Solving a Interval Fuzzy Linear Programming Problem using Alpha-Cut Operation

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ABSTRACT
In our day to day activities modeling and solving of an optimization problem is one of the most important aspect. In this paper a new method is proposed to solve a fuzzy linear programming problem using interval arithmetic based on $Alpha$- cut.

Keywords
Triangular fuzzy number, Fuzzy linear programming, Interval arithmetic.

1. INTRODUCTION
The concept of fuzzy linear programming was first proposed by Zimmermann [10]. Then Bellman and Zadeh [1] proposed the concept of decision making in fuzzy environment. Tanaka et.al, [9] adopted this concept for solving mathematical programming problems. Compos and Vardegay [2] considered linear programming problems with fuzzy constraints and fuzzy coefficients in both left and right hand of the constraints set. Maleki [6] introduced a new method for solving linear programming problem with vagueness in constraints by using ranking function. Pandian and Jayalakshmi [8] proposed a new method for solving fully fuzzy linear programming problem with fuzzy variables. Jayalakshmi and Pandian[4] introduced a new method for finding an optimal fuzzy solution for fuzzy linear programming problems. In this paper we propose the basic definitions of fuzzy set and interval arithmetic operation on fuzzy numbers based on α- cut to solve a fuzzy linear programming problem.

2. PRELIMINARIES
2.1 Definition [5]
Let A be a classical set .$\mu_A(x)$ be a real valued function defined from R→ [0, 1]. A fuzzy set A^* with the function $\mu_A(x)$ is defined by $A^* = \{(x, \mu_A(x));\ \mathbf{x} \in A \text{ and } \mu_A(x) \in [0, 1]\}$. The function $\mu_A(x)$ is known as the membership function of A^*.

2.2 Definition [5]
Given a fuzzy set A defined on X and any number $\alpha \epsilon [0,1]$,the α -cut α_A is the crisp set

$\alpha_A = \{x/A(x) \geq \alpha\}$

2.3 Definition [10]
Given a fuzzy set A defined on X and any number $\alpha \epsilon [0,1]$, the strong α -cut α_{+A} is the crisp set $\alpha_{+A} = \{x/A(x) > \alpha\}$

2.4 Definition [3]
A fuzzy number is a convex normalized fuzzy set of the real line R whose membership function is piecewise continuous.

2.5 Definition [10]
A fuzzy number Ã in R is said to be a triangular fuzzy number if its membership function

$\mu_{\tilde{A}}$: R →[0,1] has the following characteristics

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

3. INTERVAL ARITHMETIC
Interval arithmetic, interval mathematics, interval analysis, or interval computation methods was developed by various mathematicians in between 1950 and 1960.This is an approach to putting bounds on rounding errors and measurement of errors in mathematical computation and thus developing numerical methods that yield reliable results**.** This represents each value as a range of possibilities.

The following are the basic operations of interval arithmetic, for two variables [a, b] and [c,d] that are subsets of the real line (-∞,∞)

(i) Addition: [a ,b]+[c,d]=[a+c, b+d]

(ii) Subtraction: [a,b]-[c,d]=[a-d,b-c]

(iii) Multiplication: [a,b].[c,d] = [min (ac,ad,bc,bd), max(ac,ad,bc,bd)]

(iv) Division: $[\frac{[a,b]}{[c,d]} = \min(\frac{a}{c}\frac{a}{d}\frac{b}{c}\frac{b}{d})$,$\max(\frac{a}{c}\frac{a}{d}\frac{b}{c}\frac{b}{d})$],

When 0 is not in [c,d].

4. ARITHMETIC OPERATION OF FUZZY NUMBERS USING α - CUT METHOD [7]
In this section we consider Addition, Subtraction, Multiplication and Division of fuzzy numbers using α- cut method.

4.1 Addition of Fuzzy Numbers
Let X=[a,b,c] and Y=[p,q,r] be two fuzzy numbers whose membership functions are defined by

$$\mu_x(x) = \begin{cases} \frac{x-a}{b-a}, & a \leq x \leq b \\ \frac{c-x}{c-b}, & b \leq x \leq c \\ 0 & \text{otherwise} \end{cases}$$

$$\mu_y(x) = \begin{cases} \frac{x-p}{q-p}, & p \leq x \leq q \\ \frac{r-x}{r-q}, & q \leq x \leq r \\ 0 & \text{otherwise} \end{cases}$$

Then, α_x= [(b-a) α+a,c-(c-b) α] and α_y=[(q-p) α+p, r-(r-q)α] are the α-cuts of fuzzy numbers X and Y respectively. To calculate the addition of fuzzy numbers X and Y using interval arithmetic

i.e, $\alpha_x + \alpha_y$ = [(b-a) α+a , c-(c-b) α]+ [(q-p) α+p , r-(r-q) α]

= [a+p+(b-a+q-p) α , c+r-(c-b+r-q) α]

4.2 Subtraction of Fuzzy Numbers

Let X=[a,b,c] and Y=[p,q,r] be two fuzzy numbers. Then α_x= [(b-a) α+a, c-(c-b) α] and α_y= [(q-p) α+p, r-(r-q) α] are the α –cuts of fuzzy numbers X and Y respectively. To calculate subtraction of fuzzy numbers X and Y using interval arithmetic.

i.e., α_x -α_y =[(b-a) α+a , c-(c-b) α]- [(q-p) α+p , r-(r-q) α]

=[(a-r)+(b-a+r-q) α , (c-p)-(c-b+q-p) α]

4.3 Multiplication of Fuzzy Numbers

Let X=[a,b,c] and Y=[p,q,r] be two positive fuzzy numbers. Then α_x=[(b-a) α+a , c-(c-b) α] and α_y=[(q-p) α+p , r-(r-q) α] are the α –cuts of fuzzy numbers X and Y respectively. To calculate the multiplication of fuzzy numbers X and Y using interval arithmetic.

i.e, α_x *α_y=[(b-a) α+a , c-(c-b) α]* [(q-p) α+p , r-(r-q) α]

=[((b-a) α+a)*((q-p) α+p), (c-(c-b) α)*(r-(r-q) α)]

4.4 Division of Fuzzy Numbers

Let X=[a,b,c] and Y=[p,q,r] be two positive fuzzy numbers. Then α_x=[(b-a) α+a , c-(c-b) α] and α_y=[(q-p) α+p , r-(r-q) α] are the α –cuts of fuzzy numbers X and Y respectively. To calculate division of fuzzy numbers X and Y using interval arithmetic.

i.e., $$\frac{\boldsymbol{\alpha_x}}{\boldsymbol{\alpha_y}} = \frac{[(\mathbf{b-a})\,\boldsymbol{\alpha}+\mathbf{a}\,,\mathbf{c}-(\mathbf{c-b})\,\boldsymbol{\alpha}\,]}{[(\mathbf{q-p})\,\boldsymbol{\alpha}+\mathbf{p}\,,\mathbf{r}-(\mathbf{r-q})\,\boldsymbol{\alpha}\,]}$$

$$= \left[\frac{(b-a)\alpha+a}{r-(r-q)\alpha}, \frac{c-(c-b)\alpha}{(q-p)\alpha+\mathrm{p}} \right]$$

5. PROPOSED ALGORITHM

The arithmetic operations of fuzzy numbers using α- cut operations discussed in the earlier sections are used below to solve the fuzzy linear programming problem.

The steps for the computation of an optimum solution are as follows:

Step 1*:* Find the value of α_X and α_Y

Step 2*:* Add the α-cuts of X and Y using interval arithmetic

Step 3*:* The values obtained in step1 and 2 is converted into a crisp LPP and formulated as.

$$\text{Maximize } Z = \sum_{j=1}^{n} c_j x_j$$

subject to,

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad (i = 1,2,..., m)$$

$$x_j \ge 0 \qquad (j = 1,2,..., n)$$

The function to be Maximized is called an objective function. This is denoted by Z. C=($c_1, c_2, \dots, c_n$) is called a cost vector. The matrix [a_{ij}] is called a constraint matrix, and the vector b_i = $(b_1, b_2, \dots, b_m)^T$ is called right hand side vector.

Step 4: By solving the above linear programming problem we obtain the optimal solution.

6. NUMERICAL EXAMPLE

Using the proposed algorithm an interval arithmetic linear programming problem with triangular fuzzy numbers is considered.

Maximize z = $5x_1+3x_2$

subject to

[(0,1,2)+(2,3,4)] x_1+[(3,4,5)+(1,2,3)] x_2 ≤

[(10,12,14)+ (14,16,18)]

[(0,1,2)+(2,3,4)] x_1+[(2,3,4)+(4,5,6)] x_2 ≤

[(12,14,16)+(16,18,20)]

$x_1,x_2 \geq 0$

Step 1*:* Determine α_X and α_Y

$$The\ \alpha - Cut\ of\ the\ fuzzy\ number\ (0, 1, 2)\ is$$

$$\alpha = \frac{x-0}{1}\ , \frac{2-x}{1}$$

$$\alpha_x = [\alpha\ , 2\text{-}\alpha]$$

$The\ \alpha - Cut\ of\ the\ fuzzy\ number$(2, 3, 4) is

$$\alpha = \frac{y-2}{1}, \frac{4-y}{1}$$

$$\alpha_Y = [\ \alpha+2\ ,4\text{-}\ \alpha]$$

Step 2*:* Adding the α-cuts of X and Y using interval arithmetic we obtain

$$\alpha_X + \alpha_Y = [\alpha\ ,2\text{-}\alpha]+ [\ \alpha+2\ ,4\text{-}\ \alpha] =8$$

Similarly the constraint matrix a_{ij} and the right hand side number b_i are

a_{12} =[(3,4,5)+(1,2,3)] =12,

a_{21} = [(0,1,2)+(2,3,4)] =8,

a_{22} =[(2,3,4)+(4,5,6)] =16,

b_1 =[(10,12,14)+(14,16,18)]=56,

b_2 =[(12,14,16)+(16,18,20)]=64

Step 3: Now the given problem becomes the crisp LPP as

Maximize Z =$5x_1+3x_2$

subject to ,

$8x_1+12x_2 \leq 56$

$8x_1+16x_2 \leq 64$

$x_1,x_2 \geq 0$

Step 4: By solving the above linear programming problem we obtain the optimal solution.

Max Z = 35, x_1=7 ,x_2=0

7. CONCLUSION

In this paper interval arithmetic LPP with triangular fuzzy number is solved by using α –cut operation without converting them in to a classical LPP. This is an easy approach for solving FLPP using interval arithmetic when compared to the earlier approaches. This also can be extended to multiobjective linear programming and fully fuzzy linear programming in various management, industrial and one's individual day to day applications. This also can be applied in trapezoidal, hexagonal, octagonal fuzzy numbers.

8. REFERENCES

[1] Bellman R.E and Zadeh L.A. "Decision Making in A Fuzzy Environment",ManagementScience,17(1970),141-164.

[2] Campos,J.L ,Vardegay, "Linear Programming Problem and ranking of fuzzy numbers", fuzzy sets and system.32(1989)1-11.

[3] Dubois,D.,and Prade,H.,(2000) "Fundamentals of fuzzy sets", Kluwer Academic Publishers,Boston.

[4] Jayalakshmi.M,Pandian P ., "A New Method for finding an optimal fuzzy solution for fully fuzzy linear programming problems", International Journal of Engineering Research and Applications (IJERA),2(4) 2012,247-254.

[5] Kaufman A.,and Gupta,M.,(1984) "Introduction to Fuzzy Arithmetic,Theory and applications",Van Nostrand Reinhold Co.Inc.,Workingham,Berkshire.

[6] Maleki H.R.Tata.M and Mashinchi.M, "Linear Programming with Fuzzy variables" ,Fuzzy Sets and system ,109(2000),21-33.

[7] PalashDutta,HrishikeshBoruah,TazidAli,"Fuzzy Arithmetic with and without using α -cut method" International journal of latest trends in computing,(IJLTC) 2(2011),99-107.

[8] Pandian ,P.and Jayalakshmi,M., "A new method for solving Integer linear programming problems with fuzzy variable".,AppliedMathematics Sciences,. 4(20)(2010), 997-1004.

[9] Tanaka .H ,Okuda.T and Asai.K "On Fuzzy Mathematical programming" ,Journal of cybemetics and systema 3(1973),37-46.

[10] Zimmermann,H.J "Fuzzy programming and linear programming with several objective functions". Fuzzy Sets and Systems,1(1978),45-55.