

Comparative Study on Various Random Walk Techniques for Left Ventricle Cavity Segmentation

Ghada Abdel-Aziz
Computer science and
Engineering Dept., Faculty of
Electronic Engineering,
Menoufia University, Egypt

Hamdy M. Kelash
Computer science and
Engineering Dept., Faculty of
Electronic
Engineering, Menoufia
University, Egypt

Osama S. Faragallah
Computer science and
Engineering Dept., Faculty of
Electronic Engineering,
Menoufia University, Egypt

ABSTRACT

Objective of image segmentation is to group regions with coherent characteristics. This paper presents a comparative study on Random walk techniques for Left ventricle segmentation. In this paper, LV cavity segmentation is demonstrated for each technique using multi-slice MR cardiac images. The quality of segmentation process is measured by comparing the resulted images of different Random walk techniques using DICE coefficient, PSNR and Hausdorff distance. The comparison includes measuring the execution time of each technique.

General Terms

Image segmentation, Random walk techniques, LV cavity segmentation

Keywords

Image segmentation, Random walks, LV cavity extraction, segmentation techniques

1. INTRODUCTION

The problem of localizing regions of an image relative to its content is known as image segmentation [1]. Cardiovascular Disease (CVD) is defined as any disorder in the function of the heart and circulatory system. CVD may cause death. In the fact, CVD is the main cause of death worldwide. Millions of deaths occur every year due to CVDs in high income countries like in Europe as well as in middle and low income countries. Nearly half of all deaths in Europe are from CVDs; 52% of deaths in women and 42% of deaths in men. About 85% of overall mortality of middle- and low-income countries is due to CVDs. It is expected that the number of mortality due to CVDs will increase in the next 15 years and it may reach 23.3 millions. CVDs are projected to remain the single leading cause of death [2, 3].

The Left Ventricle (LV) is the main cavity of the heart. Most of the CVDs cause changes in the left ventricular shape. Temporal changes in left ventricular (LV) shape over the cardiac cycle provide fundamental information regarding systolic and diastolic function of the heart. The LV function analysis requires the accurate description or segmentation of ventricular shape. The segmentation process divides an image into homogeneous regions considering one or more similar characteristics [4, 5].

The manual segmentation and analysis for 3D cardiac MR images are difficult because of the increasing resolution and it is extremely time-consuming. In order to further aid the cardiologist in the segmentation process for the extraction of LV cavity, accurately and fast semi-automated segmentation methods are needed. The goal of the semi-automation of segmentation is to shorten the time and improve the reliability of the process [6].

Table 1 segmentation techniques

Segmentation Technique	Prior knowledge	Boundary	Region	Unsupervised model	Supervised model
Bottom-up		√	√	√	
Active contours	√	√	√	√	
Deformable templates	√	√	√	√	
ASM		√			√
AAM			√		√
Level Set	√	√	√	√	
DB-guided			√		√
Random walk		√	√	√	

From the full scope of segmentation methods in the medical analysis, semi-automatic techniques can be divided into three major levels [7, 8]. At the first level, sequential points are predetermined by the user on or near the boundary, the boundary is found between these points using a minimal path method [9, 10]. At the second level, the algorithms require the specification of an initial boundary, then by minimizing a cost function of shape priors and image information the final segmentation can be obtained [11, 12]. At the third level, the algorithms require predetermined marked nodes, or seeds within specific regions [1, 13].

From general view of different segmentation methods, Random walk techniques are considered to be the main concern of research in this paper. Comparison between general characteristics of LV segmentation methods is shown in Table 1 [1, 14].

Basic Random walk (BRW) image segmentation which is known as Random walk with seeds was first presented in a general form by Leo Grady. Basic Random walk is a multi-label image segmentation method. It can be used for medical applications. It is based on Graph-Theoretic Electrical Potentials [6, 1].

The rest of this paper is organized as follows: Section 2 briefly describes the basic random walk technique and its mathematical model, section 3 briefly describes the high-speed random walk technique, Section 4 briefly describes the extended random walk technique, section 5 discusses the experimental results, and finally Section 6 discusses the conclusion.

2. BASIC RANDOM WALK (BRW)

In basic random walker algorithm, the image is considered as a purely discrete object. It is a graph with a fixed number of vertices and edges. Each edge is assigned a real valued weight which is calculated using the weighting function [15].

Basic Random walker algorithm uses user-defined marked nodes as seeds to indicate the regions of the image. Each seed specifies a location with a user-defined label. The algorithm can give labels to an unmarked pixel using the calculation of the probability that random walker starting at this unmarked node first reaches each of the seed points. The probability that a random walker starting at pixel first reaches a seed with specific user predefined marked label, can be calculated by solving the circuit theory problem that is corresponding to a combinatorial Dirichlet problem. All seed points belonging to labels other than the specific label are grounded and this specific label node is considered to be with a unit potential. The electric potentials at each unmarked node provide the probabilities. Harmonic function solves the Dirichlet problem for a given set of boundary conditions. The established connections between random walk and graph-theoretic electrical potentials are used to get convenient method to calculate the values of these probabilities. Sparse, symmetric positive-definite system of linear equations is required to be solved. Final segmentation may be derived by using the highest probability for each pixel. [13-16].

Basic Random walk algorithm can detect weak boundaries. This algorithm is formulated in discrete space (graph) using combinatorial analogues of standard operators and principles from continuous potential theory [1, 8].

2.1 Edge Weights

A graph $G = (V, E)$ with vertices $v \in V$, edges $e \in E$, $E \subseteq V \times V$. An edge e , spanning two vertices, v_i and v_j , is denoted by e_{ij} . A weighted graph assigns a value to each edge called a weight. The weight of an edge e_{ij} , is denoted by $w(e_{ij})$ or w_{ij} . The degree of a vertex is $d_i = \sum w(e_{ij})$ for all edges e_{ij} incident on v_i . $w_{ij} > 0$ [17].

Weighting function plays an important role for a successful segmentation in the approach. Gaussian weighting function that represents a change in image intensities in the edge weights is used [18].

$$w_{ij} = \exp(-\beta(I_i - I_j)^2), \quad 1$$

Where I_i indicates the image intensity at pixel i . The value of β is a free parameter in this algorithm [1]. Performance of the weighting method depends on the selection of the free parameter β , which is decided manually. The accuracy of the output significantly depends on this parameter [19].

2.2 Combinatorial Dirichlet Problem

The combinatorial Dirichlet problem has the same solution as the desired random walker probabilities. The Dirichlet integral may be defined as:

$$D[\mu] = \frac{1}{2} \int_{\Omega} |\nabla \mu|^2 d\Omega, \quad 2$$

For a field μ and region Ω

A harmonic function is a function that satisfies the Laplace equation:

$$\nabla^2 \mu = 0 \quad 3$$

Dirichlet problem is defined as the problem of finding a harmonic function subject to its boundary values. The harmonic function that satisfies the boundary conditions minimizes the Dirichlet integral since the Laplace equation is the Euler-Lagrange equation for the Dirichlet integral [1, 13].

2.3 Build the Combinatorial Laplacian Matrix

The combinatorial Laplacian matrix as:

$$L_{ij} = \begin{cases} d_i & \text{if } i = j \\ -w_{ij} & \text{if } v_i \text{ and } v_j \text{ are adjacent nodes,} \\ 0 & \text{otherwise,} \end{cases} \quad 4$$

Where L_{ij} is indexed by vertices v_i and v_j [16].

2.4 Partition of the Laplacian matrix

Partition the vertices into two sets, V_M (marked/seed nodes) and V_U (unmarked/unseeded nodes) such that $V_M \cup V_U = V$ and $V_M \cap V_U = \emptyset$. While V_M contains all seed points, regardless of their labels [14, 16].

The nodes in L and x are ordered such that marked nodes are first and unmarked nodes are second. When seeds are given, divide L , x into:

$$L = \begin{bmatrix} L_M & B \\ B^T & L_U \end{bmatrix} \quad 5$$

$$x = \begin{bmatrix} x_M \\ x_U \end{bmatrix} \quad 6$$

Where x_M and x_U are corresponding to the potentials of the marked and unmarked nodes, respectively. The reordering and block decomposition of L is important to describe the problem in a linear algebra framework and for applying the Dirichlet boundary conditions to the problem [20].

2.5 Finding and Solving the Linear System

The composition of $D[x_U]$:

$$\begin{aligned} D[x_U] &= \frac{1}{2} [x_M^T \quad x_U^T] \begin{bmatrix} L_M & B \\ B^T & L_U \end{bmatrix} \begin{bmatrix} x_M \\ x_U \end{bmatrix} \\ &= \frac{1}{2} [x_M^T L_M x_M + 2x_U^T B^T x_M + x_U^T L_U x_U] \end{aligned} \quad 7$$

Differentiating $D[x_U]$ with respect to x_U and finding the critical point [1]:

$$L_U x_U = -B^T x_M \quad 8$$

The aforementioned technique can be interpreted within circuit theory. There the graph is represented as an electric circuit; the nodes are represented as electric potentials, the edges are represented as resistors, and the weights are represented using electrical conductance. The previously described solution gives the potentials at the nodes, subject to the boundary conditions by fixing the electric potential at the boundary /labelled nodes.

3. HIGH SPEED RANDOM WALK (HSRW)

Random walk with pre-computations is a fast approximate random walk method. The objective of this technique is to shift the burden of computations of an interactive algorithm to an offline procedure. This offline mode may be performed before any user interaction has taken place. In many cases, the offline mode can be very important especially in medical segmentation such as left ventricle cavity segmentation because there is substantial time between the acquisition and segmentation of the image. Medical images often exist for days or weeks on a data server before a user interacts with the image. Eigenvectors of the graph Laplacian matrix are used to approximate the Random Walker solution. It is unknown where the user will choose to place seeds so; Pre-computation phase or offline mode procedure is very difficult. Pre-computing a small number of eigenvectors of the graph Laplacian matrix is sufficient to allow for a good and in fast approximation of the solution to the interactive Random Walker image segmentation algorithm, regardless of where the seeds are placed [21, 22]. In this paper, Random walk with pre-computations method will be referred as High Speed Random Walk technique (HSRW).

From [21], computations can be performed before seeds or marked nodes are defined (offline), calculating eigenvalue / eigenvector pairs can be performed offline to find the high speed solution of eq. 8.

The eigenvalue decomposition of L is:

$$L = Q\Lambda Q^T \quad 9$$

Where:

Λ is a diagonal matrix of the Laplacian k smallest eigenvalues of L.

Q is a matrix whose columns are the k corresponding eigenvectors.

By increasing k the solution will be close to the exact solution.

f is defined by:

$$Lx = f \quad 10$$

Decomposition of f corresponding to marked / unmarked nodes:

$$f = \begin{bmatrix} f_M \\ f_U \end{bmatrix} \quad 11$$

By performing an eigenvector decomposition of L from eq.9 in eq. 10:

$$Q\Lambda Q^T x = f \quad 12$$

E is the pseudo inverse of L.

$$E = Q\Lambda^{-1}Q^T \quad 13$$

Decompose the E, in corresponding to marked and unmarked nodes:

$$E = \begin{bmatrix} E_M & R \\ R^T & E_U \end{bmatrix} \quad 14$$

$$R^T = Q_U\Lambda^{-1}Q_M^T \quad 15$$

When, g is eigenvector of L with corresponding to the zero eigenvalue and it is a constant vector since L's column sum is 0.

Decompose the g in corresponding to marked and unmarked nodes:

$$g = \begin{bmatrix} g_M \\ g_U \end{bmatrix} \quad 16$$

$$EL = (I - gg^T) \quad 17$$

From eq. 10, 13, 17, Approximate x using E :

$$ELx = (I - gg^T)x = Ef = Q\Lambda^{-1}Q^T f \quad 18$$

When $f_U = 0$ calculating f_M as follows:

$$L_M x_M + Bx_U = f_M \quad 19$$

Substitution of eq. 6, 7 in eq. 10, 18, gives:

$$-g_U g_M^T x_M + x_U - g_U g_U^T x_U = R^T f_M \quad 20$$

Multiply B by eq. 20, and then subtract it from eq. 19:

$$(L_M + Bg_U g_M^T)x_M + Bg_U g_U^T x_U = (I - BR^T)f_M \quad 21$$

when $(Bg_U g_M^T)$ is a very small amount and ≈ 0

let $\alpha = g_U^T x_U$, $P = (I - BR^T)$

Then:

$$L_M x_M + Bg_U \alpha = P f_M \quad 22$$

$$\text{let } f_M = \widehat{f}_M + \overline{f}_M \alpha \quad 23$$

From 23, 22 can be divided into:

$$P\widehat{f}_M = L_M x_M \quad 24$$

$$P\overline{f}_M = Bg_U \quad 25$$

Note that by definition of g and from eq. 10:

$$g^T f = g^T Lx = 0 \quad 26$$

$$g^T f = [g_M^T \ g_U^T] \begin{bmatrix} f_M \\ f_U \end{bmatrix} \quad 27$$

$$g_M^T f_M = -g_U^T f_U = 0 \quad 28$$

$$g_M^T [\widehat{f}_M + \overline{f}_M \alpha] = 0 \quad 29$$

Then, find α ,

$$\alpha = -\frac{g_M^T \widehat{f}_M}{g_M^T \overline{f}_M}, \quad 30$$

From eq. 20,

$$x_U = R^T f_M + g_U \alpha + g_U g_M^T x_M \quad 31$$

$$x_U \approx R^T f_M + g_U \alpha \quad 32$$

4. EXTENDED RANDOM WALK (ERW)

Image intensity may be used to extract information from images. Every pixel in the image has relationships with neighbours. Most of traditional segmentation methods which depend on machine learning do not take intensity

relationships in consideration. On the other hand, the objects of interest may be reasonably characterized by the intensity distribution. Random walk segmentation algorithm (BRW) is a multi label image segmentation method that can especially be used with medical images. It performs well on a wide variety of images but this algorithm has properties that could be problematic for certain segmentation tasks. Each segment must be connected to a seed and only intensity gradients were used instead of employing absolute intensity information. The goal of the extended random walker (ERW) is to combine the intensity profiling and long-range aspects of the density estimation approach with the spatial cohesion of the random walker. ERW uses an intensity model obtained via density estimation so that it could be able to integrate intensity information into the random walk spatial algorithm [6, 23].

ERW method is an extended method from the basic random walk BRW approach by employing image priors. Each seed node is connected to every other node with weight corresponding to the prior density of the pixel intensity to that label,

$$w_{ij} = \exp\left(-\beta(I_i - I_j)^2\right)$$

A set of real-valued priors λ_i^s , that represent the probability density that the intensity at node v_i and belong to the intensity distribution of label g^s

$$x_i^s = \frac{\lambda_i^s}{\sum_{q=1}^k \lambda_i^q} \quad 33$$

This equation in vector notation may be written as,

$$\left(\sum_{q=1}^k A^q\right) x^s = \lambda^s \quad 34$$

Where A^s is a diagonal matrix with the values of λ^s on the diagonal.

λ is a vector of the prior probabilities for each node, weighted by a scalar, γ . The Laplacian is augmented with auxiliary nodes.

The Laplacian matrix is divided into components corresponding to the marked, unmarked, and auxiliary nodes:

$$L = \begin{bmatrix} L_M & B & 0 \\ B^T & L_U + \gamma I_U & -\gamma \lambda \\ 0 & -\gamma \lambda & \gamma I_U \end{bmatrix} \quad 35$$

Using the same way as eq. 8,

$$(L_U + \gamma I_U)x_U = -B^T x_M + \gamma \lambda \quad 36$$

5. EXPERIMENTAL RESULTS

The proposed methods are implemented in MATLAB with processor 2.27 GHz Intel® core™ i3 CPU and 3 GB memory. The seeds are defined by the user. Where $K = 80$ and $\beta=30$.

Similarity is computed between the result image of the algorithms and the manual segmented image as a reference.

$$DICE = \frac{2(A \cap B)}{A + B}$$

Where A, B are the reference mask region and the result mask region of an algorithm.

$$PSNR = 10 \log_{10} \left(\frac{d}{MSE(A, B)} \right)$$

Where d is the maximum possible value of the image and MSE(A, B) is the mean square error computed between A and B.

$$MSE(A, B) = \frac{1}{MN} \sum_{m=1}^M \sum_{n=1}^N \|A(m, n) - B(m, n)\|^2$$

$$Hausdorff = \max\{D(A, B), D(B, A)\}$$

Where A, B are the reference contour and the result contour of an algorithm.

$$D(A, B) = \max_{x \in A} (\min_{y \in B} (\|x - y\|))$$

Multi-slice MRI cardiac images are used. All images were stored in the standard digital imaging and communications in medicine (DICOM) format [24].

Figure 1, figure 2 and figure 3 show the resulted images of Basic Random Walk segmentation on a sample sets.

High speed random walk technique is applied on the same sample sets as shown in figure 4, figure 5 and figure 6.

Figure 7, figure 8 and figure 9 demonstrate the high quality segmentation using Extended random walk on the same sample sets where $\gamma=0.0001$.

Experimental results show that basic random walk algorithm can perform LV segmentation process of good quality and its similarity may reach values over 97% in average time of 0.85 seconds per slice.

HSRW algorithm results have very close similarity values as BRW but with a little bit lower quality and much more execution speed. The pre-computations in the offline mode reduce the execution online time. The average execution time of HSRW is 0.09 seconds per slice. When $K=80$. Increasing the value of K will increase the similarity and make the segmentation more accurate but it will also reduce the execution speed.

The results of ERW segmentation method show that this technique has the best quality segmentation. Similarity may exceed 98%. The average execution time is about 0.86 seconds per slice.

The average of quality measurements of segmentation are calculated and recorded in Table 2, Table 3 and Table 4 for each technique.

Figure 10, figure 11 and figure 12 are column charts showing the results of comparison between random walk techniques using DICE, PSNR and Hausdorff distance.

Comparison of speed using the average execution time in seconds is shown in Table 5. Each technique is applied 10 times per one slice and the average running time is taken.

Table 2 : Comparison of similarity using DICE coefficients

SEGMENTATION TECHNIQUE	DICE
BRW	0.966721
HSRW	0.964817
ERW	0.972700

Table 3 Comparison using PSNR

SEGMENTATION TECHNIQUE	PSNR
BRW	27.82928
HSRW	27.49958
ERW	28.96782

Table 4 : Comparison using Hausdorff distance

SEGMENTATION TECHNIQUE	HAUSDORFF DISTANCE
BRW	4.363283
HSRW	5.309596
ERW	3.542988

Table 5 : Comparison of speed using the average of Execution time

SEGMENTATION TECHNIQUE	EXECUTION TIME
BRW	0.848492
HSRW	0.090574
ERW	0.858959

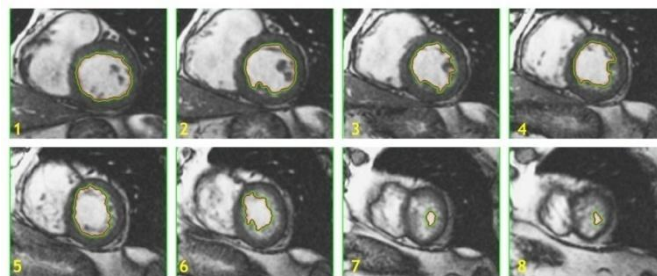


Fig 1: Results of Basic Random Walk on sample set 1

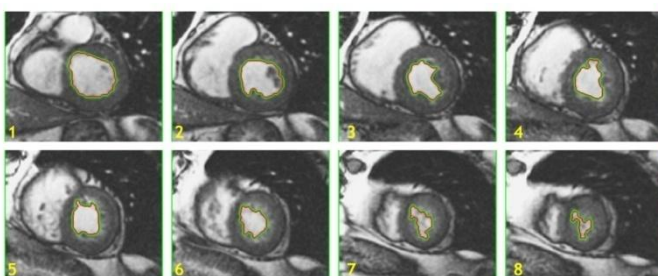


Fig 2 : Results of BRW on sample set 2

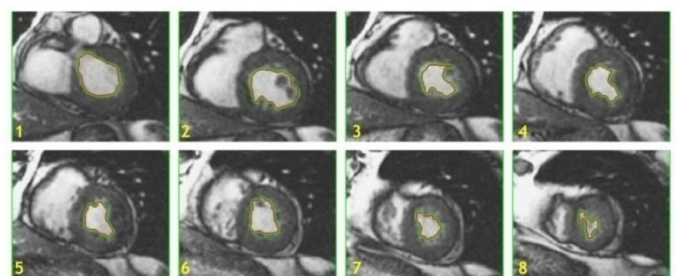


Fig 3 : Results of BRW on sample set 3

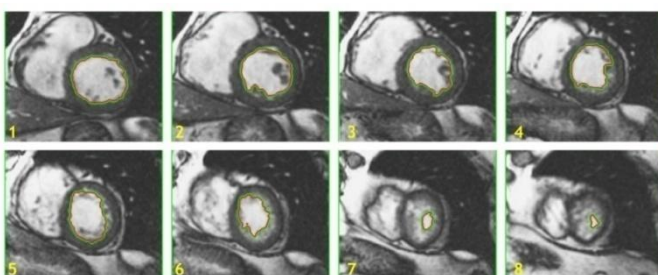


Fig 4 : Results of HSRW on sample set 1

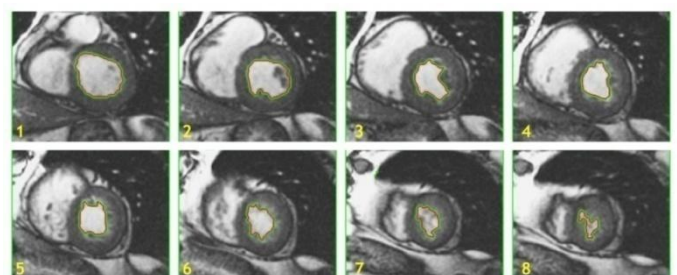


Fig 5 : Results of HSRW on sample set 2

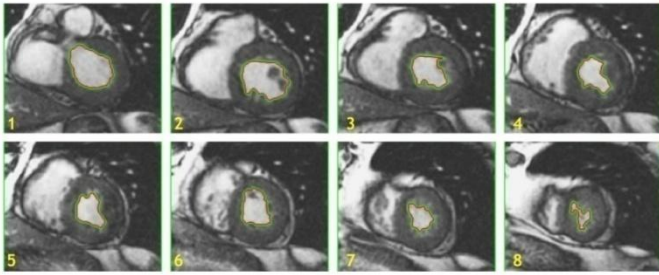


Fig 6 : Results of HSRW on sample set 3

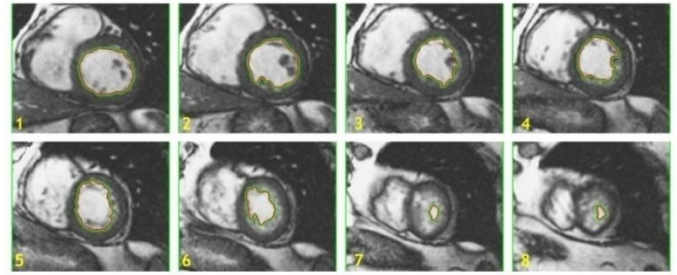


Fig 7 : Results of ERW on sample set 1

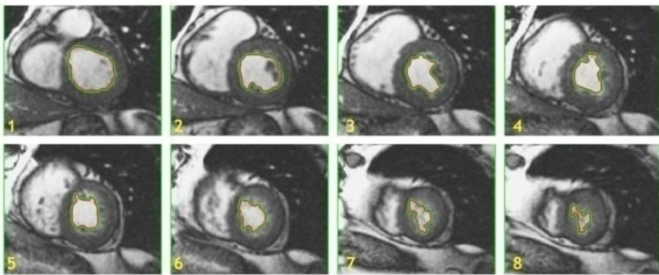


Fig 8 : Results of ERW on sample set 2

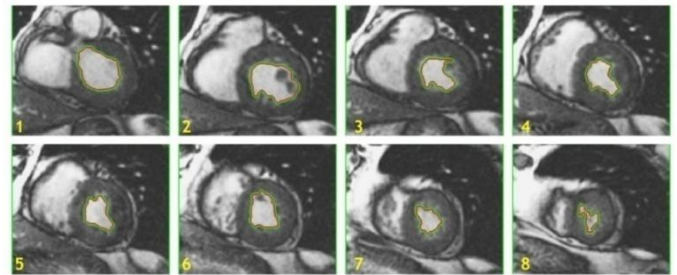


Fig 9 : Results of ERW on sample set 3

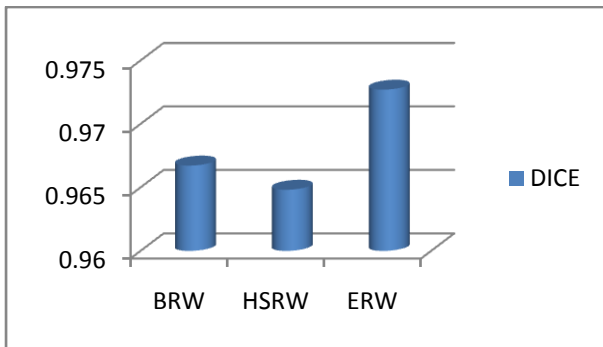


Fig 10: Comparison of similarity using DICE coefficients



Fig 13 : Comparison of Execution running time in seconds

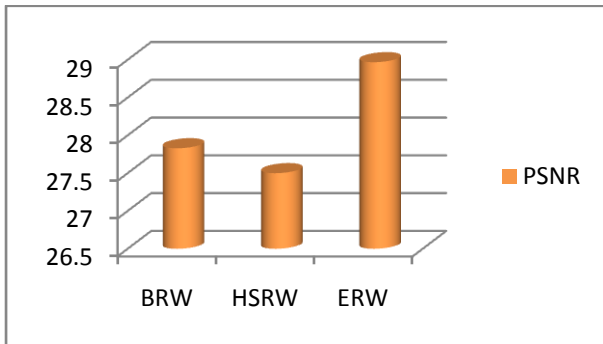


Fig 11: Comparison of similarity using PSNR

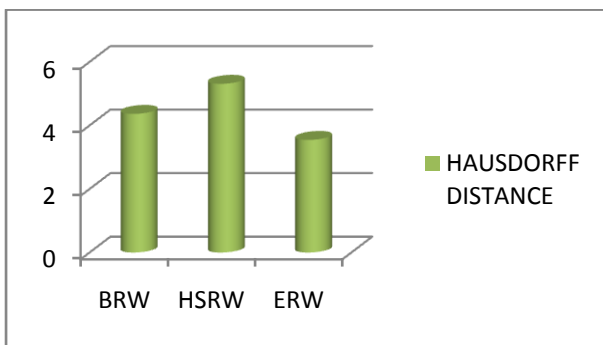


Fig 12 : Comparison of similarity using Hausdorff distance

6. CONCLUSION

In this paper, a comparative study has been carried out on three main Random walk techniques. Each technique is applied to extract the left ventricle cavity of the heart. The comparison is done by measuring the quality of segmentation using DICE coefficient, PSNR and Hausdorff distance. The execution time is measured for each technique.

Basic Random walker technique uses user-defined marked nodes to indicate regions of the image. The algorithm can give labels to an unmarked node using the calculation of the probability that random walker starting at this location first reaches each of the user defined points.

High speed random walk technique is a fast approximate random walk method. An offline procedure that may be performed before the user interaction has taken place. Eigenvectors of the graph Laplacian matrix are calculated to get the approximate random walk solution.

Extended random walk technique uses an intensity model obtained via density estimation in order to integrate intensity information into the random walk spatial algorithm. The objects of interest may be reasonably characterized by the intensity distribution of the image.

The Experimental results show that Extended random walk technique is the one of the highest quality results compared to

the other random walk techniques. Indeed, Extended random walk technique is the most accurate when it comes to similarity measurements with the corresponding manual segmentation results. The results show that the DICE coefficient of the extended random walk could reach over 98%. In High speed random walk technique, similarity results are close to the corresponding similarity measurements of the Basic random walk technique. High speed random walk results are of a little bit lower quality and much more execution speed than the basic random walk technique. The pre-computations of High speed random walk in the offline mode reduce the execution online running time.

Future work may include further studies on other different characteristics of each random walk technique. Also, two or more of the random walk techniques characteristics and features may be mixed together in order to improve the reliability of the left ventricle cavity segmentation process in order to make new hybrid random walk technique that could have the advantages such as accuracy and speed of execution to be enhanced and exclude the different disadvantages of each technique.

7. REFERENCES

- [1] L. Grady, "Random Walks for Image Segmentation" IEEE Transactions On Pattern Analysis and Machine Intelligence, vol. 28, no. 11, pp. 1768–1783, November 2006.
- [2] Nichols M, Townsend N, Luengo-Fernandez R, Leal J, Gray A, Scarborough P and Rayner M. "European Cardiovascular Disease Statistics 2012". European Heart Network, Brussels, European Society of Cardiology, Sophia Antipolis, 2012.
- [3] <http://www.who.int/mediacentre/factsheets/fs317/en/index.html>
- [4] K. S. Fu and J. K. Mui, "A survey on image segmentation," Pattern Recognition, vol. 13, no. 1, pp. 3-16, 1981.
- [5] R. Duda, P. Hart and D. Stork, "Pattern Classification", New York: Wiley-Interscience, 2000.
- [6] L. Grady and G. Funka-Lea, "Multi-Label Image Segmentation for Medical Applications Based on Graph-Theoretic Electrical Potentials", Proc. Workshop Computer Vision and Math. Methods in Medical and Biomedical Image Analysis, pp. 230-245, May 2004.
- [7] S. D. Olabarriaga and A. W. M. Smeulders, "Interaction in the segmentation of medical images": A survey. Medical Image Analysis, 5(2):127–142, 2001.
- [8] Y. Kang, K. Engelke, and W. A. Kalender. "Interactive 3D editing tools for image segmentation". Medical Image Analysis, 8(1):35–46, 2004.
- [9] L. Cohen and R. Kimmel, "Global minimum for active contour models: A minimal path approach", International Journal of Computer Vision, 24:57–78, 1997.
- [10] W. Barrett and E. Mortensen, "Interactive live-wire boundary extraction". Medical Image Analysis, 1:331–341, 1997.
- [11] M. Kass, A. Witkin, and D. Terzopoulos, "Snakes: Active contour models", Int'l J on Computer Vision, 1(4):321–331, 1987.
- [12] T. F. Chan and L. A. Vese, "Active contours without edges". IEEE Trans. on Image Processing, 10(2):266–277, 2001.
- [13] P. Doyle and L. Snell, "Random Walks and Electric Networks," no. 22, Washington, D.C.: Math. Assoc. of Am., 1984.
- [14] Osama S. Faragallah, "Enhanced Semi-Automated Method to Identify the Endo-Cardium and Epi-Cardium Borders", J. Electron. Imaging, 21(2), April 2012.
- [15] L. Grady, "Space-Variant Computer Vision: A Graph-Theoretic Approach," PhD dissertation, Boston Univ., 2004.
- [16] S. Aharon, L. Grady, T. Schiwietz. "GPU accelerated multi-label digital photo and video editing". US Patent 7630549. 8 Dec. 2009.
- [17] F. Harary, "Graph Theory," Addison-Wesley, 1994.
- [18] S. P. Dakua and J. S. Sahambi, "Weighting Function in Random Walk Based Left Ventricle Segmentation", IEEE international conference of image processing, 2011.
- [19] S. P. Dakua, J. S. Sahambi, "Effect of β in Random Walk Approach for LV Contour Extraction", World Congress on Nature & Biologically Inspired Computing, 2009.
- [20] Strang, G., "Computational science and engineering". Wellesley-Cambridge Press, 2007.
- [21] L. Grady and AK Sinop. "Fast approximate random walker segmentation using eigenvector precomputation". In IEEE Conf. CVPR, pages 1–8, 2008.
- [22] Y. Boykov and M. P. Jolly, "Interactive graph cuts for optimal boundary and region segmentation of objects in n-D images". Proc. ICCV, pages 105–112, 2001.
- [23] L. Grady, "Multilabel random walker image segmentation using prior models", IEEE Conf. CVPR, 1:763–770, June 2005.
- [24] Radau P, Lu Y, Connelly K, Paul G, Dick AJ, Wright GA. "Evaluation Framework for Algorithms Segmenting Short Axis Cardiac MRI." The MIDAS Journal - Cardiac MR Left Ventricle Segmentation Challenge.