Effects of Variable Viscosity and Thermal Conductivity on Unsteady Free Convective Heat and Mass Transfer MHD Flow of Micropolar Fluid with Constant Heat Flux through a Porous Medium

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ABSTRACT

The effects of temperature dependent viscosity and thermal conductivity on magneto-hydrodynamic unsteady free convective heat and mass transfer flow of an incompressible micropolar fluid through a porous medium along an infinite vertical plate is studied. A magnetic field of uniform strength is assumed to be applied which makes an angle α with the plane transverse to the direction of the main flow. A similarity parameter σ has been introduced and the suction velocity is considered to be inversely proportional to this time dependent parameter. The partial differential equations governing the flow, heat and mass transfer of the problem are transformed into dimensionless form of ordinary differential equations by using similarity substitutions. The governing boundary value problems are then solved numerically using Runge-Kutta shooting method. The effects of various parameters, viz. viscosity parameter, thermal conductivity parameter, mass transfer parameter, coupling constant parameter, Grashoff number, Prandtl number, Schmidt number and magnetic parameter on velocity, secondary velocity, micro-rotation, temperature and concentration field are obtained and presented graphically. The Skin-friction, Nusselt number and Sherwood number are also computed and presented in table.

Keywords
Micropolar fluid, variable viscosity, thermal conductivity, heat and mass transfer, MHD Flow, Hall Current

1. INTRODUCTION
The micropolar fluid, a subclass of micro-fluid, developed by Eringen [1], derives from the need to model the flow of fluids that contain rotating micro-constituents. The micro polar fluids can support couple stress and body couple only in which each particle has a finite size and constitutes a micro structure that can rotate about the centre of the volume element described by the micro-rotation vector. If a conductor or a semi conductor has current flowing in it because of an applied electric field and a transverse magnetic field is applied, there develops a component of electric field in the direction orthogonal to both the applied electric field and magnetic field, resulting in a voltage difference between the sides of the conductor. Due to the gyration and drift of charged particles, the conductivity parallel to the electric field is reduced and the current is induced in the direction normal to both electric and magnetic fields. This phenomenon is known as the Hall Effect. When the strength of applied magnetic field is very strong, one cannot neglect the effect of Hall currents. This effect can be taken into account within the range of magneto-hydro dynamical approximation. Several investigations have made theoretical and experimental studies of micropolar flow in the presence of a transverse magnetic

field during the last decades. Gorla *et al.* [2] investigated the magneto hydrodynamic free convection boundary layer flow of a thermo micropolar fluid over a vertical plate. The effect of hall currents on the magneto hydrodynamic boundary layer flow past a semi-infinite flat plate was studied by Katagiri [4]. Hossain [5] investigated the effect of hall current on unsteady hydro magnetic free-convection flow near an infinite vertical porous plate. Sattar and Hossain [6] studied the unsteady hydromegnetic free convection flow with hall current and mass transfer along an accelerated porous plate with time dependent temperature and concentration. Sarma and Hazarika [8] studied effects of variable viscosity and thermal conductivity on combined free-forced convection and mass transfer flow passed a vertical porous plate. Following the works of Sattar *et al.* [7] Sarma and Hazarika [9] investigated the effects of variable viscosity and thermal conductivity on free convective heat and mass transfer flow with constant heat flux through a porous medium.

The main objective of our present work is to extend the work of Sarma and Hazarika [9] for the study of effects of variable viscosity and thermal conductivity on unsteady free convective heat and mass transfer MHD flow of micropolar fluid with constant heat flux through a porous medium**.** Viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. The governing partial differential equations are reduced in to ordinary differential equations by similarity transformations. The problem is then solved numerically using Runge-kutta shooting algorithm with iteration process.

2. MATHEMATICAL FORMULATION OF THE PROBLEM:
We consider an unsteady free convective flow of an incompressible micropolar electrically conducting fluid through a porous medium along a semi-infinite porous vertical plate in presence of magnetic field. The plate is at constant temperature T_w. The temperature of the fluid far from the plate is T_∞. The x −axis is taken along the plate in upward direction and the y −axis is taken normal to it. The plate is parallel to the direction which is also the direction of gravity but directed vertically upward. N is the component of micro-rotation perpendicular to the xy −plane. The level of species concentration is assumed to be very low and hence species thermal diffusion and diffusion thermal energy effects can be neglected. A strong magnetic field of intensity B is imposed in a direction that makes an angle α with the plane transverse to the vertical plate which is

assumed to be non-conducting, such that $B = (0, \delta_1 B_0 , \sqrt{(1 - \delta_1{}^2)} B_0)$ where $\delta_1 = \cos \alpha$, $B_0 =$ applied magnetic field. If $\delta_1 = 1$ the imposed magnetic field is parallel to the $y -$direction and if $\delta_1 = 0$ the magnetic field is parallel to the plate. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is negligible compared to the applied magnetic field and the magnetic lines of force are fixed relative to the fluid, Shercliff [11]. The plate is assumed to be non-conducting hence $J_y = 0$ at the plate and hence zero everywhere. Neglecting electron pressure and ion slip we have from Ohm's law

$J_x = \frac{p\delta_1 \mu_c B_0}{\rho(1+m^2\delta_1{}^2)} (m\delta_1 u - w), J_y = \frac{p\delta_1 \mu_c B_0}{\rho(1+m^2\delta_1{}^2)} (u + m\delta_1 w)$, where $m = \omega_e \tau_e$ is the hall parameter.

2.1. Basic Equations:
Equation of continuity:

$$\frac{\partial v}{\partial y} = 0 \qquad \text{… (1)}$$

Momentum equations:

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = g_0\beta(T - T_\infty) + g_0\beta^*(C - C_\infty)$$
$$+ \frac{\partial}{\partial y}\left(v \frac{\partial u}{\partial y}\right) + \frac{\kappa}{\rho}\left(\frac{\partial N}{\partial y} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{vu}{K'}$$
$$- \frac{p\delta_1 \mu_c{}^2 B_0{}^2}{\rho(1+m^2\delta_1{}^2)} (u + m\delta_1 w) \qquad \text{…. (2)}$$

$$\frac{\partial w}{\partial t} + v \frac{\partial w}{\partial y} = \frac{\partial}{\partial y}\left(v \frac{\partial w}{\partial y}\right) + \frac{\kappa}{\rho}\left(\frac{\partial N}{\partial y} + \frac{\partial^2 w}{\partial y^2}\right)$$
$$+ \frac{p\delta_1 \mu_c{}^2 B_0{}^2}{\rho(1+m^2\delta_1{}^2)} (m\delta_1 u - w) \qquad \text{… (3)}$$

Angular momentum equation:

$$\frac{\partial N}{\partial t} + v \frac{\partial N}{\partial y} = \frac{\gamma}{\rho j}\frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j}\left(2N + \frac{\partial u}{\partial y}\right) \quad \text{… (4)}$$

Energy equation:

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + \frac{\kappa+\mu}{\rho c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right] \qquad \text{… (5)}$$

Equation of mass transfer:

$$\frac{\partial C}{\partial t} + v \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial}{\partial y}\left(v \frac{\partial C}{\partial y}\right) \qquad \text{… (6)}$$

Following Gorla [2] we have supposed that $\gamma = \left(\mu_\infty + \frac{\kappa}{2}\right)j = \mu_\infty\left(1 + \frac{K_1}{2}\right)j$, where $K_1 = \frac{\kappa}{v_\infty \rho}$, coupling constant parameter.

The Appropriate boundary conditions are

$$\left.\begin{array}{l} y = 0: \quad u = w = 0, N = 0, \frac{\partial T}{\partial y} = -\frac{q}{\lambda_\infty}, \quad C = C_w \\ y \to \infty: \quad u \to 0, w \to 0, \ N \to 0, \ T \to T_\infty, \ C \to C_\infty \end{array}\right\} \text{(7)}$$

A similarity parameter σ = σ (t) is introduced in order to make equations (1) — (6) similar where σ is in fact a time dependent length scale so that the governing equations could be transformed into a similar form in time. Using this length scale the solution of the equation (1) is considered to be

$$v = -v_0 \frac{v_\infty}{\sigma} \qquad \text{… (8)}$$

where $v =$ velocity vector , $v_0 =$ suction parameter and $v_0 > 0$.The negative sign indicates that the suction is towards the plate. It is to be mentioned that the basic flow in the medium is entirely due to buoyancy force caused by temperature difference between the wall and the medium.

We introduce the following non-dimensional quantities:

$$\left.\begin{array}{l} \eta = \frac{y}{\sigma}, f(\eta) = \frac{u}{U_0}, \varphi(\eta) = \frac{w}{U_0}, \theta(\eta) = \frac{\lambda_\infty(T-T_\infty)}{q\,\sigma}, \\ g(\eta) = \frac{C-C_\infty}{C_w-C_\infty}, h(\eta) = \frac{\sigma N}{U_0} \end{array}\right\} \text{… (9)}$$

Following Lai and Kulacki [3] we assume that the viscosity and thermal conductivity are inverse linear functions of temperature, i.e.

$$\left.\begin{array}{l} \frac{1}{\mu} = \frac{1}{\mu_\infty}[1 + \delta(T - T_\infty)] , \text{or } \frac{1}{\mu} = a(T - T_c) \\ \text{where } a = \frac{\delta}{\mu_\infty} \text{ and } T_c = T_\infty - \frac{1}{\delta} \\ \frac{1}{\lambda} = \frac{1}{\lambda_\infty}[1 + \xi(T - T_\infty)] , \text{or } \frac{1}{\lambda} = b(T - T_r) \\ \text{where } b = \frac{\xi}{\lambda_\infty} \text{ and } T_r = T_\infty - \frac{1}{\xi} \end{array}\right\} \text{… (10)}$$

where a, b, T_r and T_c are constants and their values depend on the reference state and thermal properties of the fluid i.e. v and λ.

Then we get $\left.\begin{array}{l} v = -v_\infty \frac{\theta_c}{\theta-\theta_c} , \quad \lambda = -\lambda_\infty \frac{\theta_r}{\theta-\theta_r}, \\ \theta_c = \frac{T_c-T_\infty}{T_w-T_\infty}, \quad \theta_r = \frac{T_r-T_\infty}{T_w-T_\infty} \end{array}\right\} \text{… (11)}$

Introducing the above transformations in equations (2) — (6), we have the following non dimensional equations

$$\left(1 + K_1\frac{\theta_c-\theta}{\theta_c}\right) f'' = \left[G_r\,\theta + G_c\,g + \left(\frac{\eta\sigma}{v_\infty}\frac{d\sigma}{dt} + v_0\right)f' + K_1 h' - M\delta_1\left(\frac{f}{1+m^2\delta_1{}^2} + \frac{m\delta_1\varphi}{1+m^2\delta_1{}^2}\right)\right]\frac{\theta-\theta_c}{\theta_c} - Kf + \frac{\theta' f'}{\theta-\theta_c} \qquad \text{… (12)}$$

$$\left(1 + K_1\frac{\theta_c-\theta}{\theta_c}\right) \varphi'' = \left[\left(\frac{\eta\sigma}{v_\infty}\frac{d\sigma}{dt} + v_0\right)\varphi' + K_1 h' + M\delta_1\left(\frac{m\delta_1 f}{1+m^2\delta_1{}^2} - \frac{\varphi}{1+m^2\delta_1{}^2}\right)\right]\frac{\theta-\theta_c}{\theta_c} + \frac{\theta' \varphi'}{\theta-\theta_c} \qquad \text{… (13)}$$

$$(2 + K_1) h'' = K_2 (4h + 2f') - 2\left(\frac{\eta\sigma}{v_\infty}\frac{d\sigma}{dt} + v_0\right)h' \qquad \text{… (14)}$$

$$\theta'' = P_r\left[(\eta\theta' - \theta)\frac{\sigma}{v_\infty}\frac{d\sigma}{dt} + v_0\theta'\right]\frac{\theta - \theta_r}{\theta_r} + \frac{\theta'{}^2}{\theta - \theta_r}$$
$$+ P_r E_c \frac{\theta-\theta_r}{\theta_r}\left(\frac{\theta_c}{\theta_c-\theta} + K_1\right) \qquad \text{… (15)}$$

$$g'' = S_c\left[\left(\frac{\eta\sigma}{v_\infty}\frac{d\sigma}{dt} + v_0\right)g'\right]\frac{\theta-\theta_c}{\theta_c} + \frac{g'\theta'}{\theta-\theta_c} \qquad \text{… (16)}$$

The dimensionless numbers are as follows:

$$K_1 = \frac{\kappa}{\rho \nu_\infty} \; , \; G_r = \frac{g_0 \beta q \sigma^3}{\nu_\infty \lambda_\infty U_0} \; , \; G_c = \frac{g_0 \beta^* (C_w - C_\infty)\sigma^2}{\nu_\infty U_0} ,$$

$$M = \frac{p {\mu_c}^2 \sigma^2 {B_0}^2}{\rho \nu_\infty}, K = \frac{\sigma^2}{K'}$$

$$S_c = \frac{\nu_\infty}{D} , \; P_r = \frac{\rho c_p \nu_\infty}{\lambda_\infty} \; , \; E_c = \frac{\lambda_\infty {U_0}^2}{\sigma q c_p} \qquad K_2 = \frac{\sigma^2 \kappa}{j \rho \nu_\infty}$$

$$\quad \dots (17)$$

Following the works of Sattar and Hussain [6] the

similarity condition requires that

$$\frac{\sigma}{\nu_\infty} \frac{d\sigma}{dt} = 2 \qquad \dots (18)$$

Integrating (18) we get, $\sigma = 2\sqrt{\nu_\infty t}$, which corresponds with the usual scaling factor for unsteady boundary layer flows, Schlichiting [10]. Using (18) in (12) — (16) and writing $2\zeta = 2\eta + v_0$, finally we get

$$\left(1 + K_1 \frac{\theta_c - \theta}{\theta_c}\right) f'' = \begin{bmatrix} G_r \, \theta + G_c \, g + 2\zeta f' + K_1 h' \\ -M\delta_1 \left(\frac{f}{1+m^2 \delta_1^2} + \frac{m\delta_1 \varphi}{1+m^2 \delta_1^2}\right) \end{bmatrix} \frac{\theta - \theta_c}{\theta_c}$$
$$-Kf + \frac{\theta' f'}{\theta - \theta_c} \qquad \dots (19)$$

$$\left(1 + K_1 \frac{\theta_c - \theta}{\theta_c}\right) \varphi'' = \left[2\zeta \varphi' + K_1 h' + M\delta_1 \left(\frac{m\delta_1 f}{1+m^2 \delta_1^2} - \frac{\varphi}{1+m^2 \delta_1^2}\right)\right] \frac{\theta - \theta_c}{\theta_c} + \frac{\theta' \varphi'}{\theta - \theta_c} \qquad \dots (20)$$

$$(2 + K_1) \, h'' = K_2 \, (4h + 2f') - 4\zeta h' \qquad \dots (21)$$

$$\theta'' = P_r (2\zeta \theta' - 2\theta) \frac{\theta - \theta_r}{\theta_r} + \frac{\theta'^2}{\theta - \theta_r} + P_r E_c \frac{\theta - \theta_r}{\theta_r} \left(\frac{\theta_c}{\theta_c - \theta} + K_1\right) \left(f'^2 + \varphi'^2\right) \qquad \dots (22)$$

$$g'' = 2S_c \zeta g' \frac{\theta - \theta_c}{\theta_c} + \frac{g' \theta'}{\theta - \theta_c} \qquad \dots (23)$$

The corresponding boundary conditions are

As $\eta = 0$: $f = 0, \varphi = 0, \; \theta' = -1, g = 1, \; h = 0$

As $\eta \to \infty$: $f \to 0, \; \varphi \to 0, \; g \to 0, \; \theta \to 0, \; h \to 0$

$$\qquad \dots (24)$$

The physical quantities of interest in this problem are the skin –friction coefficient c_f, Nusselt number Nu and Sherwood number S_h which indicate physically wall shear stress, rate of heat transfer and rate of mass transfer respectively. For micropolar boundary layer flow, the wall shear stress τ_w is given by

$$\tau_w = \left[(\mu + k)\frac{\partial u}{\partial y} + kN\right]_{y=0} = \rho \nu_\infty \left(\frac{\theta_c}{\theta_c - \theta(0)} + K_1\right) f'(0)$$
$$\qquad \dots (25)$$

The skin –friction coefficient c_f can be defined as

$$c_f = \frac{2 \, \tau_w}{\rho {U_0}^2} = 2\left(\frac{\theta_c}{\theta_c - \theta(0)} + K_1\right) Re^{-1/2} f'(0) \qquad \dots (26)$$

The Nusselt number is given by

$$Nu = \frac{x \, q}{\lambda_\infty (T_w - T_\infty)} = \frac{1}{2} Re^{1/2} \left[\frac{\theta_r}{\theta_r - \theta(0)}\right] \frac{1}{\theta(0)} \qquad \dots (27)$$

The mass flux at the wall is given by

$$M_w = -D \left[\frac{\partial C}{\partial y}\right]_{y=0}$$

$$S_h = \frac{S_c \, M_w \, x}{\nu_\infty (C_w - C_\infty)} = -\frac{1}{2} Re^{1/2} \left[\frac{\theta_c}{\theta_c - \theta(0)}\right] g'(0) \qquad \dots (28)$$

3. RESULTS AND DISCUSSIONS

The equations (19) — (23) together with the boundary conditions (24) are solved for various combination of the parameters involved in the equations using an algorithm based on the shooting method and presented results for the dimensionless primary velocity distribution, secondary velocity distribution , micro-rotation distribution, species concentration distribution, temperature distribution with the variation of different parameters.

Initially solution was taken for constant values of Sc = 2, Ec = 0.10, Pr = 0.70, δ_1 = 0.10, m = 1, Gr = 0.50, K1 = 0.50, K2 = 0.25, v_0 = 0.25, K = 0.50, Re = 0.50 with the viscosity parameter θ_c ranging from -15 to -1 at certain value of $\theta_r = -10$. Similarly solutions have been found with varying the thermal conductivity parameter $\boldsymbol{\theta_r}$ ranging from -15 to -1 at certain value of $\theta_c = -10$ keeping the other values remaining same. Solutions have also been found for different values of Magnetic parameter (*M*), dimensionless reference temperature corresponding to thermal conductivity parameter (θ_r) , dimensionless reference temperature corresponding to viscosity parameter (θ_c) , Prandtl number (P_r), Eckert Number (Ec), the coupling constant parameter (K_1) Suction parameter(v_0), permeability parameter (K) and Schmidt number (Sc).

The variations in primary velocity distribution, secondary velocity distribution, temperature distribution, species concentration distribution and micro-rotation distribution are illustrated in figures (1) — (15) with the variation of different parameters.

The figures (1) — (9) represent the variations in dimensionless primary velocity distribution and secondary velocity distribution with Magnetic parameter, dimensionless reference temperature corresponding to viscosity parameter and thermal conductivity parameter due to heating (Gr<0) and cooling of the plate (Gr>0), coupling constant parameter, permeability parameter and suction parameter. Figures (1) and (2) represent the variations of primary velocity and secondary velocity distribution with the variation of Magnetic parameter. We observe that primary velocity decreases with the increasing values of M. It is because that the application of transverse magnetic field will result a resistive force (Lorentz force) similar to drag force, which tends to resist the fluid flow and thus reducing its velocity. We have seen that secondary velocity increases for $\eta < 1$ and then decreases for increasing value of M . Figures (3) — (6) represent the variations of primary velocity with the variation of θ_c and θ_r due to heating (Gr<0) and cooling of the plate (Gr>0) respectively. We observe that both primary and secondary velocity decreases with the increasing values of m. From figure (7) we observe that primary velocity decreases with increasing values of v_0 . From figure (8) it is seen that primary velocity decreases when $\eta < 1.5$ and then increases with the increasing value of coupling parameter K_1 . From figure (9) we have observed that primary velocity increases with increasing values of permeability parameter. The figures (10) and (11) represent the variations in dimensionless temperature profile $\boldsymbol{\theta}(\eta)$ with the variation of Prandtl number and Eckert number respectively. From figure (10) we have observed that temperature decreases with the increasing values of P_r .It is due to the reason that with the increasing values of the Prandtl number the viscosity increases and as a result temperature decreases. From figure (11)) we have seen that temperature increases when both the values of Ec. Figure (12) displays the distribution representing concentration

profile within the boundary layer with the variation of Sc, It is observed that concentration decreases with the increasing values of Sc The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. As Sc is the connecting link between velocity and concentration

profiles, therefore with the increasing value of Sc molecular mass diffusivity decreases and as a result concentration decreases. Figures (13) — (15) display the distribution representing micro-rotation within the boundary layer with the variation of K_1, θ_c and θ_r.

3.1 Figures
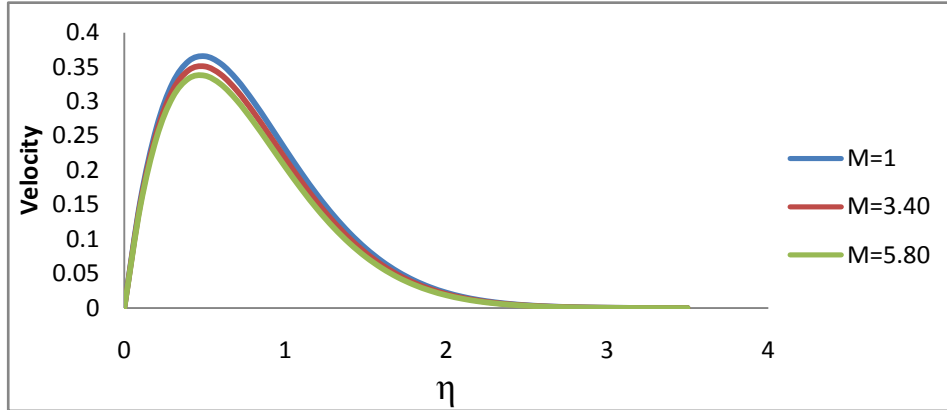
Figure 1: Variation of primary velocity distribution with M
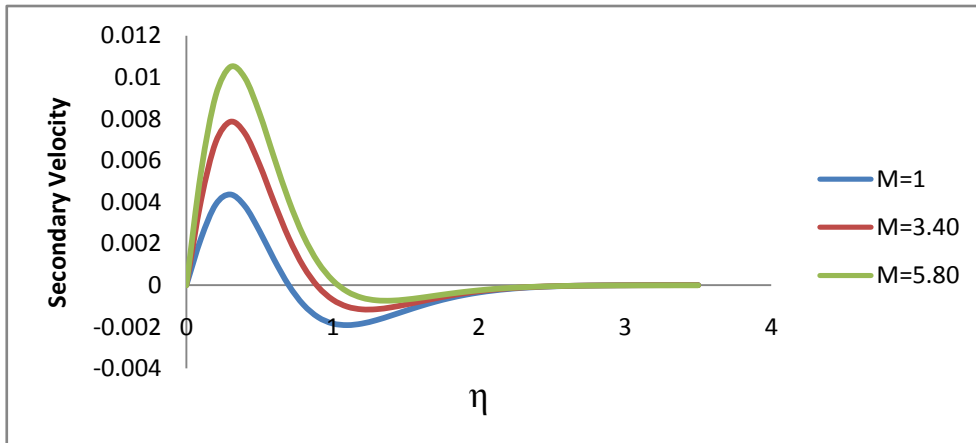
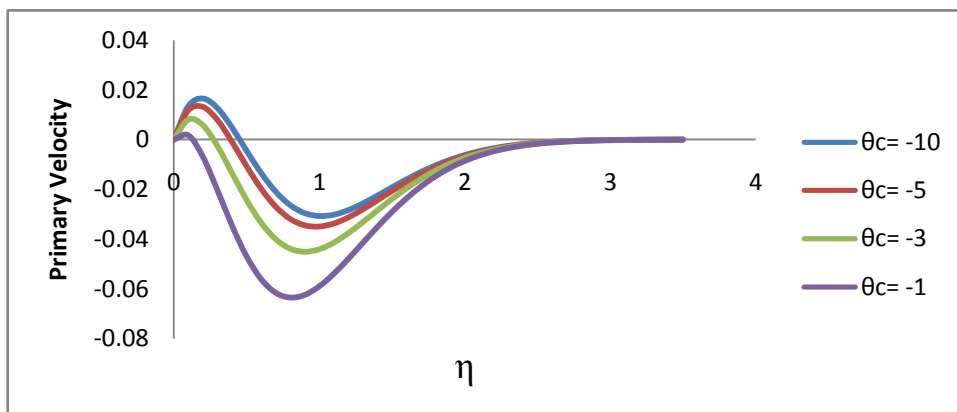**Figure 2: Variation of secondary velocity distribution with M**

Figure 3: Variation of primary velocity distribution with θ_c due to heating of the plate (Gr<0)
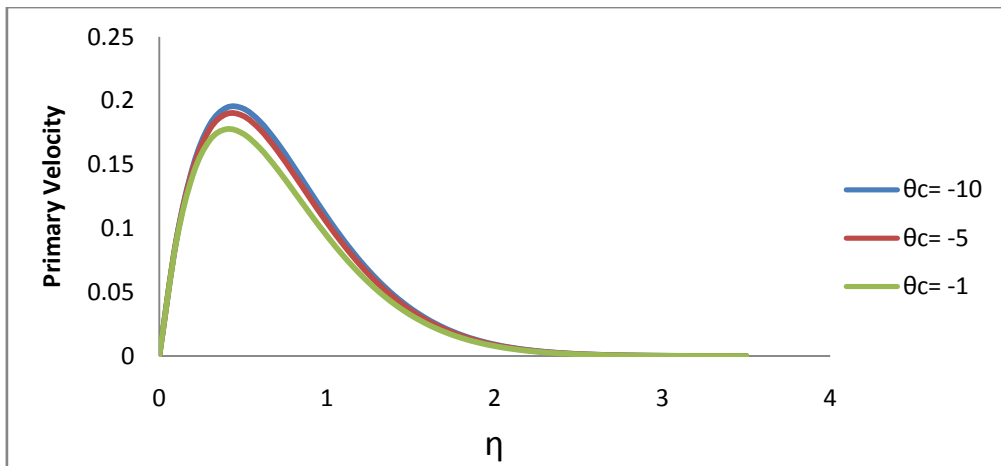
Figure 4: Variation of primary velocity distribution with θ_c due to cooling of the plate (Gr>0)
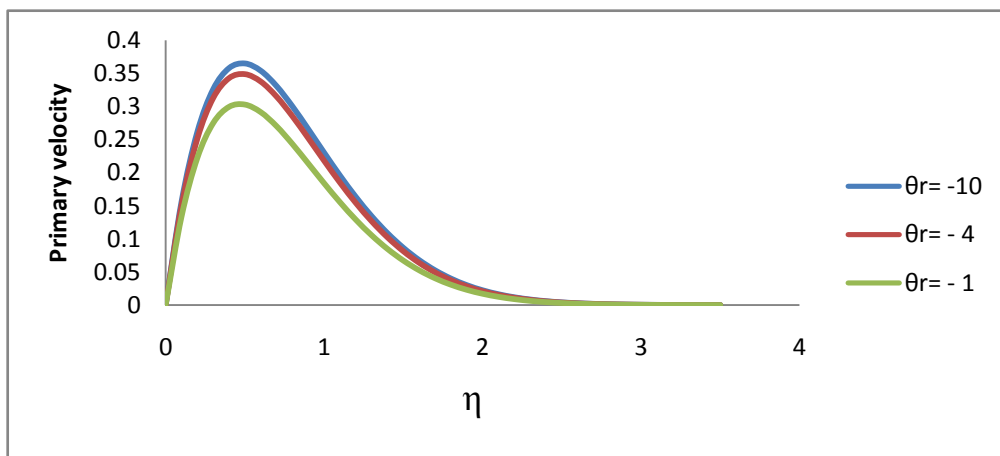

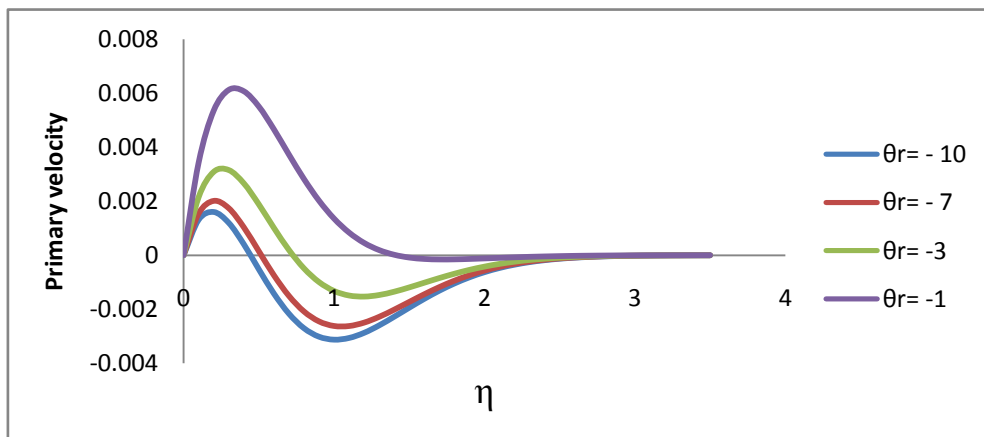**Figure 5: Variation of primary velocity distribution with θ_r due to cooling of the plate (Gr>0)**

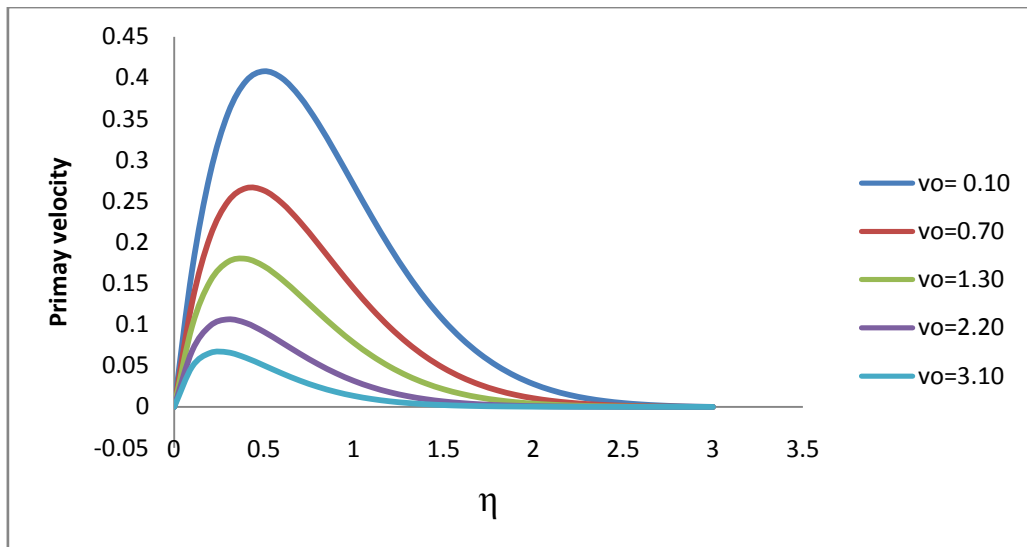**Figure 6: Variation of primary velocity distribution with θ_r due to heating of the plate (Gr<0)**

Figure 7: Variation of primary velocity distribution with suction parameter v_0
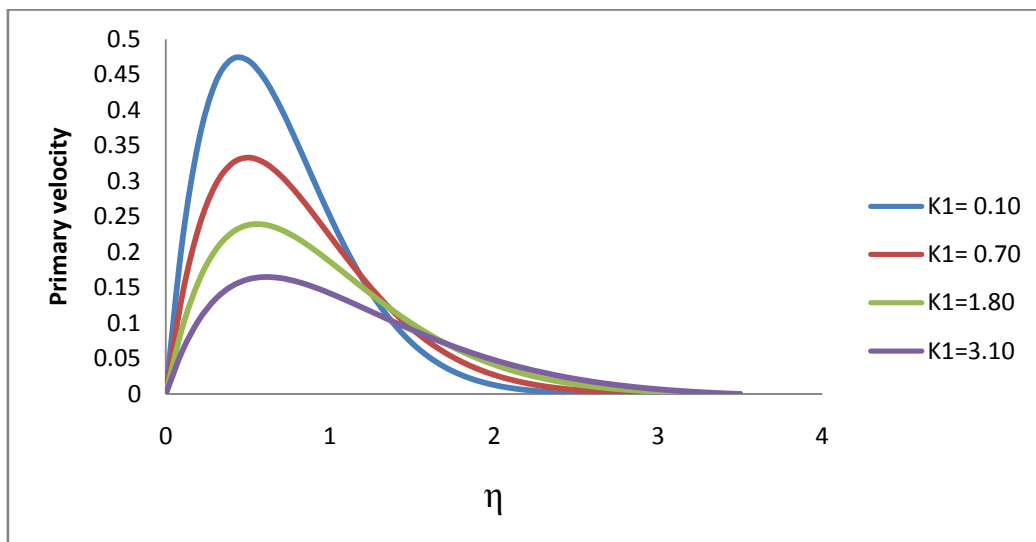

Figure 8: Variation of primary velocity distribution with K1
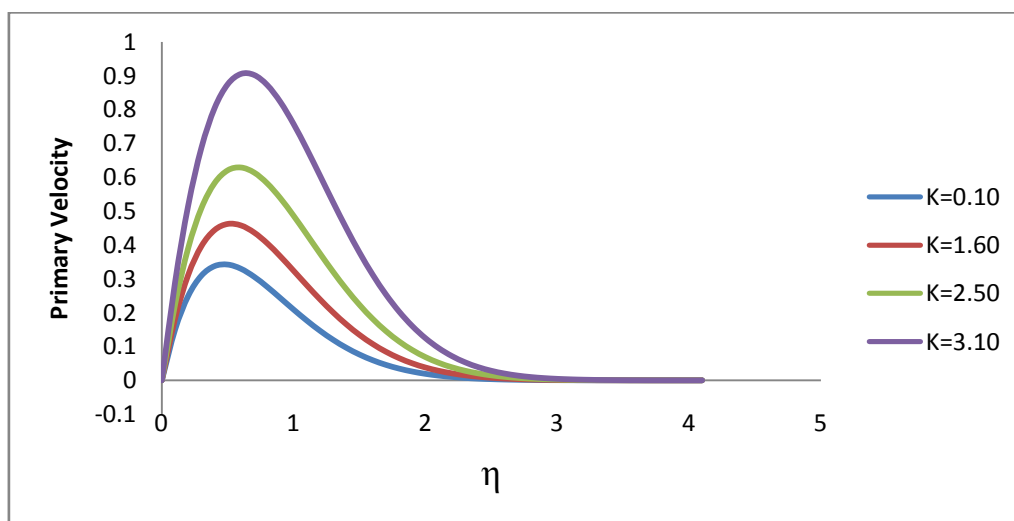

Figure 9: Variation of primary velocity distribution with K
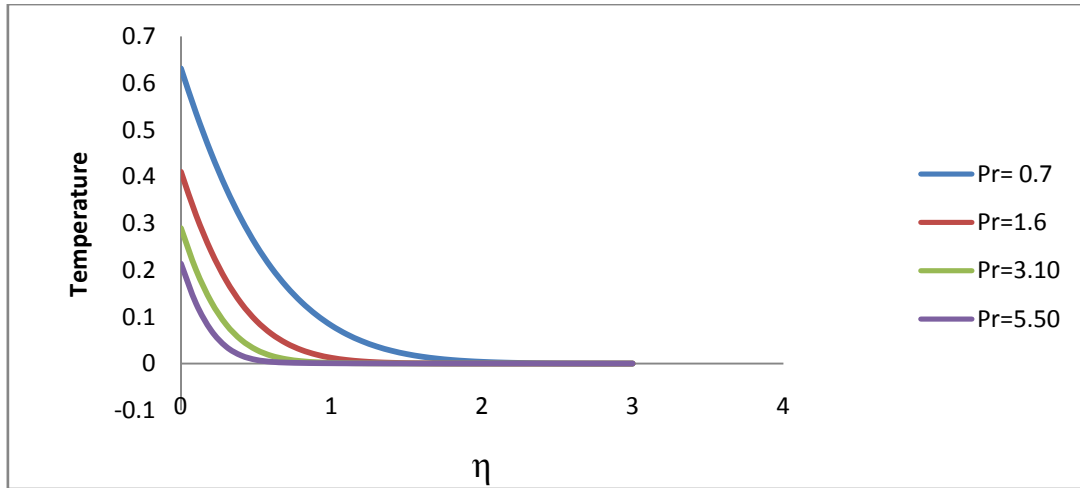
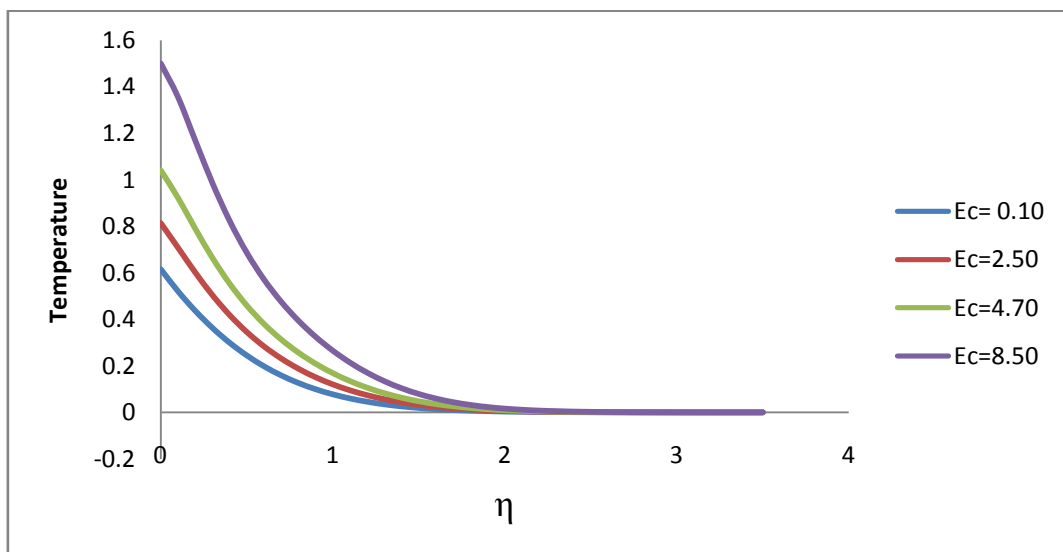**Figure 10: Variation of temperature distribution with Pr**

Figure 11: Variation of temperature distribution with Ec
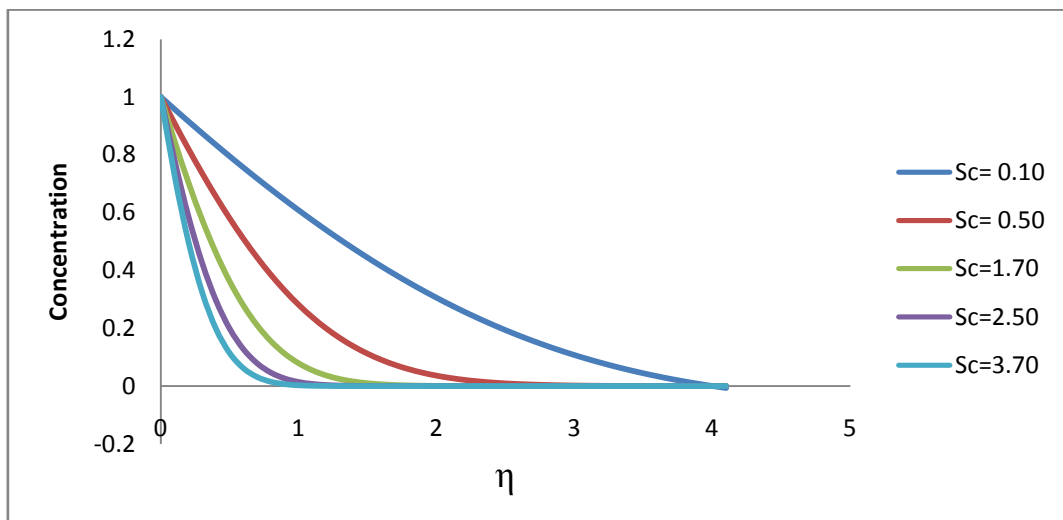

Figure 12: Variation of concentration distribution with Sc
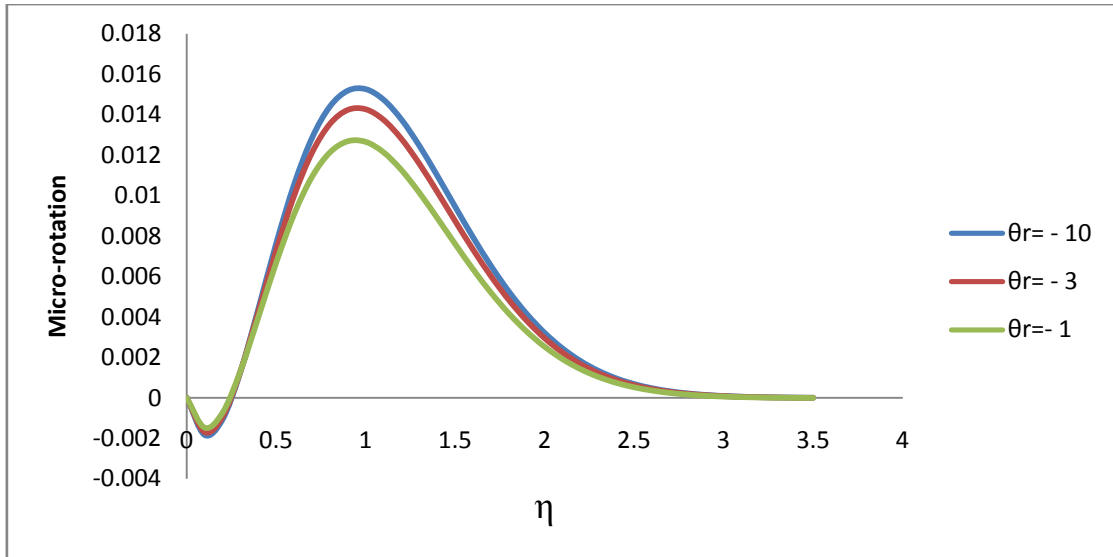
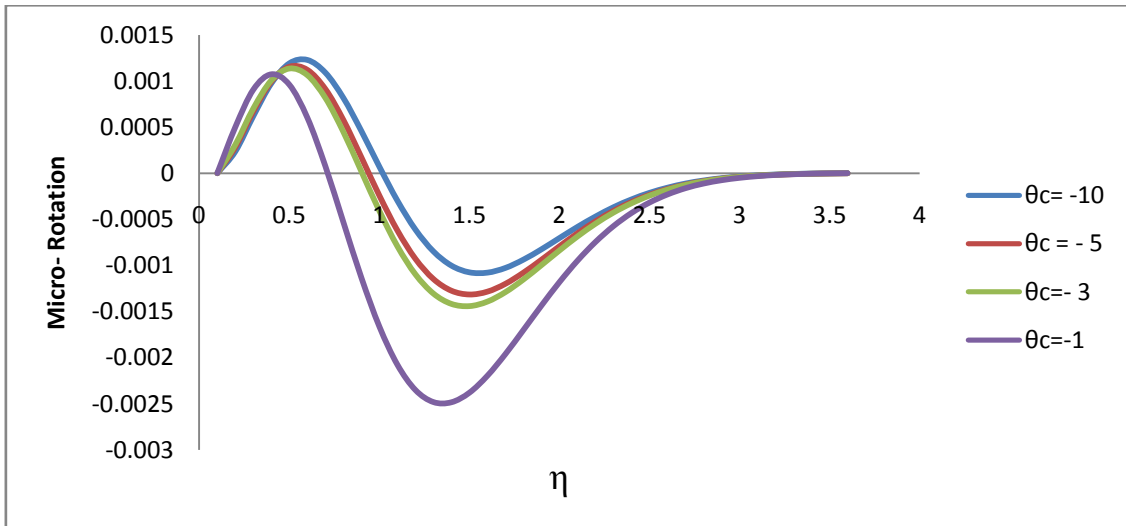**Figure 13: Variation of micro-rotation distribution with θ_r.**

Figure 14: Variation of micro-rotation distribution with θ_c
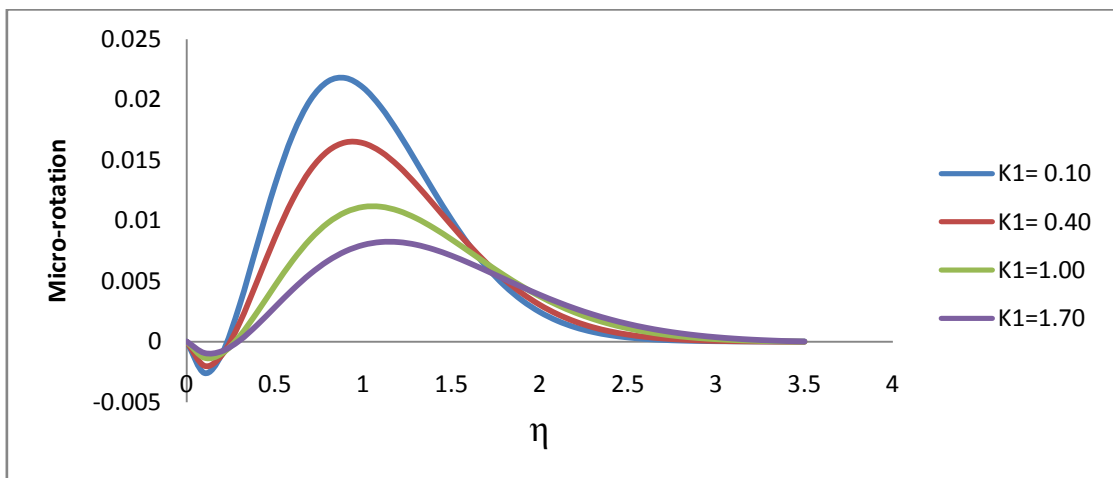

Figure 15: Variation of Micro-rotation distribution with K$_1$

3.2 Tables

Finally the effects of the above mentioned parameters on the values of $f'(0)$, g'(0), $\varphi'(0)$, h'(0), $\theta(0)$, C_f, Nu and S_h are shown in the tables (1) — (3). We consider Sc=2, θ_c=-10, Ec =0.10, θ_r=-10, Pr=0.70, δ_1=0.10, m=1.00, $Gc = 0.05$, Gr=0.50, K_1=0.50, K_2=0.25, v0=0.25, K=0.50, Re=0.50 .The behavior of these parameters is self evident from the tables and hence any further discussions about them seem to be redundant.

Table 1: Numerical values with the variation of M and Gc

M	Gc	f'(0)	$\varphi(0)$	$\theta(0)$	g(0)	h(0)	c_f	Nu	S_h
1	0.1	0.099866	0.001317	0.607654	-2.0101	-0.00209	0.423756	-227.226	0.234495
	1	0.279443	0.003346	0.608122	-2.01016	-0.00509	1.185979	-93.0704	0.234477
	2.5	0.578903	0.006732	0.609849	-2.01038	-0.0101	2.457733	-46.8875	0.234414
	4	0.878577	0.010121	0.612761	-2.01075	-0.01512	3.731248	-31.3227	0.234306
2.5	0.1	0.098366	0.002128	0.607651	-2.0101	-0.00203	0.417389	-233.551	0.234495
	1	0.275861	0.005324	0.608103	-2.01016	-0.00496	1.170766	-95.4922	0.234478
	2.5	0.571842	0.010654	0.609774	-2.01037	-0.00985	2.427717	-48.0792	0.234416
	4	0.868027	0.015991	0.612593	-2.01073	-0.01475	3.686355	-32.113	0.234312
4.5	0.1	0.096494	0.003	0.607647	-2.0101	-0.00196	0.409444	-241.875	0.234495
	1	0.271375	0.007473	0.60808	-2.01015	-0.0048	1.151717	-98.6718	0.234479
	2.5	0.562991	0.014935	0.609683	-2.01036	-0.00954	2.390091	-49.6427	0.23442
	4	0.854798	0.022405	0.612388	-2.0107	-0.01429	3.630061	-33.1496	0.23432

Table 2: Numerical values with the variation of M and P_r

M	Pr	f'(0)	$\varphi(0)$	$\theta(0)$	g(0)	h(0)	c_f	Nu	S_h
2	0.7	0.17757	0.003113	0.605664	-2.00986	-0.00335	0.753533	-140.664	0.234567
	2.7	0.119329	0.001834	0.295559	-1.97216	-0.00193	0.506336	-252.139	0.246251
	5.7	0.107829	0.00165	0.193626	-1.95841	-0.00175	0.457532	-280.941	0.25046
	7.2	0.102641	0.001493	0.167307	-1.95475	-0.00152	0.435514	-323.121	0.251578
4	0.7	0.174502	0.004635	0.605654	-2.00986	-0.00324	0.74051	-145.5	0.234567
	2.7	0.117626	0.002703	0.295554	-1.97216	-0.00187	0.499108	-260.071	0.246251
	5.7	0.106284	0.00243	0.19362	-1.95841	-0.00169	0.450976	-289.808	0.25046
	7.2	0.101448	0.002184	0.167361	-1.95476	-0.00149	0.43045	-330.438	0.251576

Table 3: Numerical values with the variation of M and θ_r

M	θ_r	f'(0)	$\varphi(0)$	$\theta(0)$	g(0)	h(0)	c_f	Nu	S_h
1	-10	0.17926	0.002212	0.60568	-2.00986	-0.00341	0.760707	-138.224	0.234566
	-5	0.175161	0.00215	0.584224	-2.00726	-0.00331	0.74331	-135.266	0.235347
	-1	0.156217	0.001867	0.480793	-1.99455	-0.00285	0.6629	-118.387	0.239183
4	-10	0.174555	0.004646	0.605664	-2.00986	-0.00324	0.740732	-145.538	0.234567
	-5	0.170603	0.00451	0.584209	-2.00726	-0.00314	0.72396	-142.395	0.235347
	-1	0.152322	0.003892	0.480783	-1.99455	-0.00271	0.646363	-124.506	0.239183

4. CONCLUSION:

In this study the effects of temperature dependent viscosity and thermal conductivity on magneto hydrodynamic unsteady free convective heat and mass transfer flow of an incompressible micropolar fluid through a porous medium along an infinite vertical plate is studied. The results demonstrate clearly that the viscosity and thermal conductivity parameters along with the other parameters viz. mass transfer parameter, coupling constant parameter, Prandtl number, Schmidt number, Hall parameter and magnetic parameter have significant effects on velocity, secondary velocity, temperature, concentration and micro-rotation distributions within the boundary layer. Thus assumption on constant properties may cause a significant error in flow problem.

5. NOMENCLATURES

β = Volumetric coefficient of thermal expansion

β^* = Volumetric co-efficient of expansion with concentration

g_0 = Gravitational acceleration

λ = Thermal conductivity

μ = Dynamic viscosity

μ_∞ = Dynamic viscosity of the ambient fluid

ν = Kinematic viscosity

ν_∞ = Kinematic Viscosity of the ambient fluid

κ = Vortex viscosity

γ = Spin-gradient or micro rotation viscosity

c_p = Specific heat

v_0 = Suction parameter

η = Dimensionless co-ordinates

u = Velocity in the x −direction

v = Velocity in the y −direction

w = Secondary velocity

U_0 = Reference velocity

f = Dimensionless velocity

φ = Dimensionless Secondary velocity

h = Dimensionless micro-rotation

g = Dimensionless species concentration

θ = Dimensionless temperature

θ_c = Dimensionless reference temperature corresponding to viscosity parameter

θ_r = Dimensionless reference temperature corresponding to thermal conductivity parameter

T = Temperature

C = Species concentration

T_∞ = Ambient temperature

T_w = Wall temperature

C_w = Species concentration at the wall

C_∞ = Species concentration far from the wall

j = Micro-inertia density

σ = Electrical conductivity

ρ = Density

D = Thermal molecular diffusivity

G_r = Local Grashoff number for heat transfer

G_c = Local Grashoff number for mass transfer

S_c = Schmidt number

P_r = Prandtl number

R_e = Local Reynolds number

M = Hartmann Number

m = Hall parameter

B_0 = Constant magnetic field intensity

K = Permeability parameter

K_1 = Coupling constant parameter

q_w = Heat transfer from the plate

C_f = Skin-friction coefficient

Nu = Nusselt number

S_h = Sherwood number

Subscripts:

w, the condition at the wall

∞, the condition far away from the surface

Superscripts:

ʹ, Differentiation with respect to η

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