Soft Bitopological Spaces

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ABSTRACT

In this paper, introduce and study the concept of soft bitopological spaces which are defined over an initial universe with a fixed set of parameters. Also introduce and investigate some new separation axioms called pairwise soft T_0, pairwise soft T_1 and pairwise soft T_2 spaces and study some of their basic properties in soft bitopological spaces.

General Terms:

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Keywords:

Soft sets, Soft topology, Soft bitopology.

1. INTRODUCTION

In the year 1999, Russian researcher Molodtsov[6], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In 2003, Maji, Biswas and Roy[5], studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union and intersection were also defined. In 2005, D. Chen[2], presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory.

Topological structures of soft set have been studied by some authors in recent years. In 2011, M. Shabir et al. and Naim Cagman et al. initiated the study of soft topology and soft topological spaces independently. M. Shabir and M. Naz[7], introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. They introduced the definitions of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. Also they obtained some interesting results for soft separation axioms which are really valuable for research in this field. N. Cagman, S. Karatas and S. Enginoglu[1], defined the soft topology on a soft set and presented its related properties and foundations of the theory of soft topological spaces.

In 1963, J. C. Kelly[4], first initiated the concept of bitopological spaces. He defined a bitopological space (X, τ_1, τ_2) to be a set X equipped with two topologies τ_1 and τ_2 on X and initiated the systematic study of bitopological space. Later work done by C. W. Patty[8], I. L. Reilly[9] and others. Reilly discussed separation axioms properties in bitopological spaces.

The following definitions which are prerequisites for present study.

1.1 Definition[6]: Let U be an initial universe and E be a set of parameters. Let $P(U)$ denote the power set of U and A be a non-empty subset of E. A pair (F, A) is called a soft set over U, where F is a mapping given by $F : A \to P(U)$.

In other words, a soft set over U is a parametrized family of subsets of the universe U. For $\epsilon \in A$, $F(\epsilon)$ may be considered as the set of ϵ-approximate elements of the soft set (F, A). Clearly, a soft set is not a set.

1.2 Definition[5]: The complement of a soft set (F, A) is denoted by $(F, A)^c$ and is defined by $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\alpha)$ for all $\alpha \in \neg A$.

Let us call F^c to be the soft complement function of F. Clearly $(F^c)^c$ is the same as F and $((F, A)^c)^c = (F, A)$.

1.3 Definition[5]: A soft set (F, A) over U is said to be a $NULL$ soft set denoted by ϕ if for all $\epsilon \in A$, $F(\epsilon) = \phi$(null set).

1.4 Definition[5]: The union of two soft sets of (F, A) and (G, B) over the common universe U is the soft set (H, C), where $C = A \cup B$ and for all $e \in E$,

$$H(e) = \begin{cases} F(e) & if e \in A - B \\ G(e) & if e \in B - A \\ F(e) \cup G(e) & if e \in A \cap B \end{cases}$$

and is written as $(F, A) \cup (G, B) = (H, C)$

1.5 Definition[3]: The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe U, denoted $(F, A) \cap (G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

1.6 Definition[7]: Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(\alpha)$ for all $\alpha \in E$.

Note that for any $x \in X, x \notin (F,E)$ if $x \notin F(\alpha)$ for some $\alpha \in E$.

1.7 Definition[7]: Let Y be a non-empty subset of X, then Y denotes the soft set (Y,E) over X for which $Y(\alpha) = Y$, for all $\alpha \in E$.

1.8 Definition[7]: Let $x \in X$, then (x,E) denotes the soft set over X for which $x(\alpha) = \{x\}$, for all $\alpha \in E$.

1.9 Definition[7]: Let (F,E) be a soft set over X and Y be a non-empty subset of X. Then the sub soft set of (F,E) over Y denoted by $({}^YF,E)$, is defined as follows

${}^YF(\alpha) = Y \cap F(\alpha)$, for all $\alpha \in E$. In other words $({}^YF,E) = Y \cap (F,E)$.

1.10 Definition[7]: The relative complement of a soft set (F,A) is denoted by $(F,A)^c$ and is defined by $(F,A)^c = (F^c,A)$ where $F^c : A \to P(U)$ is a mapping gives by $F^c(\alpha) = U - F(\alpha)$ for all $\alpha \in A$.

1.11 Definition[7]: Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X, if
(1) ϕ, X belong to τ
(2) the union of any number of soft sets in τ belongs to τ
(3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X.

1.12 Definition[7]: Let (X,τ,E) be a soft topological space over X, then the members of τ are said to be soft open sets in X.

1.13 Definition[7]: Let (X,τ,E) be a soft topological space over X. A soft open set (F,E) over X is said to be a soft closed set in X, if its relative complement $(F,E)^c$ belongs to τ.

1.14 Definition[7]: Let (X,τ,E) be a soft topological space over X and Y be a non-empty subset of X. Then $\tau_Y = \{({}^YF,E) : (F,E) \in \tau\}$ is said to be the soft relative topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).

1.15 Definition[7]: Let (X,τ,E) be a soft topological space over X and $x,y \in X$ such that $x \neq y$.

i) If there exist soft open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ or $y \in (G,E)$ and $x \notin (G,E)$, then (X,τ,E) is called a soft T_0 space.

ii) If there exist soft open sets (F,E) and (G,E) such that $x \in (F,E)$ and $y \notin (F,E)$ and $y \in (G,E)$ and $x \notin (G,E)$, then (X,τ,E) is called a soft T_1 space.

iii) If there exist soft open sets (F,E) and (G,E) such that $x \in (F,E), y \in (G,E)$ and $(F,E) \cap (G,E) = \phi$, then (X,τ,E) is called a soft T_2 space.

1.16 Definition[4]: Let X be a non-empty set and τ_1 and τ_2 be two different topologies on X. Then (X,τ_1,τ_2) is called a bitopological space.

2. SOFT BITOPOLOGICAL SPACES

Let X be an initial universe set and E be the non-empty set of parameters.

2.1 Definition: Let (X,τ_1,E) and (X,τ_2,E) be the two different soft topologies on X. Then (X,τ_1,τ_2,E) is called a soft bitopological space.

The two soft topologies (X,τ_1,E) and (X,τ_2,E) are independently satisfy the axioms of soft topology. The members of τ_1 are called τ_1 soft open sets and the complements of τ_1 soft open sets are called τ_1 soft closed sets.

Similarly, The members of τ_2 are called τ_2 soft open sets and the complements of τ_2 soft open sets are called τ_2 soft closed sets.

Throughout this paper (X,τ_1,τ_2,E) denote soft bitopological space over X on which no separation axioms are assumed unless explicitly stated.

2.2 Example: Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ and $\tau_1 = \{\phi, X, (F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E)\}$ and $\tau_2 = \{\phi, X, (G_1,E), (G_2,E), (G_3,E), (G_4,E), (G_5,E), (G_6,E), (G_7,E)\}$, where $(F_1,E), (F_2,E), (F_3,E), (F_4,E), (F_5,E)$, $(G_1,E), (G_2,E), (G_3,E), (G_4,E), (G_5,E), (G_6,E), (G_7,E)$ are soft sets over X, defined as follows

$F_1(e_1) = \{h_2\}$ $F_1(e_2) = \{h_1\}$
$F_2(e_1) = \{h_2,h_3\}$ $F_2(e_2) = \{h_1,h_2\}$
$F_3(e_1) = \{h_1,h_2\}$ $F_3(e_2) = X$
$F_4(e_1) = \{h_1,h_2\}$ $F_4(e_2) = \{h_1,h_3\}$
$F_5(e_1) = \{h_2\}$ $F_5(e_2) = \{h_1,h_2\}$

$G_1(e_1) = \{h_1,h_2\}$ $G_1(e_2) = \{h_1,h_2\}$
$G_2(e_1) = \{h_2\}$ $G_2(e_2) = \{h_1,h_3\}$
$G_3(e_1) = \{h_2,h_3\}$ $G_3(e_2) = \{h_1\}$
$G_4(e_1) = \{h_2\}$ $G_4(e_2) = \{h_1\}$
$G_5(e_1) = \{h_1,h_2\}$ $G_5(e_2) = X$
$G_6(e_1) = X$ $G_6(e_2) = \{h_1,h_2\}$
$G_7(e_1) = \{h_2,h_3\}$ $G_7(e_2) = \{h_1,h_3\}$

Then (X,τ_1,τ_2,E), is a soft bitopological space.

2.3 Example: Let X be an initial universe set and E be the non-empty set of parameters. Soft indiscrete topology $\tau_1 = \{\phi, X\}$ and soft discrete topology τ_2 is the collection of all soft sets defined over X. Then (X,τ_1,τ_2,E) is a soft bitopological space.

2.4 Definition: Let (X,τ_1,τ_2,E) be a soft bitopological space over X and Y be a non-empty subset of X. Then $\tau_{1Y} = \{({}^YF,E) : (F,E) \in \tau_1\}$ and $\tau_{2Y} = \{({}^YG,E) : (G,E) \in \tau_2\}$ are said to be the relative topologies on Y. Then $\{Y,\tau_{1Y},\tau_{2Y},E\}$ is colled a relative soft bitopological space of (X,τ_1,τ_2,E).

2.5 Theorem: If (X,τ_1,τ_2,E) is a soft bitopological space then $\tau = \tau_1 \cap \tau_2$ is a soft topological space over X.

Proof: 1). ϕ, X belong to $\tau_1 \cap \tau_2 = \tau$
2). Let $\{(F_i,E) : i \in I\}$ be a family of soft sets in $\tau_1 \cap \tau_2 = \tau$.
Then $\{(F_i,E) \in \tau_1\}$ and $\{(F_i,E) \in \tau_2\}$ for all $i \in I$.
Therefore $\bigcup_{i\in I}(F_i,E) \in \tau_1$ and $\bigcup_{i\in I}(F_i,E) \in \tau_2$.
Thus $\bigcup_{i\in I}(F_i,E) \in \tau_1 \cap \tau_2 = \tau$.
3). Let $(F,E),(G,E) \in \tau_1 \cap \tau_2 = \tau$.
Then $(F,E),(G,E) \in \tau_1$ and $(F,E),(G,E) \in \tau_2$.
Since $(F,E) \cap (G,E) \in \tau_1$ and $(F,E) \cap (G,E) \in \tau_2$.
Therefore $(F,E) \cap (G,E) \in \tau_1 \cap \tau_2 = \tau$.
Thus $\tau_1 \cap \tau_2 = \tau$ defines a soft topology on X.

2.6 Remark: If (X,τ_1,τ_2,E) is a soft bitopological space then $\tau_1 \cup \tau_2$ is not a soft topological space over X.

2.7 Example: Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ and $\tau_1 = \{\phi, X, (F_1,E), (F_2,E), (F_3,E), (F_4,E)\}$ and $\tau_2 = \{\phi, X, (G_1,E), (G_2,E), (G_3,E), (G_4,E), (G_5,E)\}$

where $(F_1, E), (F_2, E), (F_3, E), (F_4, E), (G_1, E), (G_2, E),$ $(G_3, E), (G_4, E), (G_5, E)$ are soft sets over X, defined as follows

$F_1(e_1) = \{h_1, h_2\}$ $F_1(e_2) = \{h_2, h_3\}$
$F_2(e_1) = \{h_2\}$ $F_2(e_2) = \phi$
$F_3(e_1) = \{h_2\}$ $F_3(e_2) = \{h_3\}$
$F_4(e_1) = \{h_1, h_2\}$ $F_4(e_2) = \{h_2\}$

$G_1(e_1) = \{h_1, h_2\}$ $G_1(e_2) = \{h_2, h_3\}$
$G_2(e_1) = \phi$ $G_2(e_2) = \{h_2, h_3\}$
$G_3(e_1) = \{h_1, h_2\}$ $G_3(e_2) = \{h_3\}$
$G_4(e_1) = X$ $G_4(e_2) = \{h_2, h_3\}$
$G_5(e_1) = \phi$ $G_5(e_2) = \{h_3\}$

Then (X, τ_1, τ_2, E) is a soft bitopological space. Now define

$\tau = \tau_1 \cup \tau_2 = \{\phi, X, (F_1, E), (F_2, E), (F_3, E), (F_4, E), (G_1, E),$ $(G_2, E), (G_3, E), (G_4, E), (G_5, E)\}$.
If take $(F_2, E) \cup (G_2, E) = (H, E)$, then
$H(e_1) = F_2(e_1) \cup G_2(e_1) = \{h_2\} \cup \phi = \{h_2\}$ and
$H(e_2) = F_2(e_2) \cup G_2(e_2) = \phi \cup \{h_2, h_3\} = \{h_2, h_3\}$
but $(H, E) \notin \tau$. Thus τ is not a soft topology on X.

3. SEPARATION AXIOMS OF SOFT BITOPOLOGICAL SPACES

3.1 Definition: In a soft bitopological space (X, τ_1, τ_2, E)
i) τ_1 is said to be soft T_0 space w. r. t. τ_2 if for each distinct points x, y of X then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$.
Similarly τ_2 is said to be soft T_0 space w. r. t. τ_1 if for each distinct points x, y of X then there exist a τ_2 soft open set (F, E) and a τ_1 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$.
(X, τ_1, τ_2, E) is said to be pairwise soft T_0 space if τ_1 is soft T_0 space w. r. t. τ_2 and τ_2 is soft T_0 space w. r. t. τ_1.

ii) τ_1 is said to be soft T_1 space w. r. t. τ_2 if for each distinct points x, y of X then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$.
Similarly τ_2 is said to be soft T_1 space w. r. t. τ_1 if for each distinct points x, y of X then there exist a τ_2 soft open set (F, E) and a τ_1 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$.
(X, τ_1, τ_2, E) is said to be pairwise soft T_1 space if τ_1 is soft T_1 space w. r. t. τ_2 and τ_2 is soft T_1 space w. r. t. τ_1.

iii) τ_1 is said to be soft T_2 space w. r. t. τ_2 if for each distinct points x, y of X then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E), y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$.
Similarly τ_2 is said to be soft T_2 space w. r. t. τ_1 if for each distinct points x, y of X then there exist a τ_2 soft open set (F, E) and a τ_1 soft open set (G, E) such that $x \in (F, E), y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$.
(X, τ_1, τ_2, E) is said to be pairwise soft T_2 space if τ_1 is soft T_2 space w. r. t. τ_2 and τ_2 is soft T_2 space w. r. t. τ_1.

3.2 Thereom: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and Y be a non-empty subset of X. If (X, τ_1, τ_2, E) is pairwise soft T_0 space then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pairwise soft T_0 space.

Proof: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and $x, y \in Y$ such that $x \neq y$. If (X, τ_1, τ_2, E) is pairwise soft T_0 space, then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. Now $x \in Y$ and $x \in (F, E)$. Hence $x \in Y \cap (F, E) = ({}^Y F, E)$ where $(F, E) \in \tau_1$. Consider $y \notin (F, E)$ this means that $\alpha \in E$, then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$. Therefore $y \notin Y \cap (F, E) = ({}^Y F, E)$. Therefore τ_{1Y} is soft T_0 space w. r. t. τ_{2Y}.
Similarly it can be prove that τ_{2Y} is soft T_0 space w. r. t. τ_{1Y}, that is $y \in (G, E)$ and $x \notin (G, E)$ then $y \in ({}^Y G, E)$ and $x \notin ({}^Y G, E)$. Thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pairwise soft T_0 space.

3.3 Thereom: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and Y be a non-empty subset of X. If (X, τ_1, τ_2, E) is pairwise soft T_1 space then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pairwise soft T_1 space.

Proof: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and $x, y \in Y$ such that $x \neq y$. If (X, τ_1, τ_2, E) is pairwise soft T_1 space, then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Now $x \in Y$ and $x \in (F, E)$. Hence $x \in Y \cap (F, E) = ({}^Y F, E)$ where $(F, E) \in \tau_1$. Consider $y \notin (F, E)$ this means that $\alpha \in E$, then $y \notin Y \cap F(\alpha)$ for some $\alpha \in E$. Therefore $y \notin Y \cap (F, E) = ({}^Y F, E)$. Now $y \in Y$ and $y \in (G, E)$. Hence $y \in Y \cap (G, E) = ({}^Y G, E)$ where $(G, E) \in \tau_2$. Consider $x \notin (G, E)$ this means that $\alpha \in E$, then $x \notin Y \cap G(\alpha)$ for some $\alpha \in E$. Therefore $x \notin Y \cap (G, E) = ({}^Y G, E)$. Therefore τ_{1Y} is soft T_1 space w. r. t. τ_{2Y}.
Similarly it can be prove that τ_{2Y} is soft T_1 space w. r. t. τ_{1Y}, that is $y \in (G, E)$ and $x \notin (G, E)$ then $y \in ({}^Y G, E)$ and $x \notin ({}^Y G, E)$. Thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pairwise soft T_1 space.

3.4 Thereom: Let (X, τ_1, τ_2, E) be a soft bitopological space over X. If (x, E) is a soft closed set in τ_2 for each $x \in X$ and (y, E) is a soft closed set in τ_1 for each $y \in X$ then (X, τ_1, τ_2, E) is pairwise soft T_1 space.

Proof: Suppose that for each $x \in X$, (x, E) is a soft closed set in τ_2 then $(x, E)^c$ is soft open set in τ_2. Let $x, y \in X$ such that $x \neq y$. For each $x \in X$, $(x, E)^c$ is a soft open set in τ_2 such that $y \in (x, E)^c$ and $x \notin (x, E)^c$.
Similarly for each $y \in X$, (y, E) is a soft closed set in τ_1 then $(y, E)^c$ is soft open set in τ_1 such that $x \in (y, E)^c$ and $y \notin (y, E)^c$. Thus (X, τ_1, τ_2, E) is pairwise soft T_1 space.

3.5 Thereom: Every pairwise soft T_1 space is pairwise soft T_0 space.

Proof: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and $x, y \in X$ such that $x \neq y$. If (X, τ_1, τ_2, E) is pairwise soft T_1 space. That is (X, τ_1, τ_2, E) is pairwise soft T_1 space if τ_1 is soft T_1 space w. r. t. τ_2 and τ_2 is soft T_1 space w. r. t. τ_1. If τ_1 is soft T_1 space w. r. t. τ_2, then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Obviously $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. Therefore τ_1 is soft T_0 space w. r. t. τ_2
Similarly if τ_2 is soft T_1 space w. r. t. τ_1, then there exist a τ_2 soft open set (F, E) and a τ_1 soft open set (G, E) such that $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Obviously $x \in (F, E)$ and $y \notin (F, E)$ or $y \in (G, E)$ and $x \notin (G, E)$. Therefore τ_2 is soft T_0 space w. r. t. τ_1. Thus (X, τ_1, τ_2, E) pairwise soft T_0 space.

3.6 Remark: The converse of the Theorem 3.5 is not true.

3.7 Example: Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau_1 = \{\phi, X, (F, E)\}$ and $\tau_2 = \{\phi, X\}$, where (F, E) is soft set over X, defined as follows

$$F(e_1) = \phi \; F(e_2) = \{h_1\}$$

Then (X, τ_1, τ_2, E) is a soft bitopological space. Also (X, τ_1, τ_2, E) is a pairwise soft T_0 space but which is not a pairwise soft T_1 space, because $h_1, h_2 \in X$ with $h_1 \neq h_2$ and there do not exist τ_1 soft open set (F_1, E) and τ_2 soft open set (G_1, E) such that $h_1 \in (F_1, E)$ and $h_2 \notin (F_1, E)$ and $h_2 \in (G_1, E)$ and $h_1 \notin (G_1, E)$.

3.8 Thereom: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and Y be a non-empty subset of X. If (X, τ_1, τ_2, E) is pairwise soft T_2 space then $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pairwise soft T_2 space.

Proof: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and $x, y \in Y$ such that $x \neq y$. If (X, τ_1, τ_2, E) is pairwise soft T_2 space, then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E), y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. So for each $\alpha \in E, x \in F(\alpha), y \in G(\alpha)$ and $F(\alpha) \cap G(\alpha) = \phi$. This implies that $x \in Y \cap F(\alpha), y \in Y \cap G(\alpha)$ and $F(\alpha) \cap G(\alpha) = \phi$. Hence $x \in ({}^Y F, E), y \in ({}^Y G, E)$ and $({}^Y F, E) \cap ({}^Y G, E) = \phi$, where $({}^Y F, E)$ is soft open set in τ_{1Y}, and $({}^Y G, E)$ is soft open set in τ_{2Y}. Therefore τ_{1Y} is soft T_2 space w. r. t. τ_{2Y}.
Similarly it can be prove that τ_{2Y} is soft T_2 space w. r. t. τ_{1Y}. Thus $(Y, \tau_{1Y}, \tau_{2Y}, E)$ is pairwise soft T_2 space.

3.9 Thereom: Every pairwise soft T_2 space is pairwise soft T_1 space.

Proof: Let (X, τ_1, τ_2, E) be a soft bitopological space over X and $x, y \in X$ such that $x \neq y$. If (X, τ_1, τ_2, E) is pairwise soft T_2 space. That is (X, τ_1, τ_2, E) is pairwise soft T_2 space if τ_1 is soft T_2 space w. r. t. τ_2 and τ_2 is soft T_2 space w. r. t. τ_1. If τ_1 is soft T_2 space w. r. t. τ_2, then there exist a τ_1 soft open set (F, E) and a τ_2 soft open set (G, E) such that $x \in (F, E), y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. Obviously $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Therefore τ_1 is soft T_1 space w. r. t. τ_2
Similarly if τ_2 is soft T_2 space w. r. t. τ_1, then there exist a τ_2-soft open set (F, E) and a τ_1-soft open set (G, E) such that $x \in (F, E), y \in (G, E)$ and $(F, E) \cap (G, E) = \phi$. Obviously $x \in (F, E)$ and $y \notin (F, E)$ and $y \in (G, E)$ and $x \notin (G, E)$. Therefore τ_2 is soft T_1 space w. r. t. τ_1. Thus (X, τ_1, τ_2, E) pairwise soft T_1 space.

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