_4 is damaged
Azimuth:±180 Elevation:±90 Dependent control of the north and south thrusters with opposite direction in azimuth		4 dependent thruster's if 2 eastern TH_1&TH_3 damaged
Azimuth:±180 Elevation:±90 Dependent control of the north and south thrusters with opposite direction in azimuth		4 dependent thruster's if 2 southern TH_3&TH_4 damaged

5. ATTITUDE CONTROL DESIGN

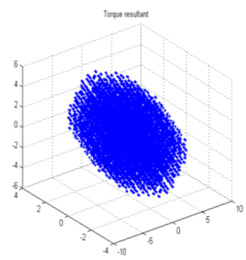
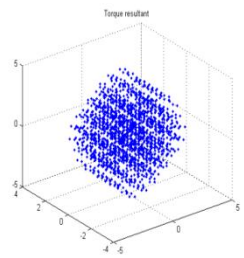
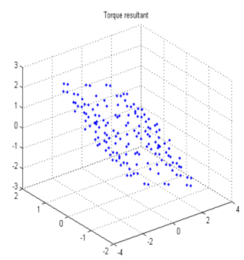
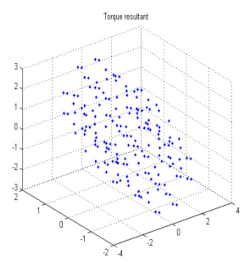
5.1 Attitude Dynamics and Kinematics

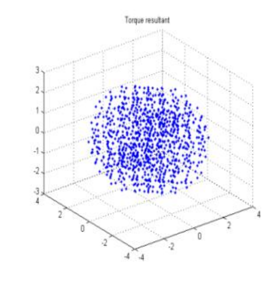
Based on the Euler's equation and Newton's third law, the dynamic equation of a satellite motion can be written as [2,9]:

$$I.\dot{\omega} = T - \omega \times H \tag{5}$$

Where $I = diag[I_x \quad I_y \quad I_z]$ is the moments of inertia of the satellite's body, $\omega = [\omega_x \quad \omega_y \quad \omega_z]^T$ is the inertially referenced satellite angular rate vector of the satellite body relative to the inertial coordinate system. H is momentum vector and equals to: $H = I.\omega$. $T = [T_x \quad T_y \quad T_z]^T$ is the total torques applied on the satellite consists of control and external disturbance torques. The satellite dynamic equations (roll, yaw and pitch) can be represented as follows:

Table 7: Lose of torque workspace in 4 independent thrusters configuration

Angle Constraint	Product Torque	Config
Azimuth: ±180 Elevation:±90 Independent control of thrusters		4 independent thruster's
Azimuth: ±180 Elevation:±90 Independent control of thrusters		4 independent thruster's ifTH_4 is damaged
Azimuth: ±180 Elevation:±90 Independent control of thrusters		4 independent thruster's if 2 eastern TH_1&TH_3 damaged
Azimuth: ±180 Elevation:±90 Independent control of thrusters		4 independent thruster's if 2 southern TH_3&TH_4 damaged

Azimuth: ±180 Elevation:±90 Independent control of thrusters		4 independent thruster's if 2 diagonal TH_2&TH_3 damaged

$$\begin{cases} \dot{\omega}_x = \dfrac{T_x - (I_y - I_z)\omega_z\omega_y}{I_x} \\ \dot{\omega}_y = \dfrac{T_y - (I_z - I_x)\omega_x\omega_z}{I_y} \\ \dot{\omega}_z = \dfrac{T_z - (I_x - I_y)\omega_y\omega_x}{I_z} \end{cases} \tag{6}$$

Quaternion is used for the attitude representation herein. Therefore, the derivatives of the Euler parameters can be updated by using the kinematics equation as follows:

$$q^{\cdot} = \frac{1}{2}\Omega[\omega].q \tag{7}$$

Where q is an attitude quaternion that represents the attitude of the satellite relative to the local-vertical–local-horizontal (LVLH) coordinate system and $\Omega[\omega]$is the skew symmetric matrix as follows:

$$\Omega[\omega] = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix} \tag{8}$$

5.2 Design of Control Law

The command control torques is related to the quaternion and angular rate errors:

$$q_E = \frac{1}{2}\begin{bmatrix} q_{T1} & q_{T2} & q_{T3} & q_{T4} \\ -q_{T2} & q_{T1} & q_{T4} & -q_{T3} \\ -q_{T3} & -q_{T4} & q_{T1} & q_{T2} \\ -q_{T4} & q_{T3} & -q_{T2} & q_{T1} \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \tag{9}$$

Where q, q_T and q_E are (respectively) the spacecraft, target and error quaternions.

The standard PD-type controller is employed for the 3-axis attitude control. The control law can be represented as:

$$T_c = -2k_p q_E - k_d\omega_e \tag{10}$$

Where q_E is the error quaternion. Whereas, ω_e is the angular rate difference between the commanded angular rate and the current angular rate.

In order to generate sufficient control torques from the control law, the proportional gain $k_p = \omega_n^2 I$ and derivative gain $k_d = 2\xi\omega_n I$ are to be chosen accordingly. These control gains are the functions of dynamic characteristics, i.e., the natural frequency ω_n and the damping ratio ξ.

In addition, the derivation of Euler angles (roll (R), pitch (P) and yaw (Y)) error $[\varphi \quad \theta \quad \psi]^T$ from the attitudequaternion error is as follows:

$$\begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} arctan\left(\dfrac{2(q_1q_4 + q_2q_3)}{1 - 2(q_1^2 + q_2^2)}\right) \\ arcsin(2(q_4q_2 - q_3q_1)) \\ arctan\left(\dfrac{2(q_4q_3 + q_1q_2)}{1 - 2(q_2^2 + q_3^2)}\right) \end{bmatrix} \tag{11}$$

6. GIMBAL THRUSTER CONTROL STRATEGY

A spacecraft or satellite has capable of employing a control system for controlling the attitude and orbital velocity. Satellite has a body which contains a variety of electronic equipment and sensors. The electronic equipment processes information gathered by the sensors and sends the processed information back to the ground station via antennae. Also, satellite has thrusters attached thereto via a gimbaled mechanism [10].

Figure 9 shows a block diagram for a satellite attitude control system.

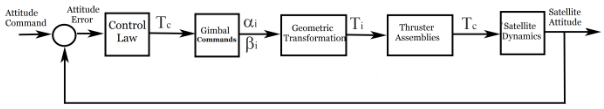


Fig 9: Satellite attitude control system block diagram.

Control of the attitude of the satellite is accomplished by sending an attitude command signal from ground control on Earth. A comparator then compares the attitude command signal with current satellite attitude signal. If the comparator determines that there is no difference in the values for the attitude command signal and the current satellite attitude signal, then no change in the direction of the thrusters is needed at that time. However, if the signals have unequal values, then the comparator produces an error signal which has a value equal to the difference between the value of the attitude command signal and the current satellite attitude signal. The error signal is then sent to a control signal generator which generates from a torque signal generator one or more torque signals representative of a satellite/body-fixed torque necessary to correct the error signal so as to have a value of Zero. The torque signals ($T_c = [T_x \quad T_y \quad T_z]^T$) are then sent to a gimbal command generator which generates control signals proportional to the angular displacements needed to produce the satellite body torque.

***Note**: the gimbal angles changes should be based on gimbal mechanism capable at stepper motors angles nominal change rate. For example, change rate of the gimbal system introduced in section 2 is 31. 625 (deg/sec).*

Thus, change rate should be less than nominal rate. Now, the procedure described as follows introduces a smooth rate change that is less than nominal rate. On the other hand, gimbal hardwares have less corrosion. In the following description, the gimbal angles ((α), (β)) and the trust level are parameters of system. The force applied by thruster can be calculated as follows [11]:

$$F_i = \begin{bmatrix} cos(\alpha_i)\,sin(\beta_i) \\ sin(\alpha_i) \\ cos(\alpha_i)\,cos(\beta_i) \end{bmatrix} f_i i = 1, \cdots, N \tag{12}$$

Where α_i, $\beta_i$and $f_i$are gimbal angles (elevation and azimuth) and thruster's thrust level respectively. N is number of thrusters. The change in the force generated by thruster due to a change in its gimbal angles and a change in its thrust level:

$$\Delta F_i = \begin{bmatrix} cos(\alpha_i)\,sin(\beta_i) & -f_i\,sin(\alpha_i)\,sin(\beta_i) & f_i\,cos(\alpha_i)\,cos(\beta_i) \\ sin(\alpha_i) & f_i\,cos(\alpha_i) & 0 \\ cos(\alpha_i)\,cos(\beta_i) & -f_i\,sin(\alpha_i)\,cos(\beta_i) & -f_i\,cos(\alpha_i)\,sin(\beta_i) \end{bmatrix}\begin{bmatrix} \Delta f_i \\ \Delta\alpha_i \\ \Delta\beta_i \end{bmatrix} \tag{13}$$

The torque may be controlled by commanding changes in the gimbal angles, without commanding changes in thrusts. The change in torque of thruster due to a change in its gimbal angles may be calculated according to equation below:

$$\Delta T_i = \begin{bmatrix} 0 & -y_i & x_i \\ y_i & 0 & -z_i \\ -x_i & z_i & 0 \end{bmatrix} \begin{bmatrix} -f_i\, sin(\alpha_i)\, sin(\beta_i) & f_i\, cos(\alpha_i)\, cos(\beta_i) \\ f_i\, cos(\alpha_i) & 0 \\ -f_i\, sin(\alpha_i)\, cos(\beta_i) & -f_i\, cos(\alpha_i)\, sin(\beta_i) \end{bmatrix} \begin{bmatrix} \Delta\alpha_i \\ \Delta\beta_i \end{bmatrix} = A_i \Delta g_i\, i = 1, \cdots, N \qquad (14)$$

Where A_i is a 3×2 matrix, Δg_i is a 2×1 vector containing $\Delta\alpha_i$ and $\Delta\beta_i$, and $r_i = [x_i \quad y_i \quad z_i]$ is the vector from thesatellite center-of-mass to the gimbal platform.

The total change in torque is the sum of the change in torque due to thrusters:

$$\Delta T = \sum_{i=1}^{N} \Delta T_i = \sum_{i=1}^{N} A_i \Delta g_i = [A_1 \quad \cdots \quad A_N] \begin{bmatrix} \Delta g_1 \\ \vdots \\ \Delta g_N \end{bmatrix} = A \Delta g_t \qquad (15)$$

Where A is a matrix and Δg_t is a vector. Each control cycle, a commanded torque change ΔT_c may be determined according to equation (16)

$$\Delta T_c = T_c - T_g \qquad (16)$$

Where T_c is the sum of the attitude control torque demand and T_g is the total torque produced by thrusters. The Δg_t that provides therequired ΔT_c is then calculated according to equation (17)

$$\Delta g_t = P \Delta T_c \qquad (17)$$

Where P is the pseudo-inverse of A computed as:

$$P = A^T (A A^T)^{-1} \qquad (18)$$

The torque control vector Δg_t, calculated using Equation (17) is a vector containing the change in gimbal angles for thrusters. Each control cycle, this change sum with gimbal angles:

$$g_{new} = g_t + \Delta g_t = \begin{bmatrix} \alpha_i \\ \beta_i \\ \vdots \\ \alpha_i \\ \beta_i \end{bmatrix} + \begin{bmatrix} \Delta\alpha_i \\ \Delta\beta_i \\ \vdots \\ \Delta\alpha_i \\ \Delta\beta_i \end{bmatrix} \qquad (18)$$

And the total torque produced by thrusters:

$$T_g = \sum_{i=1}^{N} r_i \times F_i \qquad (19)$$

The control cycle is repeated until the attitude error reaches to zero and does not require changing gimbal angles.

The control signals 36 are then sent to the thruster assemblies which angle one or more of the thrusters so that torques $T_c = [T_x \quad T_y \quad T_z]^T$ are applied to the satellite to correct the attitude.

The proposed gimbal control system make the controller capable to control the satellite even with two low thrust thrusters.

7. ATTITUDE PERFORMANCE OF VARIOUS CONFIGURATIONS

As seen in previous section, the torque may be controlled by commanding changes in the gimbal angles, without commanding changes in thrusts. In this section, changes in the gimbal angles are studied under pd (proportional-derivative) controller. But, the control law and the optimal changes in gimbal angles depend on thruster's configuration. One of the other important problems is feasibility in capable of stepper motor's angles change, i.e. the change rate must be less than nominal rate.

The standard pd control law is applied to satellite dynamics under two, three and four gimbal thrusters configuration.

Table 8. Satellite and pd controller parameters

$I_x = 1, \quad I_y = 0.5, \quad I_z = 0.7\ kgm^2$	moments of inertia
$\begin{cases} k_{px} = 1, k_{py} = 0.5, k_{pz} = 0.7 & N \\ k_{dx} = 2, k_{dy} = 1, k_{dz} = 1.4 & N \end{cases}$	Proportional and derivative gains
$\begin{bmatrix} \varphi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 40 \\ -40 \\ -60 \end{bmatrix}$	Attitude maneuver

Configuration by two thrusters:

Figure 10 represent the performance of specific attitude maneuver under two gimbal thrusters. It is clear that Euler angles are converged as well to final value and error has achieved less than10^{-8}.

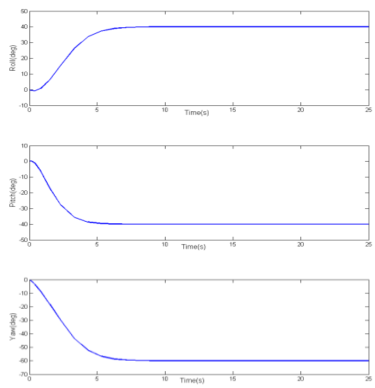


Fig 10: Performance of attitude control under two gimbal thrusters

As mentioned in previous section, the thrust levels are constant. Now it must be review that the constant thrust level can decrease to what extent that 1st: decrease fuel consumption and 2nd gimbal angles have not sudden changes. To better understanding this issue, simulations are implemented by two thrust levels 5N and 7N. Figure 11 shows gimbal angles (elevation & azimuth) changes.

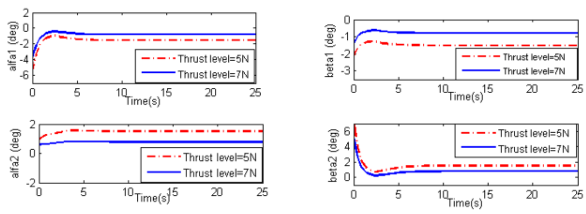


Fig 11: Gimbal angles changes by 2-thrusters

As can be seen, whatever thruster's thrust level is lower, the gimbal angles magnitude changes increases. As a result, it must be a balanced between decrease thrust level and increase gimbal angles magnitude changes. In this case, both fuel consumption decrease and avoid stepper motors destroying.

Configuration by three thrusters:
Like the previous case, the performance of specific attitude maneuver under three gimbal thrusters is desirable (Figure 12). Also, simulations of the gimbal angles changes are implemented by two thrust levels 5N and 7N (Figure 13).

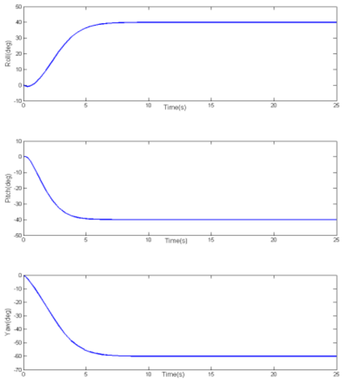


Fig 12: Performance of attitude control under three gimbal thrusters

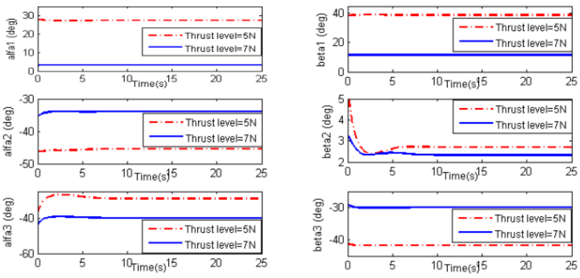


Fig 13: Gimbal angles changes by 3-thrusters

By a comparison between two proposed case (two & three thrusters), it can be seen in three thrusters configuration, the gimbal angles are deviated excessive from the origin. It may effect on the other satellite equipment like solar arrays by thruster's exhaust. More importantly that the large changes in gimbal angles may cause an early destroying in parts of stepper motors. Thus, three thrusters configuration has no advantage than two thrusters configuration in spite of more reliability and both of them have basic problems. Hence, four independent thruster's configuration is tested and results are represented.

Configuration by four independent thrusters:
Like the previous cases, the performance of specific attitude maneuver under three gimbal thrusters is desirable (Figure 14).Also, simulations of the gimbal angles changes are implemented by two thrust levels 5N and 7N (Figure 15).

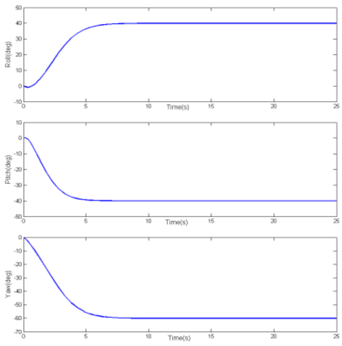


Fig 14: Performance of attitude control under four gimbal thrusters

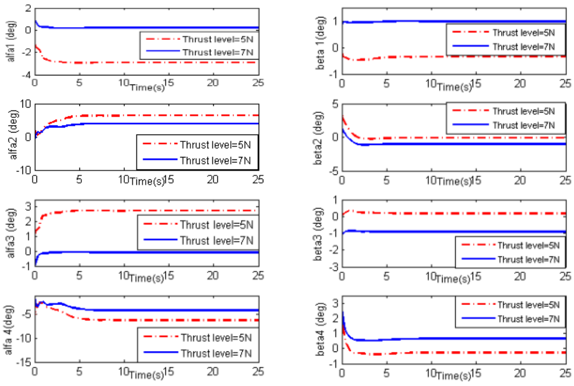


Fig 15: Gimbal angles changes by 4-thrusters

By a comparison between this configuration and the previous configurations, it can be seen that the gimbal angles have less deviation from the origin. Thus, the thruster's exhaust has a limit effect on the other satellite equipment. Also, because the time of thruster's firing decreases, the fuel efficiency increases. According to expressed items, the configuration by four independent thrusters is the best proposed configuration.

8. CONCLUSIONS

In this paper, various configurations of gimbal thrusters proposed. These configurations are designed based on 2, 3 and 4 thrusters. All of them are successful in torque workspace (controllable directions) covering. But, when one of thrusters or gimbal platforms in 2 thrusters configuration gets damaged, attitude control gets trouble. Using 3 or 4 thrusters increase reliability. Torque product is based on the gimbal angles changes. The difference between configurations is summarized in three factors. The first is fuel consumption. Probably, fuel consumption decrease by low number of thrusters, but for a large maneuver, it requires more forces by using 2 thrusters. The second factor is reliability. The configurations based on three and four thrusters are more reliable than two thrusters configuration. The third factor is gimbal angles magnitude changes. Although three thrusters configuration is reliable but the gimbal angles have more deviation from the origin. Therefore, three thrusters configuration isn't good for large maneuvers. Thus, four thrusters configuration both has more reliability and less angle deviation from the origin. Also it has optimal fuel consumption by decreasing thrust level and less angle changes. According to the all aspects, four independent thrusters configuration is the best and the most useful configuration of gimbal thrusters. As can be seen, the fuel consumption is one of the most important propulsion systems challenges. So, it can be decrease fuel consumption and magnitude changes of the gimbal angles significantly by employing optimization control laws instead of PD control law in the future works.

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