Exact Traveling Wave Solution for Nonlinear Fractional Partial Differential Equation Arising in Soliton using the exp $(-\varphi(\xi))$-Expansion Method

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ABSTRACT
The exp(($-\varphi(\xi)$)-expansion method is used as the first time to investigate the wave solution of a nonlinear the space-time nonlinear fractional PKP equation, the space-time nonlinear fractional SRLW equation, the space-time nonlinear fractional STO equation and the space-time nonlinear fractional KPP equation. The proposed method also can be used for many other nonlinear evolution equations.

Keywords
The exp (($-\varphi(\xi)$)-expansion method; The space-time nonlinear fractional PKP equation; The space-time nonlinear fractional SRLW equation; The space-time nonlinear fractional STO equation; The space-time nonlinear fractional KPP equation; Traveling wave solutions; Solitary wave solutions; Kink-antikink shaped.

AMS Subject Classifications
35A05, 35A20, 65K99, 65Z05, 76R50, 70K70

1. INTRODUCTION
The nonlinear partial differential equations of mathematical physics are major subjects in physical science [1]. Exact solutions for these equations play an important role in many phenomena in physics such as fluid mechanics, hydrodynamics, Optics, Plasma physics and so on. Recently many new approaches for finding these solutions have been proposed, for example, tanh - seen method [2]-[4], extended tanh - method [5]-[7], sine - cosine method [8]-[10], homogeneous balance method [11, 12],F-expansion method [13]-[15], exp-function method [16, 17], trigonometric function series method [18], $(\frac{G'}{G})$— expansion method [19]-[22], Jacobi elliptic function method [23]-[26] and so on.

The objective of this article is to apply The exp(($-\varphi(\xi)$)-expansion method for finding the exact traveling wave solution of the space-time nonlinear fractional PKP equation, the space-time nonlinear fractional SRLW equation, the space-time nonlinear fractional STO equation and the space-time nonlinear fractional KPP equation mathematical physics.

The rest of this paper is organized as follows: In Section 2, we give the description of The exp(($-\varphi(\xi)$)-expansion method In Section 3, we use this method to find the exact solutions of the nonlinear evolution equations pointed out above. In Section 4, conclusions are given.

2. DESCRIPTION OF METHOD
Suppose that we have the following nonlinear fractional partial differential equation:

$$f(u, D_t^\alpha, D_x^\alpha u, \dots) = 0, \quad 0 < \alpha \le 1, \qquad (2.1)$$

where $D_t^\alpha u$, $D_X^\alpha u$ are the modified Riemann-Liouville derivatives, and F is a polynomial in *u(x, t)* and its partial fractional derivatives, in which the highest order derivatives and the nonlinear terms are involved. In the following we give the main steps of this method.

Step 1. Using the nonlinear fractional complex transformation

$$u(x,t) = u(\xi), \quad \xi = \frac{Kx^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)} + \xi_0$$

where *k*, c, ξ_0 are constants with *k*, $c \neq 0$, to reduce Eq.(2.1) to the following ordinary differential equation (ODE) with integer order:

$$P(u, u', u'', u''' ,\dots) = 0, \qquad (2.2)$$

where P is a polynomial in u(ξ) and its total derivatives,while ' $= \frac{d'}{d\xi}$.

Step 2. Suppose that the solution of ODE(2.2) can be expressed by a polynomial in *exp* $(-\varphi(\xi))$

as follows

$$u(\xi) = a_m (exp(-\varphi(\xi)))^m + \cdots, \quad a_m \neq 0, \qquad (2.3)$$

where $\varphi(\xi)$ satisfies the ODE in the form

$$\varphi'(\xi) = exp(-\varphi(\xi) + \mu exp(\varphi(\xi) + \lambda \qquad (2.4)$$

the solutions of ODE (2.4) are

i. when $\lambda^2 - 4\mu > 0$, $\mu \neq 0$,

$$\varphi(\xi) = ln\left(\frac{-\sqrt{\lambda^2-4\mu}\, tan\,h\left(\frac{\sqrt{\lambda^2-4\mu}(\xi+C_1}{2}\right)-\lambda}{2\mu}\right) \qquad (2.5)$$

ii. when $\lambda^2 - 4\mu > 0$, $\mu = 0$,

$$\varphi(\xi) = ln\left(\frac{\lambda}{exp\,(\lambda(\xi+C_1))-1}\right) \qquad (2.6)$$

iii. when $\lambda^2 - 4\mu = 0$, $\mu \neq 0$, $\lambda \neq 0$,

$$\varphi(\xi) = ln\left(\frac{2(\lambda(\xi+C_1)+2}{\lambda^2(\xi+C_1)}\right) \qquad (2.7)$$

iv. when $\lambda^2 - 4\mu = 0$, $\mu = 0$, $\lambda = 0$,

$$\varphi(\xi) = ln(\xi + C_1), \qquad (2.8)$$

v. when $\lambda^2 - 4\mu < 0$,

$$\varphi(\xi) = ln\left(\frac{\sqrt{4\mu-\lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}(\xi+C_1}{2}\right)-\lambda}{2\mu}\right) \qquad (2.9)$$

where a_m, ..., λ, n are constants to be determined later,

Step 3. Substitute Eq.(2.3) along Eq.(2.4) into Eq.(2.2) and collecting all the terms of the same power *exp* (-$m\varphi(\xi)$), $m = 0$, 1, 2, 3, ... and equating them to zero, we obtain a system of algebraic equations, which can be solved by Maple or Mathematica to get the values of a_i.

Step 4. Substituting these values and the solutions of Eq.(2.4) into Eq.(2.2) we obtain the exact solutions of Eq.(2.2).

3. APPLICATION

In this section we construct the exact solutions of the following four nonlinear fractional PDEs using the proposed method of Sec. 2 as following

3.1-Example 1: The Space-Time Nonlinear Fractional PKP Equation

This equation is well-known [27] and has the form:

$$\frac{1}{4}D_x^{4\alpha}u + \frac{3}{2}D_x^{\alpha}uD_x^{2\alpha}u + \frac{3}{4}D_y^{2\alpha}u + D_t^{\alpha}(D_t^{\alpha}u) = 0 \qquad (3.1)$$

where $0 < \alpha < 1$. Eq.(3.1) has been investigated in [27] using the fractional sub-equation method. Let us now solve Eq.(3.1) using the proposed method of Sec. 2. To this end, we use the nonlinear fractional complex transformation $u(x,y,t) = u(\xi)$,

$$\xi = \frac{k_1x^{\alpha}}{\Gamma(1+\alpha)} + \frac{k_2y^{\alpha}}{\Gamma(1+\alpha)} + \frac{ct^{\alpha}}{\Gamma(1+\alpha)} + \xi_0,$$

where k_1, k_2, c, ξ_0 are constants, to reduce Eq.(3.1) to the following ODE with integer order:

$$k_1^4u''' + 3k_1^3u'^2 + (3k_2^2 + 4ck_1)u' = 0. \qquad (3.2)$$

Balancing u''' with u'^2 $\Rightarrow N + 3 = 2(N + 1) \Rightarrow N = 1$. Consequently, Eq.(3.2) has the formal solutions:

$$u = a_0 + a_1 exp(-\varphi(\xi)), \qquad (3.3)$$

where a_0, a_1 are constants to be determined later, such that, $a_1 \neq 0$. It is easy to see that

$$u'' = 2\frac{a_1}{(e^{\phi(\xi)})^3} + 2\frac{a_1u}{e^{\phi(\xi)}} + 3\frac{a_1\lambda}{(e^{\phi(\xi)})^2} + a_1\lambda\mu + \frac{a_1\lambda^2}{e^{\phi(\xi)}} \qquad (3.4)$$

Substituting (3.3) along (3.4) into (3.2), collecting all the terms of the same order *exp* (-$i\varphi(\xi)$),

i = 0,l, 2,.. and setting each coefficient to zero, we have the following set of algebraic equation:

$$-6k_1^4a_1 + 3k_1^3a_1^2 = 0, \qquad (3.5)$$

$$-12k_1^4a_1\lambda + 6k_1^3a_1^2\lambda = 0, \qquad (3.6)$$

$$-k_1^4(8\alpha_1\mu + 7\alpha_1\lambda^2) + 3k_1^3a_1^2(\lambda^2 + 2\mu) - \alpha_1(3k_2^2 + 4ck_1) = 0, \qquad (3.7)$$

$$-k_1^4(8\alpha_1\mu\lambda + 7\alpha_1\lambda^3) + 6k_1^3a_1^2\lambda\mu - \lambda\alpha_1(3k_2^2 + 4ck_1) = 0, \qquad (3.8)$$

$$-k_1^4(\alpha_1\mu\lambda^2 + 2\alpha_1\mu^2) + 3k_1^3a_1^2\mu^2 - \mu\alpha_1(3k_2^2 + 4ck_1) = 0, \qquad (3.9)$$

On solving these algebraic equation with aid of Maple or Mathematica we have

$$a_0 = a_0, \quad , a_1 = 2k_1, \quad c = \frac{4k_1^4\mu - k_1^4\lambda^2 - 3k_2^2}{4k_1}$$

So that the solution of Eq.(3.2)

$$u = a_0 + 2k_1\exp(-\varphi(\xi)), \qquad (3.10)$$

The solutions of ODE (3.2) are

I. when $\lambda^2 - 4\mu > 0$, $\mu \neq 0$,

$$u = a_0 + \left(\frac{4k_1\mu}{-\sqrt{\lambda^2-4\mu}\, tan\, h\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C_1)\right)-\lambda}\right) \qquad (3.11)$$

II. when $\lambda^2 - 4\mu > 0$, $\mu = 0$,

$$u = a_0 - \left(\frac{4k_1\lambda}{exp(\lambda(\xi+C_1))-1}\right) \qquad (3.12)$$

III. when $\lambda^2 - 4\mu = 0$, $\mu \neq 0$, $\lambda \neq 0$,

$$u = a_0 + \left(\frac{2k_1\lambda^2(\xi+C_1)}{2(\lambda(\xi+C_1))+2}\right) \qquad (3.13)$$

IV. when $\lambda^2 - 4\mu = 0$, $\mu = 0$, $\lambda = 0$,

$$u = a_0 + \left(\frac{2k_1}{\xi+C_1}\right) \qquad (3.14)$$

V. when $\lambda^2 - 4\mu <$

$$0, u = a_0 + \left(\frac{4k_1\mu}{-\sqrt{4\mu-\lambda^2}\tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C_1)\right)-\lambda}\right) \qquad (3.15)$$

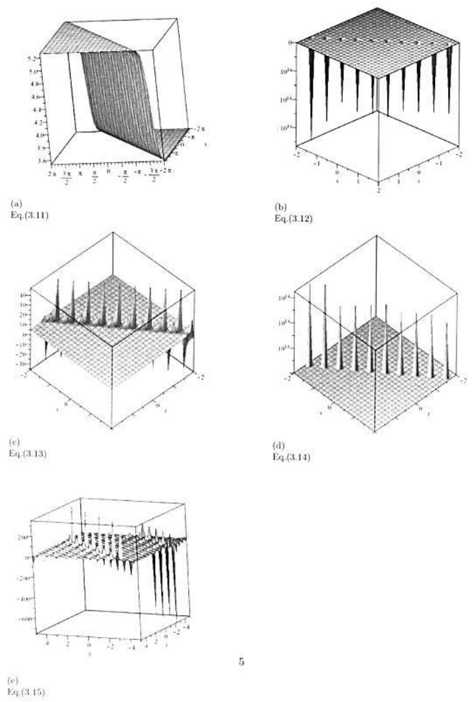


Figure 1: solution of Eqs. (3.11)-(3.15)

3.2- Example 2: The Space-Time Nonlinear Fractional SRLW Equation

This equation is well-known [27] and has the form:

$$D_t^{2\alpha}u + D_x^{2\alpha}u + uD_t^{\alpha}(D_x^{\alpha}u) + D_t^{\alpha}uD_t^{\alpha}u + D_t^{2\alpha}(D_x^{2\alpha}u) = 0, \quad (3.16)$$

where $0 < a \leq$ I. Eq.(3.16) has been investigated in [27] using the fractional sub-equation method. Let us now solve Eq.(3.16) using the proposed method of Sec. 2. To this end, we use the nonlinear fractional complex transformation $u(x,y,t) = u(\xi)$,

$$\xi = \frac{kx^{\alpha}}{\Gamma(1+\alpha)} + \frac{ct^{\alpha}}{\Gamma(1+\alpha)} + \xi_0,$$

where k, c, ξ_0 are constants, to reduce Eq.(3.16) to the following ODE with integer order:

$$k^2c^2u'' + (k^2+c^2)u + \frac{kc}{2}u^2 = 0. \quad (3.17)$$

Balancing u'' with $u^2 \Rightarrow N + 2 = 2N \Rightarrow N = 2$. Consequently, Eq.(3.17) has the formal solutions:

$$U = a_0 + a_1\exp(-\varphi(\xi)) + a_2\exp(-2\varphi(\xi)), \quad (3.18)$$

where a_0, a_1, a_2 are constants to be determined later, such that, $a_2 \neq 0$. It is easy to see that

$$u'' = 2\frac{a_1}{(e^{\phi(\xi)})^3} + 2\frac{a_1\mu}{e^{\phi(\xi)}} + 3\frac{a_1\lambda}{(e^{\phi(\xi)})^2} + a_1\lambda\mu + \frac{a_1\lambda^2}{e^{\phi(\xi)}} + 6\frac{a_2}{(e^{\phi(\xi)})^4} + 8\frac{a_2\mu}{(e^{\phi(\xi)})^2}$$

$$+10\frac{a_2\lambda}{(e^{\phi(\xi)})^3} + a_2\mu^2 + 6\frac{a_2\mu\lambda}{e^{\phi(\xi)}} + 4\frac{a_2\lambda^2}{(e^{\phi(\xi)})^2} \quad (3.19)$$

Substituting (3.18) along (3.19) into (3.17), collecting all the terms of the same order $exp(-i\varphi(\xi))$

, i = 0, 1, 2, .. and setting each coefficient to zero, we have the following set of algebraic equation:

$$6k^2c^2a_2 + \frac{1}{2}kc\,a_2^2 = 0, \quad (3.20)$$

$$k^2c^2(2a_1 + 10a_2\lambda) + kc\,a_1a_2 = 0, \quad (3.21)$$

$$k^2c^2(8a_2\mu + 3a_1\lambda + 4a_2\lambda^2) + \frac{1}{2}kc(a_1^2 + 2a_0a_2) + a_2(k^2 + c^2) = 0, \quad (3.22)$$

$$k^2c^2(2a_1\mu + a_1\lambda^2 + 6a_2\lambda\mu) + kc\,a_0a_1 + a_1(k^2+c^2) = 0, \quad (3.23)$$

$$k^2c^2a_1\lambda\mu + 2k^2c^2\,a_2\mu^2 + \frac{1}{2}kc\,a_0^2 + a_0(k^2+c^2) = 0, \quad (3.24)$$

On solving these algebraic equation with aid of Maple or Mathematica we have

$$a_0 = \frac{c}{k} + \frac{k}{c} - 3kc\lambda^2, \quad ,a_1 = -12kc\lambda, \quad a_2 = -12kc,$$

So that the solution of Eq.(3.2)

$$u = \frac{c}{k} + \frac{k}{c} - 3kc\lambda^2 - 12kc\lambda\,exp(-\varphi(\xi)) - 12kc\,exp(-2\varphi(\xi)), \quad (3.25)$$

the solutions of ODE (3.2) are

i. when $\lambda^2 - 4\mu > 0$, $\mu \neq 0$,

$$u = \frac{c}{k} + \frac{k}{c} - 3kc\lambda^2 - 3kc\left(\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\,tanh\left(\frac{\sqrt{\lambda^{2-4\mu}}}{2}(\xi+C_1)\right)-\lambda}\right) - 12kc\left(\frac{2\mu}{-\sqrt{\lambda^2-4\mu}\,tan\,h\left(\frac{\sqrt{\lambda^{2-4\mu}}}{2}(\xi+C_1)\right)-\lambda}\right)^2, \quad (3.26)$$

ii. when $\lambda^2 - 4\mu > 0$, $\mu = 0$,

$$u = \frac{c}{k} + \frac{k}{c} - 3kc\lambda^2 - 3kc\left(\frac{\lambda}{exp(\lambda(\xi+C_1)-1)}\right) - 12kc\left(\frac{\lambda}{exp(\lambda(\xi+C_1)-1)}\right)^2 \quad (3.27)$$

iii. when $\lambda^2 - 4\mu = 0$, $\mu \neq 0$, $\lambda \neq 0$,

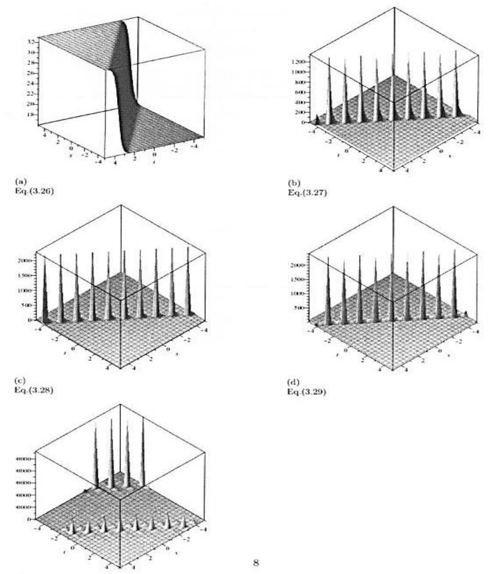
$$u = \frac{c}{k} + \frac{k}{c} - 3kc\lambda^2 - 3kc\left(\frac{\lambda^2(\xi+C_1)}{2(\lambda(\xi+C_1)+2)}\right) - 12kc\left(\frac{\lambda^2(\xi+C_1)}{2(\lambda(\xi+C_1)+2)}\right)^2 \quad (3.28)$$

iv. when $\lambda^2 - 4\mu = 0$, $\mu = 0$, $\lambda = 0$,

$$u = \frac{c}{k} + \frac{k}{c} - 3kc\lambda^2 - \frac{3kc\lambda}{\xi+C_1} - \left(\frac{12kc}{\xi+C_1}\right)^2, \quad (3.29)$$

when $\lambda^2 - 4\mu < 0$,

$$u = \frac{c}{k} + \frac{k}{c} - 3kc\lambda^2 - 3kc\lambda\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\,tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C_1)\right)-\lambda}\right) - 12kc\left(\frac{2\mu}{\sqrt{4\mu-\lambda^2}\,tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C_1)\right)-\lambda}\right)^2 \quad (3.30)$$



(a) Eq.(3.26) (b) Eq.(3.27) (c) Eq.(3.28) (d) Eq.(3.29)

3.3-Example 3: The Space-Time Nonlinear Fractional STO Equation

This equation is well-known [28] and has the form:

$$D_t^{\alpha}u + 3\beta(D_x^{\alpha}u)^2 + 3\beta u^2 D_x^{\alpha}u + 3\beta u D_x^{2\alpha}u + \beta D_x^{2\alpha}u = 0, \quad (3.31)$$

where $0 < a \leq 1$. Eq.(3.31) has been investigated in [28] using the fractional sub-equation method. Let us now solve Eq.(3.31) using the proposed method of Sec. 2. To this end, we use the nonlinear fractional complex transformation $u(x,y,t) = u(\xi)$,

$$\xi = \frac{kx^{\alpha}}{\Gamma(1+\alpha)} + \frac{ct^{\alpha}}{\Gamma(1+\alpha)} + \xi_0,$$

where $k,c,$£o are constants, to reduce Eq.(3.31) to the following ODE with integer order:

$$cu + 3\beta k^2 uu' + \beta k u^3 + \beta k^3 u'' = 0, \quad (3.32)$$

Balancing u'' with $u^3 \Longrightarrow N + 2 = 3N \Longrightarrow N = I.$ Consequently, Eq.(3.32) has the formal solutions:

$$u = a_0 + a_1 \exp(-\varphi(\xi)), \quad (3.33)$$

where a_0, a_1 are constants to be determined later, such that, $a_1 \neq 0$. It is easy to see that

$$u'' = 2\frac{a_1}{(e^{\phi(\xi)})^3} + 2\frac{a_1\mu}{e^{\phi(\xi)}} + 3\frac{a_1\lambda}{(e^{\phi(\xi)})^2} + a_1\lambda\mu + \frac{a_1\lambda^2}{e^{\phi(\xi)}}, \quad (3.34)$$

Substituting (3.33) along (3.34) into (3.32), collecting all the terms of the same order

, i = 0,1,2,.. and setting each coefficient to zero, we have the following set of algebraic equation:

$$2\beta k^3 a_1 + \beta k a_1^3 - 3\beta k^2 a_1^2 = 0, \quad (3.35)$$

$$3\beta k^3\lambda a_1 + 3\beta k a_0 a_1^2 - 3\beta k^2 a_0 a_1 - 3\beta k^2 \lambda a_1^2 = 0, \quad (3.36)$$

$$\beta k^3\lambda^2 a_1 + 2\beta k^3 \mu a_1 - 3\beta k a_0^2 a_1 - 3\beta k^2 \lambda a_0 a_1 - 3\beta k^2 \mu a_1^2 + a_1 c = 0, \quad (3.37)$$

$$\beta k^3 a_1 \lambda\mu + \beta k a_0^3 - 3\beta k^2 a_0 a_1 \mu + a_0 c = 0, \quad (3.38)$$

On solving these algebraic equation with aid of Maple or Mathematical we have

$$a_0 = \lambda k, \qquad a_1 = 2k,$$

So that the solution of Eq.(3.2)

$$u = \lambda k + 2k\, exp(-\varphi(\xi)), \quad (3.39)$$

the solutions of ODE (3.2) are

i. when $\lambda^2 - 4\mu > 0,\ \mu \neq 0,$

$$u = \lambda k + \left(\frac{4k\mu}{-\sqrt{\lambda^2-4\mu}\, tan\, h\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi + C_1)\right) - \lambda}\right) \quad (3.40)$$

ii. when $\lambda^2 - 4\mu > 0,\ \mu = 0,$

$$u = \lambda k - \left(\frac{4k\lambda}{exp\,(\lambda(\xi + C_1)) - 1}\right), \quad (3.41)$$

iii. when $\lambda^2 - 4\mu = 0,\ \mu \neq 0,\ \lambda \neq 0,$

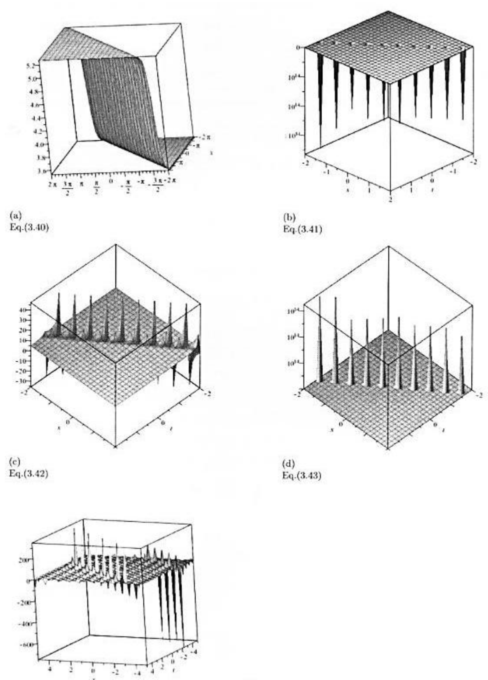
$$u = \lambda k + \left(-\frac{2\lambda^2(\xi + C_1)}{2(\lambda(\xi + C_1)) + 2}\right), \quad (3.42)$$

iv. when $\lambda^2 - 4\mu = 0,\ \mu = 0,\ \lambda = 0,$

$$u = \lambda k + \frac{2}{\xi + C_1}, \quad (3.43)$$

v. when $\lambda^2 - 4\mu < 0,$

$$u = \lambda k + \left(\frac{4\mu}{\sqrt{4\mu - \lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi + C_1)\right) - \lambda}\right) \quad (3.44)$$



3.4-Example 4: The Space-Time Nonlinear Fractional KPP Equation

This equation is well-known [29] and has the form:

$$D_t^{\alpha}u - D_x^{2\alpha}u + \mu_1 u + \gamma u^2 + \delta u^3 = 0, \quad (3.45)$$

where $0 < a < 1$. Eq.(3.45) has been investigated in [29] using the fractional sub-equation method. Let us now solve Eq.(3.45) using the proposed method of Sec. 2. To this end, we use the nonlinear fractional complex transformation $u(x,y,t) = u(\xi)$,

$$\xi = \frac{kx^{\alpha}}{\Gamma(1+\alpha)} + \frac{ct^{\alpha}}{\Gamma(1+\alpha)} + \xi_0,$$

where $k,c,\ \xi_o$ are constants, To reduce Eq.(3.16) to the following ODE with integer order:

$$k^2c^2u'' + (k^2 + c^2)u + \frac{kc}{2}u^2 = 0. \quad (3.46)$$

Balancing u'' with $u^3 \Longrightarrow N+2 = 3N \Longrightarrow N = 1$. Consequently, Eq.(3.17) has the formal

solutions:

$$u = a_0 + a_1 \exp(-\varphi(\xi)). \tag{3.47}$$

where a_o, a_1 are constants to be determined later, such that, $a_1 \neq 0$. It is easy to see that

$$u'' = 2\frac{a_1}{(e^{\phi(\xi)})^3} + 2\frac{a_{1\mu}}{e^{\phi(\xi)}} + 3\frac{a_1\lambda}{(e^{\phi(\xi)})^2} + a_1\lambda\mu + \frac{a_{1\lambda^2}}{e^{\phi(\xi)}}. \tag{3.48}$$

Substituting (3.47) along (3.48) into (3.46), collecting all the terms of the same order $exp(-i\varphi(\xi))$

, i = 0,1,2,.. and setting each coefficient to zero, we have the following set of algebraic equation:

$$\delta a_1^3 - 2k^2 a_1 = 0. \tag{3.49}$$

$$3\delta a_0 a_1^2 + \gamma a_1^2 - 3k^2\lambda a_1 - c a_1 = 0. \tag{3.50}$$

$$3\delta a_0^2 a_1 + 2\gamma a_0 a_1 + \mu a_1 - k^2\lambda^2 a_1 - 2k^2\mu a_1 - c\lambda a_1 = 0. \tag{3.51}$$

$$\delta a_0^3 + \gamma a_0^2 + \mu_1 a_0 - k^2\lambda\mu a_1 - c\mu a_1 = 0. \tag{3.52}$$

On solving these algebraic equation with aid of Maple or Mathematica we have

$$a_0 = \frac{\lambda a_1}{2} - \frac{\gamma}{2\delta}, \qquad a_1 = \frac{-2c}{\gamma},$$

So that the solution of Eq.(3.46)

$$u = \frac{\lambda a_1}{2} - \frac{\gamma}{2\delta} - \frac{2c}{\gamma}\exp(-\varphi(\xi)), \tag{3.52}$$

the solutions of ODE (3.46) are

i. when $\lambda^2 - 4\mu > 0, \mu \neq 0,$

$$u = \frac{\lambda a_1}{2} - \frac{\gamma}{2\delta} - \frac{-2c}{\gamma}\left(\frac{\mu}{-\sqrt{\lambda^2-4\mu}\, tan\, h\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+C_1)\right)-\lambda}\right) \tag{3.54}$$

ii. when $\lambda^2 - 4\mu > 0, \mu = 0,$

$$u = \frac{\lambda a_1}{2} - \frac{\gamma}{2\delta} - \frac{-2c}{\gamma}\left(\frac{2\lambda}{\exp(\lambda(\xi+C_1))-1}\right), \tag{3.55}$$

iii. when $\lambda^2 - 4\mu = 0, \mu \neq 0, \lambda \neq 0,$

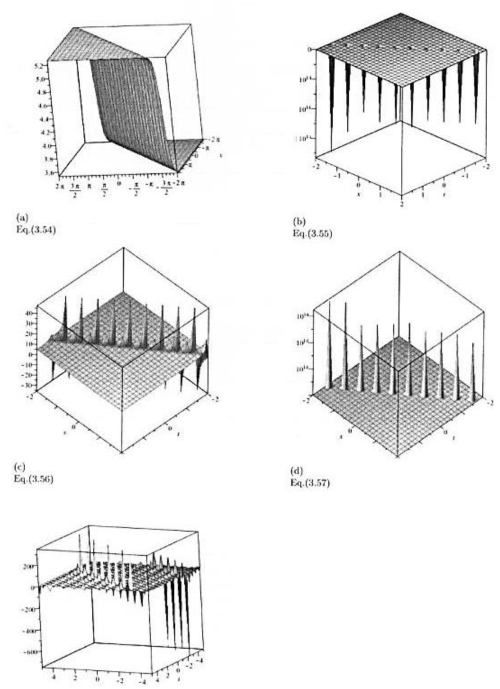
$$u = \frac{\lambda a_1}{2} - \frac{\gamma}{2\delta} - \frac{2c}{\gamma}\left(\frac{2\lambda^2(\xi+C_1)}{2(\lambda(\xi+C_1))+2}\right), \tag{3.56}$$

iv. when $\lambda^2 - 4\mu = 0, \mu = 0, \lambda = 0,$

$$u = \frac{\lambda a_1}{2} + \frac{-2c}{\gamma}\frac{\gamma}{2\delta} + \frac{2}{\xi+C_1}, \tag{3.57}$$

v. when $\lambda^2 - 4\mu < 0,$

$$u = \frac{\lambda a_1}{2} - \frac{\gamma}{2\delta} - \frac{2c}{\gamma}\left(\frac{\mu}{\sqrt{4\mu-\lambda^2}\, tan\left(\frac{\sqrt{4\mu-\lambda^2}}{2}(\xi+C_1)\right)-\lambda}\right) \tag{3.58}$$



4. CONCLUSION

We establish exact solutions for the space-time nonlinear fractional PKP equation, the space-time nonlinear fractional SRLW equation, the space-time nonlinear fractional STO equation and the space-time nonlinear fractional KPP equation with the exp(-$\varphi(\xi)$)-expansion method. The exp(-$\varphi(\xi)$)-expansion method has been successfully used to find the exact traveling wave solutions of some nonlinear evolution equations. As an application, the traveling wave solutions the space-time nonlinear fractional PKP equation, the space-time nonlinear fractional SRLW equation, the space-time nonlinear fractional STO equation and the space-time nonlinear fractional KPP equation , which have been constructed using The exp(-$\varphi(\xi)$)-expansion method. Let us compare between our results obtained in the present article with the well-known results obtained by other authors using different methods as follows: Our results of the space-time nonlinear fractional PKP equation, the space-time nonlinear fractional SRLW equation, the space-time nonlinear fractional STO equation and the space-time nonlinear fractional KPP equation , are new and different from those obtained in [27]-[29]. It can be concluded that this method is reliable and propose a variety of exact solutions NPDEs. The performance of this method is effective and can be applied to many other nonlinear evolution equations.

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