A Novel Direct Relational Heuristic Algorithm of Possibilistic Clustering

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ABSTRACT
The paper deals with the problem of a heuristic approach to possibilistic clustering. The approach is based on the concept of allotment among fuzzy clusters. The paper provides the description of basic definitions of the heuristic approach to possibilistic clustering. A new parameter for algorithms is introduced and a new relational algorithm for unknown number of fuzzy clusters is proposed. An illustrative example of application of the proposed algorithm to Tamura's portrait data set is considered. Preliminary conclusions are formulated.

General Terms
Pattern Recognition, Clustering, Algorithms.

Keywords
Possibilistic Clustering, Fuzzy Tolerance, Fuzzy Cluster, Allotment, Cluster Size.

1. INTRODUCTION
1.1 Preliminaries
Clustering is a process aiming at grouping a set of objects into classes according to the characteristics of data so that objects within a cluster have high mutual similarity while objects in different clusters are dissimilar. In fuzzy clustering the data is not only partitioned in a number of clusters, but each object is assigned a degree of membership for each cluster.

Heuristic methods, hierarchical methods and objective function-based methods are main approaches in fuzzy clustering. In objective function-based clustering the mathematical model is stated in form of an objective function that evaluates the partition of data with respect to the membership degrees and the underlying similarity or dissimilarity measure. Different assumptions and constraints lead to a variety of basic clustering concepts. If the objective function is differentiable, necessary conditions for the membership degrees and other cluster parameters used in the distance or similarity measure can be derived in order to optimize the objective function. The resulting equations are then alternatively applied in an algorithm to determine the data fuzzy partition [1].

A possibilistic approach to clustering was proposed by Krishnapuram and Keller [2] and the approach can be considered as a special case of fuzzy approach to clustering because all methods of possibilistic clustering are objective function-based methods. On the other hand, constraints in the possibilistic approach to clustering are less strong than constraints in the fuzzy objective function-based approach to clustering and values of the membership function of a possibilistic partition can be considered as typicality degrees. So, the possibilistic approach to clustering is more general and flexible approach to clustering than the fuzzy approach.

Objective function-based approach in fuzzy clustering is most common and widespread approach. However, heuristic algorithms of fuzzy clustering display low level of complexity and high level of essential clarity. Some heuristic clustering algorithms are based on a definition of the cluster concept and the aim of these algorithms is cluster detection conform to a given definition. Such algorithms are called algorithms of direct classification or direct clustering algorithms.

1.2 A Heuristic Approach to Possibilistic Clustering
A heuristic approach to possibilistic clustering is proposed in [3]. The essence of the heuristic approach to possibilistic clustering is that the sought clustering structure of the set of observations is formed based directly on the formal definition of fuzzy cluster and possibilistic memberships are determined also directly from the values of the pairwise similarity of observations. A concept of the allotment among fuzzy clusters is basic concept of the approach and the allotment among fuzzy clusters is a special case of the possibilistic partition [2].

Direct heuristic algorithms of possibilistic clustering can be divided into two types: relational versus prototype-based. A fuzzy tolerance relation matrix is a matrix of the initial data for the direct heuristic relational algorithms of possibilistic clustering and a matrix of attributes is a matrix of the initial data for the prototype-based algorithms. In particular, the group of direct relational heuristic algorithms of possibilistic clustering includes

- D-AFC(c)-algorithm: using the construction of the allotment among given number c of partially separate fuzzy clusters;
- D-PAFC-algorithm: using the construction of the principal allotment among an unknown minimal number of at least c fully separate fuzzy clusters;
- D-AFC-PS(c)-algorithm: using the partially supervised construction of the allotment among given number c of partially separate fuzzy clusters.

On the other hand, the family of direct prototype-based heuristic algorithms of possibilistic clustering includes

- D-AFC-TC-algorithm: using the construction of the allotment among an unknown number c of fully separate fuzzy clusters;
- D-PAFC-TC-algorithm: using the construction of the principal allotment among an unknown minimal number of at least c fully separate fuzzy clusters;
- D-AFC-TC(α)-algorithm: using the construction of the allotment among an unknown number c of fully separate

fuzzy clusters with respect to the minimal value α of the tolerance threshold.

The main goal of the paper is proposition of a new relational heuristic algorithm of possibilistic clustering based on the parameter that controls cluster sizes. So, the contents of this paper is as follows: in the second section basic concepts of the heuristic approach to possibilistic clustering are considered, in the third section a new parameter of clustering is introduced and a plan of the new algorithm is described, in the fourth section an illustrative example of application of the proposed algorithm to Tamura's portrait data set [4] is given, in the fifth section some final remarks are stated.

2. BASIC DEFINITIONS

Basic concepts of the heuristic method of possibilistic clustering [3] must be reminded before introducing a new parameter of classification and describing a detail plan of a new clustering procedure.

Let $X = \{x_1, \ldots, x_n\}$ be the initial set of objects. Let T be a fuzzy tolerance on X and α be α-level value of T, $\alpha \in (0,1]$. Columns or lines of the fuzzy tolerance matrix are fuzzy sets $\{A^1, \ldots, A^n\}$. Let $\{A^1, \ldots, A^n\}$ be fuzzy sets on X, which are generated by a fuzzy tolerance T. The α-level fuzzy set $A^l_{(\alpha)} = \{(x_i, \mu_{A^l}(x_i)) \mid \mu_{A^l}(x_i) \geq \alpha\}$, $l \in [1,n]$ is fuzzy α-cluster or, simply, fuzzy cluster. So $A^l_{(\alpha)} \subseteq A^l$, $\alpha \in (0,1]$, $A^l \in \{A^1, \ldots, A^n\}$ and μ_{li} is the membership degree of the element $x_i \in X$ for some fuzzy cluster $A^l_{(\alpha)}$, $\alpha \in (0,1]$, $l \in [1,n]$. Value of α is the tolerance threshold of fuzzy clusters elements.

The membership degree of the element $x_i \in X$ for some fuzzy cluster $A^l_{(\alpha)}$, $\alpha \in (0,1]$, $l \in [1,n]$ can be defined as a

$$\mu_{li} = \begin{cases} \mu_{A^l}(x_i), & x_i \in A^l_\alpha \\ 0, & otherwise \end{cases}, \tag{1}$$

where an α-level $A^l_\alpha = \{x_i \in X \mid \mu_{A^l}(x_i) \geq \alpha\}$, $\alpha \in (0,1]$ of a fuzzy set A^l is the support of the fuzzy cluster $A^l_{(\alpha)}$. So, condition $A^l_\alpha = Supp(A^l_{(\alpha)})$ is met for each fuzzy cluster $A^l_{(\alpha)}$, $\alpha \in (0,1]$, $l \in [1,n]$. Membership degree can be interpreted as a degree of typicality of an element to a fuzzy cluster.

Let T is a fuzzy tolerance on X, where X is the set of objects, and $\{A^1_{(\alpha)}, \ldots, A^n_{(\alpha)}\}$ is the family of fuzzy clusters for some $\alpha \in (0,1]$. The point $\tau^l_e \in A^l_\alpha$, for which

$$\tau^l_e = \arg\max_{x_i} \mu_{li}, \ \forall x_i \in A^l_\alpha, \tag{2}$$

is called a typical point of the fuzzy cluster $A^l_{(\alpha)}$, $\alpha \in (0,1]$, $l \in [1,n]$. A fuzzy cluster $A^l_{(\alpha)}$ can have several typical points. That is why symbol e is the index of the typical point.

Let $R^\alpha_{c(z)}(X) = \{A^l_{(\alpha)} \mid l = \overline{1,c}, 2 \leq c \leq n, \alpha \in (0,1]\}$ be a family of fuzzy clusters for some value of tolerance threshold α, $\alpha \in (0,1]$, which are generated by some fuzzy tolerance T on the initial set of elements $X = \{x_1, \ldots, x_n\}$. If a condition

$$\sum_{l=1}^{c} \mu_{li} > 0, \ \forall x_i \in X \tag{3}$$

is met for all fuzzy clusters $A^l_{(\alpha)} \in R^\alpha_{c(z)}(X)$, $l = \overline{1,c}$, $c \leq n$, then the family is the allotment of elements of the set $X = \{x_1, \ldots, x_n\}$ among fuzzy clusters $\{A^l_{(\alpha)}, l = \overline{1,c}, 2 \leq c \leq n\}$ for some value of the tolerance threshold α. It should be noted that several allotments $R^\alpha_{c(z)}(X)$ can exist for some tolerance threshold α. That is why symbol z is the index of an allotment.

Allotment $R^\alpha_I(X) = \{A^l_{(\alpha)} \mid l = \overline{1,n}, \alpha \in (0,1]\}$ of the set of objects among n fuzzy clusters for some tolerance threshold $\alpha \in (0,1]$ is the initial allotment of the set $X = \{x_1, \ldots, x_n\}$. In other words, if initial data are represented by a matrix of some fuzzy T then lines or columns of the matrix are fuzzy sets $A^l \subseteq X$, $l = \overline{1,n}$ and α-level fuzzy sets $A^l_{(\alpha)}$, $l = \overline{1,c}$, $\alpha \in (0,1]$ are fuzzy clusters. These fuzzy clusters constitute an initial allotment for some tolerance threshold α and they can be considered as clustering components.

If some allotment $R^\alpha_{c(z)}(X) = \{A^l_{(\alpha)} \mid l = \overline{1,c}, c \leq n\}$ corresponds to the formulation of a concrete problem, then this allotment is an adequate allotment. In particular, if a condition

$$\sum_{l=1}^{c} card(A^l_\alpha) \geq card(X), \ \forall A^l_{(\alpha)} \in R^\alpha_z(X), \ \alpha \in (0,1],$$
$$card(R^\alpha_z(X)) = c, \tag{4}$$

and a condition

$$card(A^l_\alpha \cap A^m_\alpha) \leq w, \ \forall A^l_{(\alpha)}, A^m_{(\alpha)}, \ l \neq m, \ \alpha \in (0,1], \tag{5}$$

are met for all fuzzy clusters $A^l_{(\alpha)}$, $l = \overline{1,c}$ of some allotment $R^\alpha_{c(z)}(X) = \{A^l_{(\alpha)} \mid l = \overline{1,c}, c \leq n\}$ for a value $\alpha \in (0,1]$, then the allotment is the allotment among partially separate fuzzy clusters.

Several adequate allotments can exist. Thus, the problem consists in the selection of the unique adequate allotment $R^*_c(X)$ from the set B of adequate allotments, $B = \{R^\alpha_{c(z)}(X)\}$, which is the class of possible solutions of the concrete classification problem. The selection of the unique adequate allotment $R^*_c(X)$ from the set $B = \{R^\alpha_{c(z)}(X)\}$ of adequate allotments must be made on the basis of evaluation of allotments. The criterion

$$F(R^{\alpha}_{c(z)}(X),\alpha)=\sum_{l=1}^{c}\frac{1}{n_l}\sum_{i=1}^{n_l}\mu_{li}-\alpha\cdot c\,, \quad (6)$$

where c is the number of fuzzy clusters in the allotment $R^{\alpha}_{c(z)}(X)$ and $n_l = card(A^l_{\alpha})$, $A^l_{(\alpha)} \in R^{\alpha}_{c(z)}(X)$ is the number of elements in the support of the fuzzy cluster $A^l_{(\alpha)}$, can be used for evaluation of allotments. Maximum of criterion (6) corresponds to the best allotment of objects among c fuzzy clusters. So, the classification problem can be characterized formally as determination of the solution $R^{*}_{c}(X)$ satisfying

$$R^{*}_{c}(X)=\arg\max_{R^{\alpha}_{c(z)}(X)\in B} F(R^{\alpha}_{c(z)}(X),\alpha)\,. \quad (7)$$

The problem of cluster analysis can be defined in general as the problem of discovering the unique allotment $R^{*}_{c}(X)$, resulting from the classification process.

3. THE D-AFC(u)-ALGORITHM

Some new parameters for direct relational heuristic algorithms of possibilistic clustering were introduced in [4]. In particular, an analyst can determine the maximal number u of elements in a fuzzy cluster. If $1 \le u < n$ is a maximal number of elements in a fuzzy cluster, then $1 \le n_l \le u$, $\forall l=\overline{1,c}$, where $n_l = card(A^l_{\alpha})$, $A^l_{\alpha} = Supp\left(A^l_{(\alpha)}\right)$ for each fuzzy cluster $A^l_{(\alpha)}$, $l=\overline{1,c}$, $\alpha\in(0,1]$. So, parameter u can be considered as the parameter that controls cluster sizes. This natural idea was developed for the FCM-algorithm by Miyamoto, Ichihashi and Honda [5].

Thus, the classification problem can be formulated as follows: detection of an unknown number c of partially separated fuzzy clusters with given maximal number of elements $1 \le u < n$ in every class can be considered as the aim of classification. So, the corresponding D-AFC(u)-algorithm for detecting the allotment among fuzzy clusters with given maximal number of elements u in every class is an eleven-step procedure of classification.

1. Calculate α-level values of the fuzzy tolerance T and construct the ordered sequence $0<\alpha_0<\alpha_1<\ldots<\alpha_{\lambda}<\ldots<\alpha_Z\le 1$ of α-levels; set $\lambda := 0$;

2. Construct the initial allotment $R^{\alpha}_{I}(X)=\{A^l_{(\alpha)}\mid l=\overline{1,n}\}$, $\alpha=\alpha_{\lambda}$;

3. The following condition is checked:

   if for some fuzzy cluster $A^l_{(\alpha)} \in R^{\alpha}_{I}(X)$, $l\in\{1,\ldots,n\}$, $\alpha=\alpha_{\lambda}$ the condition $n_l = n$ is met

   then set $\lambda := \lambda+1$ and go to step 2;

4. Construct the set U of possible clustering components as follows:

   if for some fuzzy cluster $A^l_{(\alpha)} \in R^{\alpha}_{I}(X)$, $l\in\{1,\ldots,n\}$, $\alpha=\alpha_{\lambda}$ the condition $n_l \le u$ is met

   then $A^l_{(\alpha)} \in U$

   else $A^l_{(\alpha)} \notin U$;

5. The following condition is checked:

   if condition $card(U)\ge 2$ **and** condition $\sum_{l=1}^{c} card(A^l_{\alpha}) \ge card(X)$, $\forall A^l_{(\alpha)} \in U$, $\alpha=\alpha_{\lambda}$ are not met

   then set $\lambda := \lambda+1$ and go to step 2

   else go to step 6;

6. Set $w := 0$;

7. Check if it is possible to construct allotments $R^{\alpha}_{c(z)}(X)=\{A^l_{(\alpha)}\mid A^l_{(\alpha)}\in U\}$, $c \le n$, which satisfy conditions (4) and (5) for the value $\alpha=\alpha_{\lambda}$;

8. The following condition is checked:

   if allotments $R^{\alpha}_{c(z)}(X)$ satisfying conditions (4) and (5) are not constructed

   then set $w := w+1$ and go to step 7

   else go to step 9;

9. Construct the class of possible solutions of the classification problem $B(u)=\left\{R^{\alpha}_{c(z)}(X)\right\}$, $\alpha=\alpha_{\lambda}$ for all allotments $R^{\alpha}_{c(z)}(X)$ which were obtained on the step 7;

10. Calculate the value of the criterion (6) for every allotment $R^{\alpha}_{c(z)}(X)\in B(u)$;

11. The result $R^{*}_{c}(X)$ of classification is formed as follows:

    if for some unique allotment $R^{\alpha}_{c(z)}(X)\in B(u)$ the condition (7) is met

    then the allotment is the result of classification $R^{*}_{c}(X)$

    else **if** the condition $u>1$ is met

    then set $u := u-1$ and go to step 7.

The allotment $R^{*}_{c}(X)$ among unknown number c partially separate fuzzy clusters with determined sizes and the corresponding value of tolerance threshold $\alpha\in(0,1]$ are results of classification obtained from the D-AFC(u)-algorithm.

4. AN ILLUSTRATIVE EXAMPLE

Let us consider an application of the proposed D-AFC(u)-algorithm to the classification problem for the following illustrative example. The problem of classification of family portraits coming from three families was considered by Tamura, Higuchi and Tanaka in [6].

The number of portraits was equal to 16 and the real portrait assignment among three classes is presented in Figure 1.

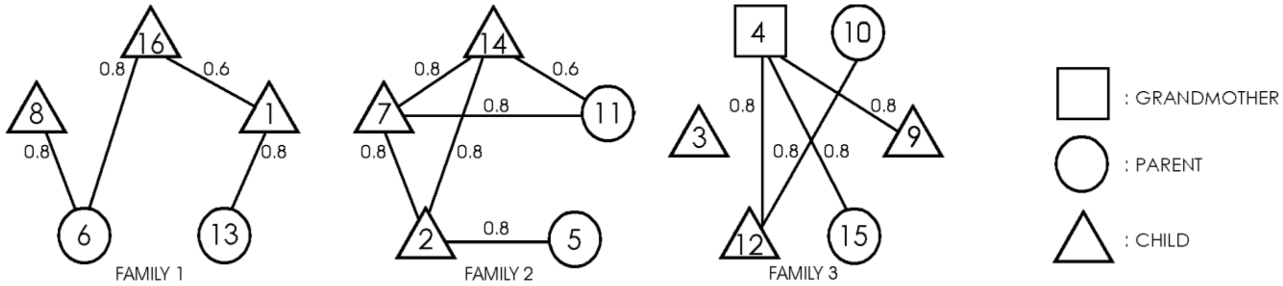

Fig 1: Real portraits classification

The data were originally analyzed in order to identify families with the technique of first transforming the matrix of a fuzzy tolerance into a matrix of a fuzzy similarity relation and then taking an appropriate α -level of the fuzzy similarity relation [6].

The partition proved to be obtained with α -level equal to 0.6. The partition identified the following three families $A^1 = \{x_1, x_6, x_8, x_{13}, x_{16}\}$, $A^2 = \{x_2, x_5, x_7, x_{11}, x_{14}\}$ and $A^3 = \{x_4, x_9, x_{10}, x_{12}, x_{15}\}$. However, person x_3 is not a member of any of the three families.

The subjective similarities assigned to the individual pairs of portraits collected in the tabular format are presented in Table 1.

Table 1. The matrix of subjective similarities

T	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
x_1	1.0															
x_2	0.0	1.0														
x_3	0.0	0.0	1.0													
x_4	0.0	0.0	0.4	1.0												
x_5	0.0	0.8	0.0	0.0	1.0											
x_6	0.5	0.0	0.2	0.2	0.0	1.0										
x_7	0.0	0.8	0.0	0.0	0.4	0.0	1.0									
x_8	0.4	0.2	0.2	0.5	0.0	0.8	0.0	1.0								
x_9	0.0	0.4	0.0	0.8	0.4	0.2	0.4	0.0	1.0							
x_{10}	0.0	0.0	0.2	0.2	0.0	0.0	0.2	0.0	0.2	1.0						
x_{11}	0.0	0.5	0.2	0.2	0.0	0.0	0.8	0.0	0.4	0.2	1.0					
x_{12}	0.0	0.0	0.2	0.8	0.0	0.0	0.0	0.0	0.4	0.8	0.0	1.0				
x_{13}	0.8	0.0	0.2	0.4	0.0	0.4	0.0	0.4	0.0	0.0	0.0	0.0	1.0			
x_{14}	0.0	0.8	0.0	0.2	0.4	0.0	0.8	0.0	0.2	0.2	0.6	0.0	0.0	1.0		
x_{15}	0.0	0.0	0.4	0.8	0.0	0.2	0.0	0.0	0.2	0.0	0.0	0.2	0.2	0.0	1.0	
x_{16}	0.6	0.0	0.0	0.2	0.2	0.8	0.0	0.4	0.0	0.0	0.0	0.0	0.4	0.2	0.0	1.0

The D-AFC(c)-algorithm was applied to the matrix of fuzzy tolerance for $u = 3,\ldots,6$. By executing the D-AFC(u)-algorithm for $u = 3$ the allotment $R_c^*(X)$ among $c = 8$ partially separated fuzzy clusters was obtained and the result obtained for the tolerance threshold $\alpha = 0.6$. The fourth portrait belongs to fourth and eighth fuzzy clusters. A matrix of memberships for eight fuzzy clusters is presented in Table 2.

Table 2. The matrix of memberships for eight partially separated fuzzy clusters

Number of fuzzy cluster	Objects															
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
1	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.8	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8
4	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	1.0	0.0	0.0	0.6	0.0	0.0
7	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0

After application of D-AFC(u)-algorithm to the matrix of fuzzy tolerance for $u = 4$ the allotment $R_c^*(X)$ among $c = 7$ partially separated fuzzy clusters was obtained for the tolerance threshold $\alpha = 0.6$. The twelfth portrait belongs to second and fifth fuzzy clusters. A matrix of memberships for seven fuzzy clusters is presented in Table 3.

Table 3. The matrix of memberships for seven partially separated fuzzy clusters

Number of fuzzy cluster	Objects															
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
1	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.8	0.0	0.0	0.8	0.0
3	0.0	0.8	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.8
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	1.0	0.0	0.0	0.6	0.0	0.0
7	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.0	0.0

By executing the D-AFC(u)-algorithm for $u = 5$, the allotment $R_c^*(X)$ among $c = 4$ partially separated fuzzy clusters which corresponds to the result, is obtained for the tolerance threshold $\alpha = 0.4$. The fourth portrait belongs to second and fourth fuzzy clusters. A matrix of memberships for four fuzzy clusters is presented in Table 4.

Table 4. The matrix of memberships for four partially separated fuzzy clusters

Number of fuzzy cluster	Objects															
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
1	0.5	0.0	0.0	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.8
2	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.4	0.8	0.0	1.0	0.0	0.0	0.0	0.0
3	0.0	0.8	0.0	0.0	0.4	0.0	0.8	0.0	0.0	0.0	0.6	0.0	0.0	1.0	0.0	0.0
4	0.0	0.0	0.4	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0

By executing the D-AFC(u)-algorithm for $u = 6$, the allotment $R_c^*(X)$ among $c = 4$ fully separated fuzzy clusters which corresponds to the result, is obtained for the tolerance threshold $\alpha = 0.4$. A matrix of memberships for four fuzzy clusters is presented in Table 5.

Table 5. The matrix of memberships for four fully separated fuzzy clusters

Number of fuzzy cluster	Objects															
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
1	0.5	0.0	0.0	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0	0.4	0.0	0.0	0.8
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0	0.8	0.0	0.0	0.0	0.0
3	0.0	1.0	0.0	0.0	0.8	0.0	0.8	0.0	0.4	0.0	0.5	0.0	0.0	0.8	0.0	0.0
4	0.0	0.0	0.4	0.8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0

It should be note, that the allotment $R_c^*(X)$ among four partially separated fuzzy clusters obtained from the D-AFC(u)-algorithm for $u = 5$ is differ from the allotment $R_c^*(X)$ among four fully separated fuzzy clusters obtained from the D-AFC(u)-algorithm for $u = 6$. On the other hand, the allotment $R_c^*(X)$ among four fully separated fuzzy clusters obtained from the D-AFC(u)-algorithm for $u = 6$ is equal to the principal allotment $R_P^{\alpha=0.4}(X)$ among four fully separated fuzzy clusters which was obtained from the D-PAFC-algorithm [3]. Membership functions of four fuzzy clusters obtained by using the D-AFC(u)-algorithm for $u = 6$ and the D-PAFC-algorithm are shown in Figure 2.

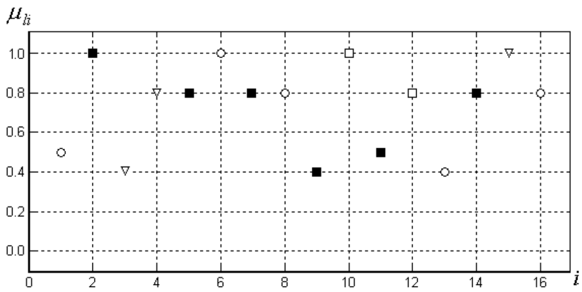

Fig 2: Membership functions of four fully separated fuzzy clusters

So, by executing the D-AFC(u)-algorithm for $u = 6$, the following result is obtained: the first class is composed of 5 elements, all belonging to Family 1; the second class contains 2 elements, all from Family 3; the third class is formed by 6 elements, where five elements correspond to Family 2 and one element corresponds to Family 3, and the fourth class consists of 3 elements, all belonging to Family 3. The sixth object is the typical point of the first class, the tenth object is the typical point of the second class, the second object is the typical point of the third class and the fifteen object is the typical point of the fourth class.

So, the union of the third and fourth classes is the class, which corresponds to Family 3 and there is one mistake of classification. The ninth element of the set of objects is the misclassified object. Membership functions of four classes are presented in Figure 2, where membership values of the first class are represented by ○, membership values of the second class are represented by □, membership values of the third class are represented by ■, and membership values of the fourth class are represented by ∇.

The result, obtained by using the D-AFC(u)-algorithm for $u = 6$ seems to be more appropriate, than results obtained for $u \in \{3, K, 5\}$ and the result seems to be satisfactory.

5. CONCLUDING REMARKS

The new direct relational heuristic D-AFC(u)-algorithm of possibilistic clustering is proposed in the paper. The D-AFC(u)-algorithm is based on the parameter u that controls cluster sizes, in other words, cluster volumes. The criterion (6) does not depends directly on the parameter u. The class of possible solutions of the classification problem depends on the value of parameter u in a concrete case and a unique allotment must be selected from the class on the basis of the criterion (6) for every allotment from the class. So, the number of fuzzy clusters in the sought allotment $R_c^*(X)$ will be determined automatically.

On the other hand, the number of clusters c in the sought fuzzy partition is the parameter for the fuzzy c-means with a variable controlling cluster sizes which is considered by Miyamoto, Ichihashi and Honda [5]. So, the proposed D-AFC(u)-algorithm has the advantage over the corresponding FCMA-algorithm [5].

It should be noted that all direct relational heuristic algorithms of possibilistic clustering can be applied for classification the interval-valued data and the three-way data [3]. Moreover, the Mamdani-type fuzzy inference system can be generated on the basis of the clustering results in corresponding cases [7], [8], [9]. That is why the proposed D-AFC(u)-algorithm can be applied to designing fuzzy inference systems for corresponding cases of the training data set.

These perspectives for investigations are of great interest both from the theoretical point of view and from the practical one as well.

6. ACKNOWLEDGMENTS

This investigation was done in framework of a project of the scientific program “Monitoring-SG” of the Union State of Russia and Belarus. We are grateful to thank the anonymous referees for their valuable comments.

7. REFERENCES

[1] Höppner, F., Klawonn, F., Kruse, R. and Runkler, T. 1999. Fuzzy Cluster Analysis: Methods for Classification, Data Analysis and Image Recognition. Chichester: John Wiley & Sons.

[2] Krishnapuram, R. and Keller, J.M. 1993. A Possibilistic Approach to Clustering. IEEE Transactions on Fuzzy Systems. 1(1), 98-110.

[3] Viattchenin, D.A. 2013. A Heuristic Approach to Possibilistic Clustering: Algorithms and Applications. Heidelberg: Springer.

[4] Viattchenin, D.A., Damaratski, A. and Yaroma, A. 2014. New Parameters for Relational Heuristic Algorithms of Possibilistic clustering. Proceedings of the 12^{th} International Conference PRIP'2014, 295-299.

[5] Miyamoto, S., Ichihashi, H. and Honda K. 2008. Algorithms for Fuzzy Clustering: Methods in C-Means Clustering with Applications. Heidelberg: Springer.

[6] Tamura, S., Higuchi, S. and Tanaka, K. 1971. Pattern Classification Based on Fuzzy Relations. IEEE Transactions on Systems, Man, and Cybernetics. 1(1), 61-66.

[7] Viattchenin, D.A. 2010. Derivation of Fuzzy Rules from Interval-Valued Data. International Journal of Computer Applications. 7(3), 13-20.

[8] Viattchenin, D.A. 2012. On Constructing Fuzzy Classifiers from Interval-Valued Data in Case of Unstable Clustering Structure. Control Engineering and Applied Informatics. 14(4), 17-26.

[9] Kacprzyk, J., Owsinski, J.W., and Viattchenin, D.A. 2014. Extracting Fuzzy Classification Rules from Three-Way Data. Journal of Automation, Mobile Robotics and Intelligent Systems. 8(2), 47-57.