Improving the Security of Image Encryption by using Two Chaotic Maps

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ABSTRACT
During the recent years several chaotic image encryption algorithms based on one dimensional (1-*D*) have been proposed, but the drawbacks of small key space, low speed, and low security in 1-*D* chaotic cryptosystems are obvious. This paper proposes a new image encryption technique based on 1-*D* and 2-*D* chaotic maps which provide more security and big key space. The 1-*D* chaotic system is induced from a mathematical combination between the logistic map and the cubic chaotic map. It has the advantage of bigger key space and high security analysis such as key space analysis, statistical analysis and sensitivity analysis were carried out. The proposed system uses 1-*D* and 2-*D* chaotic maps for simultaneous encryption of the components *R, G, B* of the colored image; these three components affect each other. So, the security and confidentiality of image data become more important. The security of digital images and performance speed has become more important since the communications of digital information over network occur more frequently. An image encryption has applications in various fields for example; the internet communication, the multimedia systems, the medical imaging, Tele-medicine and the military communication. The image encryption's aim is to convert the original image to another image that is hard to understand. However, image encryption differs from text encryption due to some intrinsic features of images which include bulk data capacities, high redundancy, strong correlations among pixels, etc. These features make conventional cipher systems such as DES, AES and RSA unsuitable for practical image encryption [1]. The chaos theory is used in cryptography due to its intrinsic features. These properties of chaos includes: sensitivity to initial condition and control parameters, random like behavior and mixing property, etc [2]. Several chaos based image encryption approaches have been described. The first is the confusion approach where the position of pixels scrambled without changing the values of pixels, the application of this approach does not guarantee a good level of security [3]. The second approach is the diffusion which aims to change the value of each pixel in the whole image [4]. Other approach shuffles the positions of plain-image in spatial domain by cat map [5-7]. To enhance the security, the majority of methods propose to mix between more than one approach simultaneously. Some conditions should be fulfilled such as a large key space, randomness of the cipher-image and a high sensitivity on the initial conditions. In [8], an image encryption scheme based on improved 3-*D* cat map is proposed. Shuffling the positions with Henon map and

changing the grey values of image pixels with improved 2-*D* Logistic map are combined simultaneously to ensure the security of our scheme. New color image encryption algorithms encrypt the three components *R, G, B* of the color image. The common drawback of these algorithms is: they neglect the correlations between the components *R, G, B* and are more vulnerable to attack [9-15]. To overcome this problem, a novel color image encryption algorithm based on 1-*D* chaos map is described for the encryption *R, G and B* components of color image at the same time and made the three components affect each other [16].

In this paper, a new chaotic map which combines the logistic map and cubic map is used as a 1-*D* chaotic system to shuffle the position of blocks of the three components of image, on the other side the 2-*D* logistic map is used to rearrange the position of image pixels. These two chaotic systems are used to encrypt *R, G,* and *B* components of color image at the same time and made the three components affect each other. The current algorithm consists of 3 phases. Phase-1 creates a 1-*D* chaotic system, its initial values and scrambles the blocks of the image. Phase-2 uses a 2-*D* chaotic system to scrambling the rows and columns and increases the quadratic coupling of the items $y_i^2, x_i^2, x_i y_i$ this provides more security to the system. Phase-3 uses the 1-D chaotic system again to change the values of pixels.

The rest of this paper is organized as follows: section 2 describes the proposed chaotic system. Section 3 describes the proposed image encryption algorithm.section4 describes the image decryption algorithm. Section 5 experimental results and performance analysis are reported. Section 6 presents the conclusion.

1. THE PROPOSED SYSTEM
The proposed chaotic system uses 1-D formula (1). This formula is written as follows:

$$x_{n+1=}\mu\, x_n(1-x_n)(2+x_n) \tag{1}$$

When $1.41 < \mu < 1.59$, the system comes into chaotic state and generate a chaotic sequence in the region (0, 1] [17]. The 2-D logistic map formula is written by the following formula:

$$x_{i+1} = \mu_1 x_i(1-x_i) + \gamma_1 y_i^2 \tag{2}$$

$$y_{i+1} = \mu_2 y_i(1-y_i) + \gamma_2(x_i^2 + x_i y_i)$$

Formula (2) increases the quadratic coupling of the items y_i^2, x_i^2, $x_i y_i$ and provides more security to the system. When *2.75 < μ_1 < 3.4, 2.7 < μ_2 < 3.45, 0.15< γ_1 < 0.21*, and *0.13< γ_2 < 0.15*, the formula (2) comes into chaotic state and generates a chaotic sequence in the region (0, 1] [8].

2. IMAGE ENCRYPTION ALGORITHME

The color image P has the size $M \times N$, where M is the number of row and N is the number of column. Decompose the color image P into three matrixes R, G, and B. The proposed algorithm generates a total shuffling P-box to shuffle the position of blocks of the three components of image, rows and columns the shuffling P-box is described as the following:

Shuffling P-box (W, $formula(n)$)

Step-1.   For a given initial values in formula (n) generate the chaotic sequence $Sx_{mW} = \{x_1, x_2, \dots, x_m, x_{m+1}, \dots, x_{m+W}\}$ of length (m+W) by iterating formula for *(m+W)* times.

Step-2.   Delete the first m values of Sx_{mW}, the chaotic sequence is defined as $Sx_W = \{x_{m+1}, \dots, x_{m+W}\}$.

Step-3.   Sort the sequence Sx_{VU} and get the sequence $Sx'_W = \{x'_{m+1}, x'_{m+2}, \dots, x'_{m+W}\}$.

Step-4.   Determine the position of values Sx'_W in Sx_W and mark the transform map $Tx_W = \{t_1, t_2, \dots, t_W\}$ where the value x'_{m+i} is the same value x_{t_i}.

The proposed algorithm is divided into three phases. The first phase is the Block shuffling phase. The blocks of pixels are shuffled using 1-*D* chaotic formula (1). The second phase is the Rows and Column*s* scrambling phase. The pixels in rows and columns are scrambled using the 2-*D* logistic map formula (2). Finally; the third phase is the Pixels Diffusion phase. The values of pixels are changed using 1-*D* chaotic system formula (1).

2.1 Block shuffling

Each matrices R, G, and B of the image P was divided into # *(V, U)* of blocks which were combined in $P1 = [R_1, G_1, B_1]$, and converted into a one-dimensional vector V_{P1}. The position of these blocks of pixels was shuffled. Phase 1 can be described as follows:

Step-1.   Call the shuffling P-box($3VU$, $formula(1)$) and get Tx_{VU}

Step-2.   Rearrange the one dimensional vector V_{P1} according to Tx_{VU}

Decompose the one dimensional vector V_{P1} into three $U \times V$ matrices R_2, G_2 and B_2 then reconstruct the blocks in matrices R_2, G_2, and B_2 to create R_3, G_3 and B_3 matrices each of size $M \times N$.

2.2 Rows and Columns Scrambling

Determine a chaotic system for the formula (2) and its initial values $\mu_1, \mu_2, \gamma_1, \gamma_2, x_0$ and y_0. Rearrange the position of image pixels using the transform maps Tx_{MN} and Ty_{MN} according to formula (2).

Step-1.   Call the shuffling P-box($3MV$, $formula(2)$) and get Tx_{MN}, Ty_{MN}

Step-2.   Combine the three matrices R_3, G_3 and B_3 vertically and get matrix P_2 with *3M* rows and N columns.

Step-3.   Rearrange the rows of P_2 according to Tx_{MN}, which move the t_1 row to the first row, t_2 row to the second row, etc. until the rows have been moved and get transformed matrix P'_2.

Step-4.   Partition P'_2 into three matrixes R_4, G_4 and B_4 from top to dawn

Step-5.   Combine, R_4, G_4 and B_4 horizontally and get matrix P_3 with M rows and *3N* columns.

Step-6.   Rearrange the columns of P_3 according to Ty_{MN}, which move the t_1 column to the first column, t_2 column to the second column, etc. until the columns have been moved and get transformed matrix P'_3.

2.3 Pixels Diffusion

The values of pixels are changed after the pixel permuted. Given the initial parameters μ and x_0 this phase can be described as follows:

Step-1.   Use the 1-*D* chaotic system formula(1) to compute the chaotic sequence

- $YMN = \{y_1, y_2, \dots, y_{3*M*N}\}$

- $z_{1n} = mod(YNM*10^{14}, 3), (n=1,2,\dots,3MN)$

- $z_{2n} = mod(YNM10^{14}, 256), (n=1,2,\dots,3MN)$

Step-2.   Partition the transformed matrix P'_3 into three $M \times N$ matrices R_5, G_5 and B_5 from left to right.

Step-3.   Convert the components R_5, G_5 and B_5 into the vectors V_{Ri}, V_{Gi}, V_{Bi} resp. (i=1,2,…,M*N).

Step-4.   The diffusion formula is $C_{now} = (P_{now} + z_{2n} + C_{pre} + P_{pre})mod256$ where C_{now} is the current ciphered value, P_{now} is the current plain value, C_{pre} is the previous ciphered value and P_{pre} is the previous plain value, and the initial values are $C_0 = 0, P_0 = 0$.

Step-5.   Repeat

If $z_{1n} = 0$ then

Diffuse the current value of V_{Ri}

If all the values of V_{Ri} have changed then

$z_{1n} = 1$

Diffuse the current value of V_{Gi}

If all the values of V_{Gi} have changed then

$z_{1n} = 2$

Diffuse the current value of V_{Bi}

If all the values of V_{Bi} have changed then

$z_{1n} = 0$

Until all values of V_{Ri}, V_{Gi}, V_{Bi} is changed

Step-6. The resulted ciphered vectors are $V'_{Ri}, V'_{Gi}, V'_{Bi}$ where $(i = 1, 2, ..., M*N)$.

Step-7. The vectors $V'_{Ri}, V'_{Gi}, V'_{Bi}$ are converted into three components R, G, B of the ciphered image C.

3. IMAGE DECRYPTION

The decryption procedure is similar to that of the encryption process but in the reversed order. The initial parameters and values are as it is in encryption process, the study uses the same methods to obtain $Tx_{VU}, Tx_{MN}, Ty_{MN}, z_{1n},$ and z_{2n}.

Step-1. Perform the reverse operations to obtain the vectors V_{Ri}, V_{Gi}, V_{Bi} $(i=1,2,..,M*N)$ from C. The sequence z_{1n} is used to remove the effect of diffusion of the ciphered vectors $V'_{Ri}, V'_{Gi}, V'_{Bi}$ according to the following diffusion formula:

$$P_{now} = (C_{now} - z_{2n} - C_{pre} - P_{pre}) \bmod 256$$

Where $P_0 = 0$, $C_0 = 0$

Step-2. Perform the reverse operations to remove effect of rows and columns scrambling using the transform maps Tx_{MN} and Ty_{MN}

Step-3. Reverse the operations of the block shuffling phase using the transform maps Tx_{VU}.

Step-4. Reconstruct the one dimensional V_{p1} into three $M \times N$ matrices R_1, G_1 and B_1, then reconstruct the blocks in each matrix R_1, G_1 and B_1 to create R, G, B matrices of the original colored image P.

4. EXPERIMENTAL RESULTSE

A The System is implemented by MATLAB R2012a on Windows7 64-bit operating system with an Intel® core™i5-2430M CPU (2.40GHz) and 4GB of RAM. For the performance evaluation of the proposed method, 'pepper.jpg' image of size 256x256 have been used. The initial parameters and values are (μ=1.57, x_0 =0.12345678912345, μ_1 = 3.33, μ_2 = 3.34, γ_1 =0.17, γ_2 =0.14, x_{10} =0.23456789876543, y_{10} =0.12345 678987654, μ_3 =1.56, y_{20} =0.34567891234567). Figure 1(a) show the original color image. Figures2 (b)-(d) show the R, G, B components of the original image. Figures2 (a)-(d) shows the encrypted image and its encrypted components R, G, B.
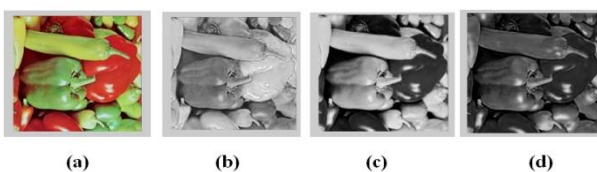

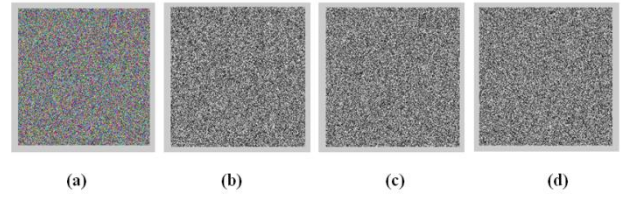**Fig1. Original image P and its R, G, B components**.

Fig2. Encrypted image P and its encrypted components R, G and B.

4.1 Key Space Analysis

The key space should be large enough to make brute-force attacks infeasible. The total number of the different keys which can be used in the encryption is defined as the size of the key space. The good encryption algorithm should be sensitive to the secret keys. In current work, the initial conditions and parameters μ, x_0, μ_1, μ_2, γ_1, γ_2, x_{10}, y_{10}, μ_3, y_{20}, are the secret key. If the precision is10^{-14}, the key space size reaches to 10^{140}, it is bigger than 2^{128}. So the key space is large enough to resist the brute-force attacks.

4.2 Key Sensitivity Analysis

A good encryption algorithm should be sensitive to the encryption keys in process of both encryption and decryption. When encrypt image, tiny change of keys receive two different cipher images and when decrypt image, if we use wrong key we receive different image. Figures 3(a)-(d) shows the decrypted image of pepper with the correct key μ_3=1.56.
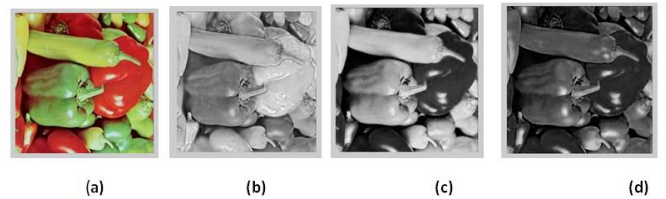

Fig3. Decrypted image and its components with correct parameters and its components.

Figure 4(a)-(d) shows the decrypted image of pepper with a wrong encryption key μ_3=1.56000000000001. So it can be concluded that the algorithm is sensitive to the key, a small change of the key will generate a completely different decryption result and cannot get the correct plain-image.
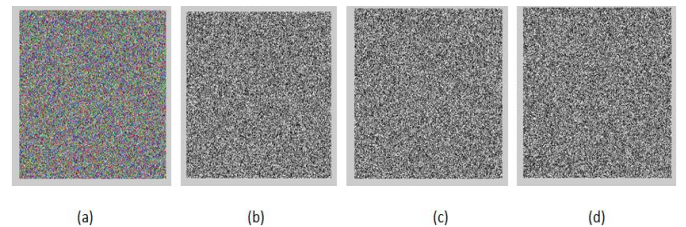

Fig4. Decrypted image with and its components incorrect parameters

4.3 Correlation Analysis

Randomly, select 3000 pairs of adjacent pixels in three directions (vertical, horizontal and diagonal) to test the correlation among pixels, and plot the corresponding distributions. The correction coefficient is calculated for each pair using the following formulas:

$$r_{xy} = \frac{cov(x,y)}{\sqrt{D(x)}\sqrt{D(y)}} \qquad (3)$$

$$cov(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - E(x))(y_i - E(y))$$

$$E(x) = \frac{1}{N}\sum_{i=1}^{N}(x_i)$$

$$E(y) = \frac{1}{N}\sum_{i=1}^{N}(y_i)$$

$$D(x) = \sum_{i=1}^{N}(x_i - E(x))^2$$

$$D(y) = \sum_{i=1}^{N}(y_i - E(y))^2$$

Where *x, y* are gray value of two adjacent pixels. The vertical and diagonal directions results are obtained, they are shown in Table 1. The correlation of two adjacent pixels in plain image closed to 1, while in ciphered image it closed to 0 which demonstrate that the image encryption could effectively resist statistical attack.

Table 1: The correlation test of the components *R, G, B* of the plain image and the encrypted image.

Image	Component	Plain image	Encrypted image
pepper	R	0.9602	0.0014
	G	0.9790	0.0009
	B	0.9557	0.00013

4.4 Information Entropy

The information entropy is defined to express the degree of uncertainties in the system [18].Using it to express uncertainties of the image information. The information entropy measures the distribution of gray value in image. If the uncertainty of image is greater, the entropy is bigger and the decrypting process of the image requires more information too. On the contrary, the more orderly of the encrypted image is the smaller of the information entropy. The information entropy equation is written as follows:

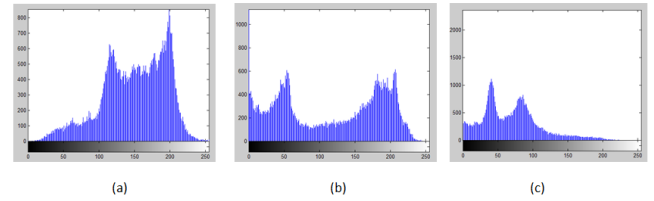$$H(m) = \sum_{i=0}^{M} P(m_i)\, log_2 1 \frac{1}{P(m_i)} \qquad (4)$$

Where *M* is the total number of symbols m_i, $P(m_i)$ represents the probability of occurrence of symbol m_i and the logarithm bases 2 so that the entropy is expressed in bits. The information entropy of an ideal random image is 8. Table 2 shows that the entropy of encryption image is very close to 8. This means that the encrypted images are close to a random source and the proposed algorithm is secure against the entropy attack.

Table2. Information entropy of original image and decrypted

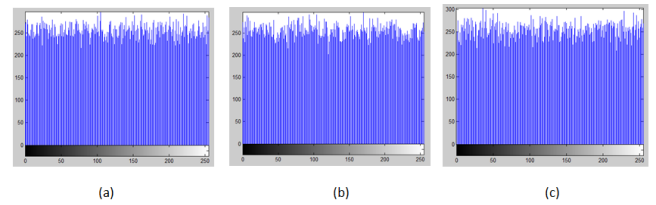| Image	Component	Plain image	Encrypted image
pepper	R	7.3921	7.9973
	G	7.6813	7.9973
	B	7.1756	7.9966

4.5 Histogram analysis

The image histogram illustrated how pixels in an image are distributed by graphing the number of pixels at each color intensity level. Figure 5(a)-(c) shows the histogram of plain components R, G, B.

(a)　　　　(b)　　　　(c)

Fig5. The histogram of original image's components R, G, and B. (a) The histogram of R component. (b) The histogram of G component. (c) The histogram of B component.

Figures 6(a)-(c) shows the histogram of encrypted components R, G, and B. The histogram of the cipher image is fairly uniform and significantly different from that of the original image

(a)　　　　(b)　　　　(c)

Fig 6: The histogram of the encrypted image's components R, G, and B. (a) The histogram of encrypted component R. (b) The histogram of encrypted component G (c) The histogram of encrypted component B.

4.6 NPCR and UACI Analysis

The measures NPCR and UACI test the different range between two images. NPCR stands for the changing rate of the number of pixels while one pixel of plain image is changed. UACI stands for the average intensity of the differences between the plain image and the cipher image. Calculate $NPCR_{R,G,B}$ and $UACI_{R,G,B}$ by using the following formulas:

$$NPCR_{R,G,B} = \frac{\sum_{ij} D_{R,G,B}(i,j)}{W \times H} \times 100\% \qquad (5)$$

$$UACI_{R,G,B} = \frac{1}{W \times H}\left[\frac{\sum_{ij}|C_{R,G,B}(i,j) - C'_{R,G,B}(i,j)|}{255}\right] \times 100\% \qquad (6)$$

Where W and H are the width and height of the plain image, $C_{R,G,B}(i,j)$ and $C'_{R,G,B}(i,j)$ are the *(i, j)* pixel values of the encrypted image before and after changing the value of one pixel the pixel *(i, j)* of the plain image respectively. The value $D_{R,G,B}(i,j)$ equals zero if $C_{R,G,B}(i,j) = C'_{R,G,B}(i,j)$, otherwise $D_{R,G,B}(i,j)$ equal one. In Table 3, $NPCR_{R,G,B}$ is over 99% and $UACI_{R,G,B}$ is over 33%. These results show that, the proposed algorithm is very sensitive to tiny changes in the plain image, even if there is a change in only one pixel, the decrypted images is completely different. Thus, the algorithm is robust against differential attack.

Table3. NPCR and UACI results

Image		Proposed algorithm		
		R	**G**	**B**
Pepper	NPCR %	99.6674	99.6338	99.5712
	UACI %	33.6665	33.5003	33.4723

4.7 PSNR analysis

The Peak Signal-to-Noise Ratio (PSNR) reflects the encryption quality. The Mean Square Error (MSE) is the cumulative squared error between the original and the decrypted image. The lower value of MSE means lesser error.

$$MSE_{R,G,B} = \sum_i \sum_j \frac{P_{R,G,B}(i,j) - C_{R,G,B}(i,j)}{MxN} \qquad (7)$$

$$PSNR_{R,G,B} = 20 * log10(\frac{255}{sqrt(MSE_{R,G,B})}) \qquad (8)$$

Where $P_{R,G,B}$ the original is image and $C_{R,G,B}$ is the encrypted image. The lower value of PSNR is the better encryption quality. The results are showed in table 4.

Table4. PSNR results

Image	Proposed algorithm		
	R	G	B
Pepper	9.0945	7.7804	7.7578

4.8 Randomness test

The security of cryptosystem has some properties such as good distribution, long period, high complexity and efficiency. In particular, the outputs of the cryptosystem must be unpredictable in the absence of knowledge of the inputs. Recently, the NIST designed a set of different statistical tests to test randomness of binary sequences produced by either hardware or software based cryptographic random or pseudorandom number generators [19]. These tests focus on a variety of different types of non-randomness that could exist in a sequence. We use the NIST tests suite in order to test the randomness of the proposed algorithm. In all tests if the computed P-value is less than 0.01, the sequence is non-random. Otherwise, the sequence is random. Table 5 depicts the NIST tests results and illustrates that the image sequences encrypted by the proposed system pass all the statistical tests with high P-values.

Table5. NIST Randomness Tests

Image	Test Name	P-value
Pepper	Approximate Entropy	0.817929
	Frequency	0.079125
	Block Frequency	0.968310
	Cumulative sums	0.149466
	FFT	0.257962
	Linear complexity	0.828726
	Run	0.840074
	Longest Run	0.166992
	Overlapping template	0.137277
	Rank	0.611852
	Serial P-vaue1	0.672378
	Serial P-value2	0.640758
	Universal	0.814821

5. CONCLUSION

In this paper, the security of the system is increased by using two chaotic systems. The proposed system consisted of three phases. In the first phase a new 1-*D* chaotic map is used to scramble the blocks of image. In the second phase a 2-*D* logistic map is used to shuffle the position in rows and columns. Finally the 1-*D* chaotic map is used again to change the values of the pixels. The simulation results show that the correlation coefficient between two adjacent pixels in the cipher image is far less than that of plain image; this indicates that the cipher image has a high performance of resisting statistical attack and high sensitive to security key. The key space of the proposed system is about 10^140, it is bigger than the key space of using only a 1-*D* chaotic map. The keys space is large enough to resist brute-force attack. 3D logistic map will be used in future work to increase the security of algorithm.

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