Effects of Variable Viscosity and Thermal Conductivity on Magnetohydrodynamic Forced Convective Boundary Layer Flow past a Stretching/Shrinking Sheet Prescribed with Variable Heat Flux in the Presence of Heat Source and Constant Suction

G.C. Hazarika
Department of Mathematics,
Dibrugarh University,Dibrugarh, Assam, India

Jadav Konch
Department of Mathematics,
Dibrugarh University,Dibrugarh, Assam, India

ABSTRACT
The aim of this paper is to analyze the effects of variable viscosity and thermal conductivity on magneto hydrodynamic forced convective boundary layer flow past a stretching/shrinking sheet prescribed with variable heat flux in the presence of heat source and constant suction. The fluid viscosity and thermal conductivity are assumed to be inverse linear functions of temperature. The boundary equations are transformed into ordinary differential equations with similarity transformations. The effects of viscosity variation parameter and thermal conductivity variation parameter on velocity profile and temperature profile are discussed numerically by solving the governing transformed ordinary differential equations with the help of Runge-Kutta shooting method and plotted graphically. Skin-friction coefficient and wall temperature are also explored for typical values of the parameter involved in the study.

Keywords
Variable viscosity, Variable thermal conductivity, skin-friction, stretching/shrinking sheet.

1. INTRODUCTION
Boundary layer behaviour over a continuous moving solid surface is an important of flow occurring in several engineering processes. The variation of viscosity and thermal conductivity of an ambient fluid is one of the thrust areas of current research. Such investigations find their application over a broad spectrum of science and engineering process, especially in the field of chemical engineering

Sakiadis[1] initiated the study of the boundary layer flow over a continuous solid surface moving with constant speed. The boundary layer problem considered by *Sakiadis*[2] differs from the classical boundary layer problem addressed by Blasius mainly due to the entrainment of the ambient fluid. Here the surface is assumed to be inextensible whereas most of the physical situations concern with extensible surfaces moving in a cooling liquid. *Crane*[3] was the first to consider the boundary layer behaviour over an extensible surface where the velocity of the surface varies linearly with the distance from the slit. The linear stretching problem for hydromagnetic case was studied by *Chakrabarti and Gupta*[4]. The effects of variable surface temperature and variable surface heat flux over the heat transfer characteristics of a continuous linear stretching surface was investigated by *Chen and Char*[5].

Thermal boundary layer on a power law stretched surface with suction or injection was investigated by *Ali*[6]. *Elbashbeshy*[7] examined the heat transfer over a stretching surface with variable surface heat flux. *Liao*[8] obtained a new branch of solution of boundary layer flow over a permeable stretching plate. The micropolar transport phenomena over a stretching sheet were discussed by *Bhargava et al*. [9]. MHD flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption was studied by *Khedr et al*. [10]. Dissipation effects on nonlinear MHD flow over a stretching surface with prescribed heat flux was examined by *Anjali Devi and Ganga* [11]. Radiative MHD flow over a non-isothermal stretching sheet in a porous medium was investigated by *Paresh Vyas* and *Nupur Srivastava* [12]. *Azeem Shahzad et al*. [13] presented the exact solution for axisymmetric flow and heat transfer over a nonlinear radially stretching sheet.

The problem in the reverse case i.e., very little is known about the shrinking sheet where the velocity on the boundary is towards the origin. For this flow configuration, the sheet is shrunk towards a slot and the flow is quite different from the stretching out case. It is also shown that mass suction is required to maintain the flow over a shrinking sheet. Literature survey indicates that the flow induced by a shrinking Sheet recently gains attention of modern researchers for its interesting characteristics.

Shrinking sheet is a surface which decreases in size to a certain area due to an imposed suction or external heat. One of the most common applications of shrinking sheet problems in industries and engineering is shrinking film. In packaging of bulk products, shrink film is very useful as it can be unwrapped easily with adequate heat. Shrinking problem can also be applied to study the capillary effects in smaller pores, the shrink-swell behaviour and the hydraulic properties of agricultural clay soils.

The existence and uniqueness of similarity solution of the equation for the flow due to a shrinking sheet with suction was established by *Miklavcic* and *Wang* [14]. MHD rotating flow of a viscous fluid over a shrinking surface was analyzed by *Sajid et al.* [15]. Closed form exact solution of MHD viscous flow over a shrinking sheet was examined by *Fang* and *Zhang* [16] without considering the heat transfer. The application of homotopy analysis method for MHD viscous flow over a shrinking sheet was examined by *Sajid* and *Hayat* [17]. An analytical solution for thermal boundary layer flow over a shrinking sheet considering prescribed wall temperature and prescribed wall heat flux cases was investigated by *Fang and Zhang* [18]. *Hayat et al*. [19] examined the analytical solution of shrinking flow of second grade fluid in a rotating frame. *Ali et al*. [20] presented MHD flow and heat transfer due to a permeable shrinking sheet of a viscous electrically conducting fluid with prescribed surface heat flux. Noor *et al*. [21] obtained simple non-perturbative solution for MHD viscous flow due to a shrinking sheet. The effect of heat source/sink on MHD flow and heat transfer over a shrinking sheet with heat transfer over a shrinking sheet with mass suction for constant surface temperature was analysed by Bhattacharyya [22]. Das [23] investigated the effects of slip on MHD mixed convection stagnation flow of a micropolar fluid towards a shrinking vertical sheet.

Recently, Anjali Devi and Raj[24] have studied on the MHD forced convective boundary layer flow past a stretching/shrinking sheet prescribed with variable heat flux in the presence of heat source and constant suction.

In most of the studies, of this type of problems, the viscosity and thermal conductivity of the fluid were assumed to be constant. However it is known from the work of Herwig and Gerstem[25] that these properties may change with temperature, especially the fluid viscosity. When the effects of variable viscosity and thermal conductivity are taken into account, the flow characteristics are significantly changed compared to the constant property case. Hence, aim of the problem is to investigate the effects of varying viscosity and thermal conductivity on the MHD forced convective boundary layer flow past a stretching/shrinking sheet prescribed with variable heat flux in the presence of heat source and constant suction. Following Lai and Kulacki[26], the fluid viscosity and thermal conductivity are assumed to vary as an inverse linear functions of temperature. The governing boundary layer equations are transformed into dimensionless forms using suitable similarity transformations and then solved numerically for the prescribed boundary layer conditions using Runge-Kutta shooting method.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a steady, laminar, two-dimensional boundary layer flow of a viscous, incompressible, electrically conducting fluid due to a stretching/shrinking sheet subjected to suction in the presence of uniform transverse magnetic field. The velocity components u and v are taken in x and y directions respectively. A magnetic field of strength B_0 is applied normal to the boundary.

The analysis is based on the following assumptions:

(i) The fluid has constant physical properties, except for the fluid viscosity and thermal conductivity, which are assumed to be inverse linear functions of temperature.

(ii) The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible.

(iii) Since the induced magnetic field is assumed to be negligible and as B_0 is independent of time, *curl* $\vec{E} = \vec{0}$. Also *div* $\vec{E}$ = 0 in the absence of surface charge density. Hence $\vec{E} = \vec{0}$ is assumed.

(iv) The energy equation involves the heat source term and variable heat flux is prescribed at the stretching/shrinking surface.

(v) The effect of the viscous and joule dissipation are assumed to be negligible in the energy equation.

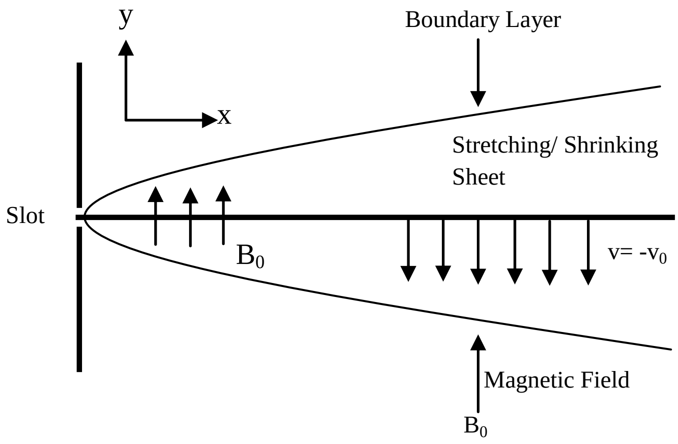

Fig.1: Schematic diagram of the problem

Under the above assumptions, the governing equations of the problem are given by:

The equation of continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{...(1)}$$

The momentum equation:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) - \frac{\sigma B_0^2}{\rho_\infty}u \quad \text{...(2)}$$

The energy equation:

$$\rho_\infty c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + Q\left(T - T_\infty\right) \quad \text{...(3)}$$

Where μ, k are the viscosity and thermal conductivity of the fluid, C_p is the specific heat at constant pressure, σ is the electrical conductivity, T is the temperature, ρ_∞ is the density of the fluid at infinity, Q is the internal heat generation.

Boundary conditions are:

at $y = 0$;

$$u = bx,\ v = -v_0,\ -k_\infty\frac{\partial T}{\partial y} = q_w = Dx^n \quad \text{...(4.a)}$$

as $y \to \infty$; $u = 0$, $T \to T_\infty$ ...(4.b)

Where k_∞ is the thermal conductivity of the fluid at infinity, q_w is the heat flux at the surface , D be a positive constant, b<0 is the shrinking constant and b>0 is the stretching constant.

The velocity components along the axes can be expressed as:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \qquad \text{...(5)}$$

where ψ is the stream function such that the continuity equation is satisfied.

Now we introduce the following similarity transformations with dimensionless stream function $F(\eta)$ given by:

$$\psi(x,y) = \sqrt{a\upsilon_\infty}\, xF(\eta),$$

$$T - T_\infty = \frac{Dx_\infty}{k_\infty}\sqrt{\frac{\upsilon_\infty}{a}}\,\theta(\eta), \quad \eta = y\sqrt{\frac{a}{\upsilon_\infty}} \qquad \text{...(6)}$$

where υ_∞ is the kinematic viscosity of the fluid at infinity.

From Eqs. (5) and (6), the velocity components become:

$$u = axF', \quad v = -\sqrt{a\upsilon_\infty}\,F \qquad \text{...(7)}$$

Where the prime (') denotes derivative with respect to η.

It is known that, viscosity and thermal conductivity are inverse linear functions of temperature, following Lai and Kulacki[25], we assume

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \delta(T - T_\infty)\right] \qquad \text{...(8)}$$

or, $\frac{1}{\mu} = \alpha(T - T_r)$,

where $\alpha = \frac{\delta}{\mu_\infty}$ and $T_r = T_\infty - \frac{1}{\delta}$

And

$$\frac{1}{k} = \frac{1}{k_\infty}\left[1 + \gamma(T - T_\infty)\right] \qquad \text{...(9)}$$

or, $\frac{1}{k} = \beta(T - T_c)$,

where $\beta = \frac{\gamma}{k_\infty}$ and $T_c = T_\infty - \frac{1}{\gamma}$

Where α, β, T_r and T_c are constants and their values depend on the reference state and thermal properties of the fluid i.e., υ (kinematic viscosity) and k (thermal conductivity). In general α >0 for liquids and α <0 for gases. The non-dimensional form of viscosity and thermal conductivity parameters θ_r and θ_c can be written as,

$$\theta_r = \frac{T_r - T_\infty}{\frac{Dx^n}{k_\infty}\sqrt{\frac{\upsilon_\infty}{a}}} \quad \text{and} \quad \theta_c = \frac{T_c - T_\infty}{\frac{Dx^n}{k_\infty}\sqrt{\frac{\upsilon_\infty}{a}}} \qquad \text{...(10)}$$

Substituting Eqs. (6)-(10) with the boundary conditions given in Eqs.(4.a)-(4.b) in the Eqs. (2)-(3), we have

$$\frac{\theta_r}{\theta - \theta_r}F''' - \frac{\theta_r}{(\theta - \theta_r)^2}\theta' F'' - FF'' + F'^2 + M^2 F' = 0 \qquad \text{...(11)}$$

$$\frac{\theta_c}{\theta - \theta_c}\theta'' - \frac{\theta_c}{(\theta - \theta_c)^2}\theta' - \Pr F\theta' + n\Pr F'\theta - \Pr B\theta = 0 \qquad \text{...(12)}$$

where $M^2 = \frac{\sigma B_0^2}{\rho_\infty a}$ is the magnetic parameter, $\Pr = \frac{\mu_\infty C_p}{\rho}$ is the Prandtl number, $B = \frac{Q}{a\rho C_p}$ is the heat source parameter, *n* is the heat flux parameter.

The boundary conditions with the new variables are:

$$F = S, \quad F' = \varepsilon, \quad \theta' = -1 \quad \text{at} \quad \eta = 0 \qquad \text{...(13.a)}$$

$$F' = 0, \quad \theta = 0 \quad \text{as} \quad \eta \to \infty \qquad \text{...(13.b)}$$

Where $S = \frac{v_0}{\sqrt{a\upsilon_\infty}}$, ($v_0$ >0) is the suction parameter and $\varepsilon = \frac{b}{a}$ is the stretching/shrinking parameter and ε >0 denotes the stretching sheet and ε <0 denotes the shrinking sheet.

The governing Eqns.(11)-(12) with boundary conditions given in Eqns.(13.a)-(13.b) are solved numerically by using the 4th order Runge-Kutta Shooting method.

3. SKIN FRICTION ANALYSIS

The parameters which we considered in this problem are very important in science and technology and the effect of the parameters plays a significant role on the skin friction coefficient C_f, which indicate physically wall shear stress. The shear stress at the wall is given by :

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}.$$

4. RESULTS AND DISCUSSION

The effects of variable viscosity and thermal conductivity on nonlinear MHD boundary flow and heat transfer over a stretching/shrinking surface with variable heat flux is presented. The viscosity-temperature variation and conductivity-temperature variation are represented by the dimensionless parameters θ_r and θ_c respectively.

The system of differential equations (11)-(12) governed by the boundary conditions are solved numerically by an efficient numerical technique based on the fourth order Runge-Kutta Shooting method. The numerical method can be programmed and applied easily. It is experienced that the convergence of the iteration process is quite rapid.

Numerical values of the solution are obtained by fixing various values for the physical parameters involved in the problem namely suction parameter *S*, magnetic parameter M^2, Prandtl number *Pr*, heat source parameter *B*, stretching/shrinking parameter *ε* and heat flux parameter *n*. The effect of pertinent parameters on the velocity, temperature, skin friction coefficient and wall temperature are presented. The skin friction at the plate and the wall temperature for prescribed wall heat flux case are found and shown by graph.

When constant viscosity and thermal conductivity are prescribed on the shrinking sheet the present results are identical to the results for the two dimensional case of *Anjali Devi and Raj*[24], which is shown in the Figs. (2– 4).

Further it is seen that when viscosity and thermal conductivity are taken as constant the results for the skin friction coefficient give us a good agreement with that of the result of Anjali Devi and Raj[24] for S=3, M^2 =2, B=0.05, Pr=0.71, ε = -1, n=2. This is noted through Table 1.

Table 1:

Present Study (Author's)	Anjali Devi and Raj[24]
3.302776	3.302775

The missing values are found for different parameters in the Tables (2-6). Tables(2-5) and in Table 6 show the values of $F''(0)$ and $\theta(0)$ for varying viscosity parameter θ_r and thermal conductivity θ_c with respect to parameter M^2 and stretching/shrinking parameter ε respectively for various values of *S, B, Pr, n.* In Table 2 and Table 3, it is observed that the missing values of $F''(0)$ increases as the values of θ_r and M^2 increases while that of $\theta(0)$ decreases. From the Table 4 and Table 5 it is seen that an increase in magnetic parameter M^2 missing values of $F''(0)$ increases while that of $\theta(0)$ decreases. On the other hand missing values of $F''(0)$ and $\theta(0)$ increases as θ_c increases. The study reveals that in case of a shrinking sheet skin friction coefficient increases as M^2, θ_r and θ_c increases while that of wall temperature decreases for increasing values of M^2 and θ_r, and increases for θ_c increases.

Table 6 depicts the effects of viscosity parameter and stretching/shrinking parameter on skin friction coefficient and wall temperature. It is cleared from the table that stretching/shrinking parameter retarded the skin friction coefficient and wall temperature. And in case of shrinking sheet for increasing values of θ_r skin friction coefficient increases while that of wall temperature decreases, whereas in case of stretching sheet reverse activity is shown.

Figs.(5-8) represent that with the increase in the value of θ_r longitudinal as well as transverse velocity profiles increases in a shrinking sheet, while that of Figs.(9-12) show that for increasing values of θ_r longitudinal and transverse velocity profiles decreases in a stretching sheet.

In Fig.13 and Fig.14 it is observed that, both the longitudinal and transverse velocity does not change significantly with the change in conductivity parameter θ_c.

Fig.15 shows that the effects of variation of thermal conductivity on the temperature profiles and it is seen that temperature of the fluid increases with increase in the value of θ_c.

The variation of missing values of $f''(0)$ are shown for various values of heat source parameter *B*, heat flux parameter *n* and suction parameter *S* against θ_r in the Figs.(16-20). And it is observed from Figs.(16-19) that the missing values of $f''(0)$ increases with the increase in the value of the parameters *B, n* and *S*. Fig.20 conveys the effect of Prandtl number *Pr* over $f''(0)$ and it is seen that as *Pr* increases, the $f''(0)$ decreases. From the unknown values of $f''(0)$ we can say about wall shear stress.

Table 2: Estimated missing values of $F''(0)$ and $\theta(0)$ for various θ_r, M_2 and θ_c= -10.00, Pr=0.71, S=3.00, n=2.00, ε = -1.00, B=0.05

$M^2 \rightarrow$ / $\theta_r \downarrow$	0		1		2		3		4	
	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)
2	1.600938	0.698207	1.988469	0.663358	2.276746	0.644527	2.516775	0.63184	2.726818	0.622391
5	2.230833	0.658044	2.612682	0.635487	2.90981	0.621874	3.161627	0.612236	3.384095	0.604844
8	2.378289	0.650863	2.760027	0.630142	3.059338	0.617408	3.31386	0.608312	3.539149	0.601297
11	2.444379	0.647842	2.826127	0.627865	3.126407	0.615495	3.38212	0.606626	3.608653	0.599769

Table 3: Estimated missing values of $F''(0)$ and $\theta(0)$ for various θ_r, M^2 and θ_c= -10.00, Pr=0.71, S=3.00 , n=2.00, ε = -1.00, B=0.05

$M^2 \rightarrow$ / $\theta_r \downarrow$	0		1		2		3		4	
	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)
-11	2.78863	0.633775	3.170769	0.617044	3.475897	0.606322	3.737573	0.598498	3.970349	0.592381
-8	2.851841	0.631457	3.234083	0.615228	3.54006	0.60477	3.802781	0.597116	4.036659	0.591121
-5	2.98963	0.626647	3.372111	0.611432	3.679884	0.601515	3.944832	0.594212	4.18106	0.588469

Table 4:Estimated missing values of $F''(0)$ and $\theta(0)$ for various θ_c, M^2 and θ_r = -10.00, Pr=0.71, S=3.00, n=2.00, ε = -1.00, B=0.05

$M^2 \rightarrow$ / $\theta_c \downarrow$	0		1		2		3		4	
	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)
2	2.761062	0.486794	3.146127	0.479159	3.452648	0.474099	3.715125	0.470321	3.948406	0.467315
5	2.784345	0.563573	3.168071	0.551609	3.474073	0.543821	3.736336	0.538078	3.96955	0.533553
8	2.790021	0.582243	3.173356	0.569084	3.479198	0.560555	3.741387	0.554283	3.974569	0.54935
11	2.792535	0.59051	3.17569	0.576809	3.481458	0.567944	3.743613	0.561432	3.976778	0.556316

Table 5: Estimated missing values of $F''(0)$ and $\theta(0)$ for various θ_c, M^2 and θ_r = -10.00, Pr=0.71, S=3.00 , n=2.00, ε = -1.00 , B=0.05

$M^2 \rightarrow$ / $\theta_c \downarrow$	0		1		2		3		4	
	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)
-11	2.804955	0.631285	3.187167	0.614819	3.492541	0.604248	3.754507	0.596526	3.987584	0.590485
-8	2.807058	0.638177	3.189101	0.621234	3.494405	0.610371	3.756338	0.602441	3.989398	0.596242
-5	2.811395	0.652382	3.193085	0.634456	3.498243	0.622988	3.760104	0.614632	3.993131	0.608106
-2	2.824592	0.695476	3.20516	0.674635	3.509859	0.66138	3.771502	0.651761	4.004428	0.644274

Table 6: Estimated missing values of $F''(0)$ and $\theta(0)$ for various θ_r, ε and θ_c = -10.00, M^2=2.00, Pr=0.71, S=3.00, n=2.00, B=0.05

$\varepsilon \rightarrow$ / $\theta_r \downarrow$	-1		-0.5		0.5		1	
	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)	F''(0)	θ(0)
-10	3.493045	0.605904	1.803389	0.534855	-1.9127	0.445234	-3.92922	0.414704
-8	3.54006	0.60477	1.82455	0.534518	-1.93091	0.445386	-3.96368	0.414927
-6	3.617959	0.602936	1.859713	0.533967	-1.96124	0.445638	-4.02107	0.415295
-4	3.772149	0.599457	1.929659	0.532903	-2.02181	0.446133	-4.13578	0.416024
-2	4.223585	0.590301	2.136758	0.52997	-2.20286	0.447563	-4.47933	0.418152

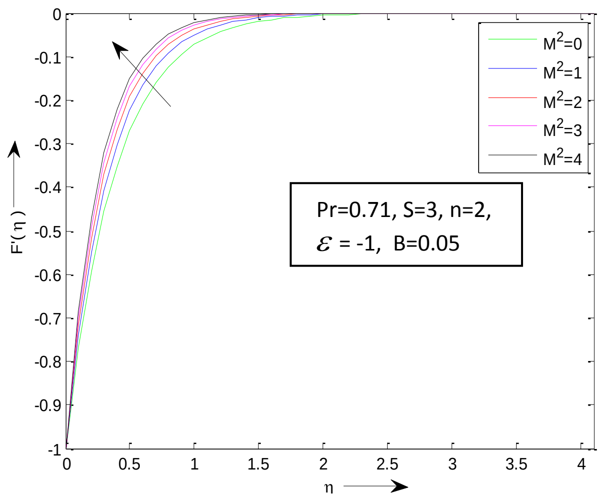

Fig. 2: Longitudinal velocity profiles for different M^2

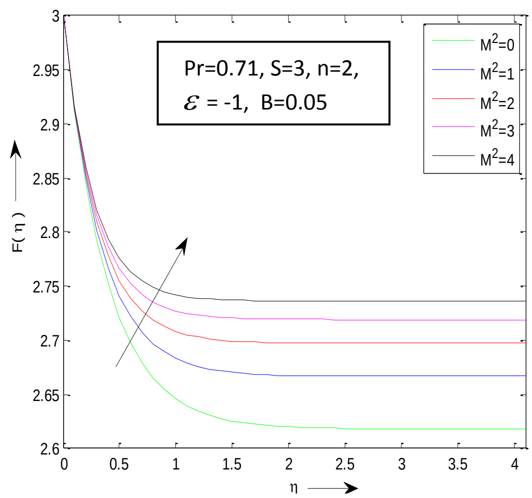

Fig. 3: Transverse velocity profiles for different M^2

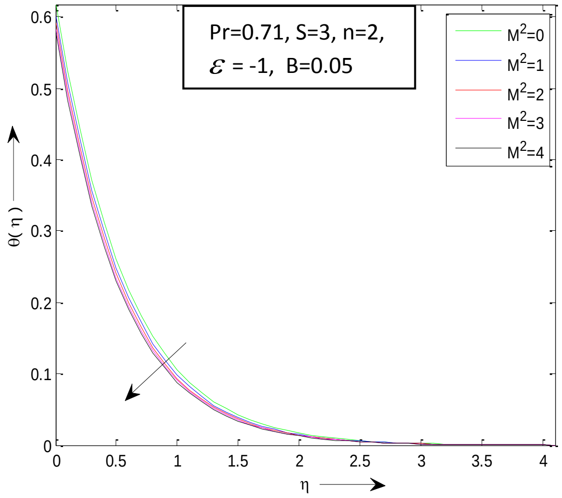

Fig. 4: Temperature profiles for different M^2

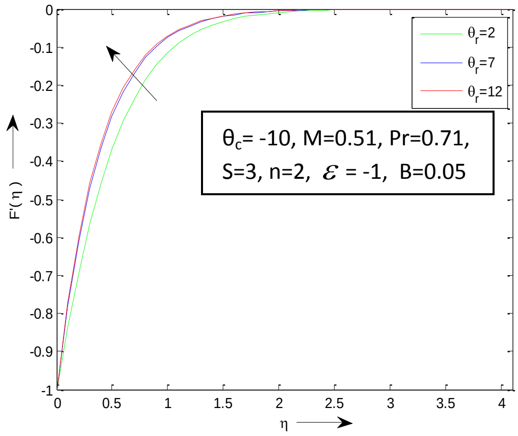

Fig. 5: Longitudinal velocity profiles for different positive values of θ_r

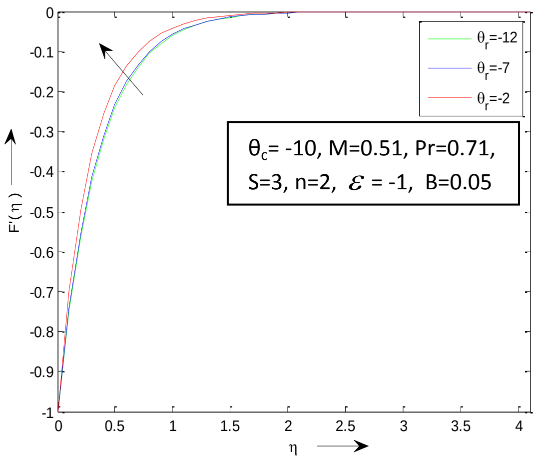

Fig. 6: Longitudinal velocity profiles for different negative values of θ_r

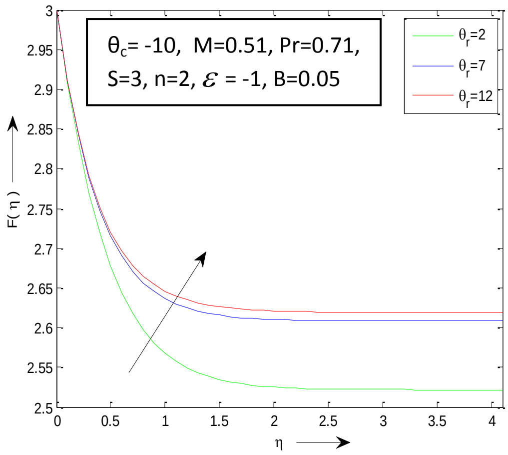

Fig. 7: Transverse velocity profiles for different positive values of θ_r

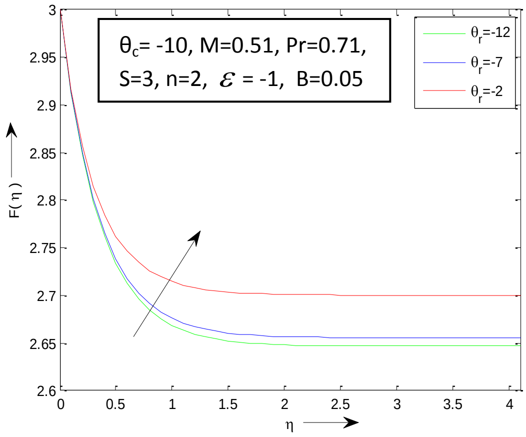

Fig. 8: Transverse velocity profiles for different negative values of θ_r

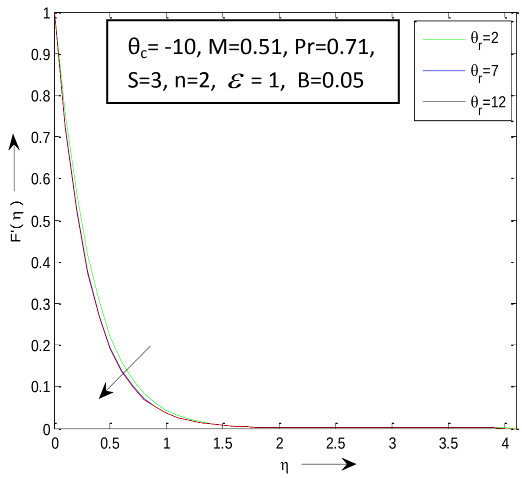

Fig. 9: Longitudinal velocity profiles for different positive values of θ_r

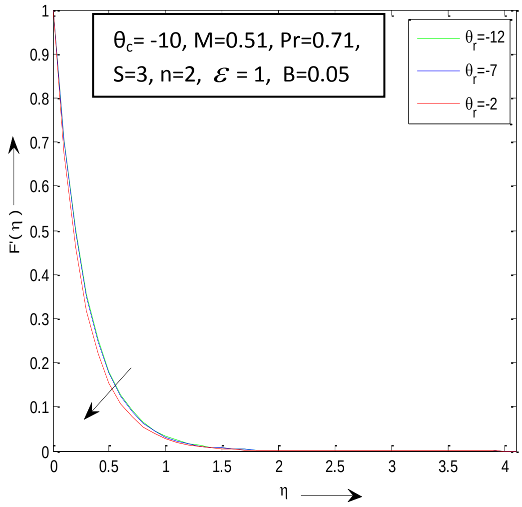

Fig. 10: Longitudinal velocity profiles for different negative values of θ_r

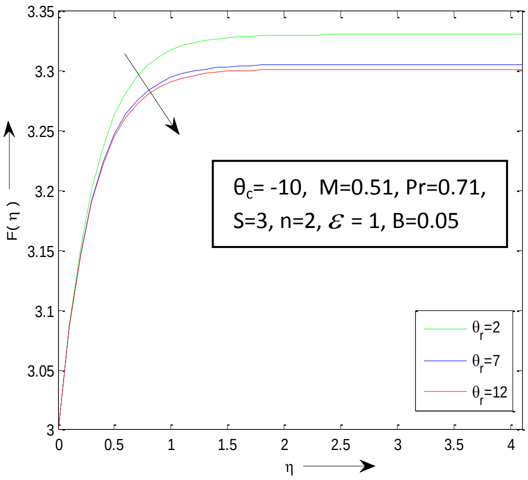

Fig.11: Transverse velocity profiles for different positive values of θ_r

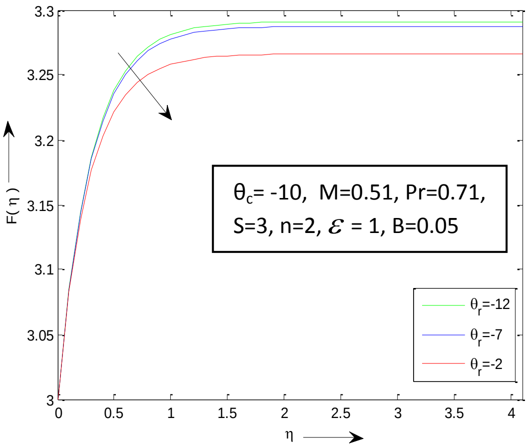

Fig. 12: Transverse velocity profiles for different negative values of θ_r

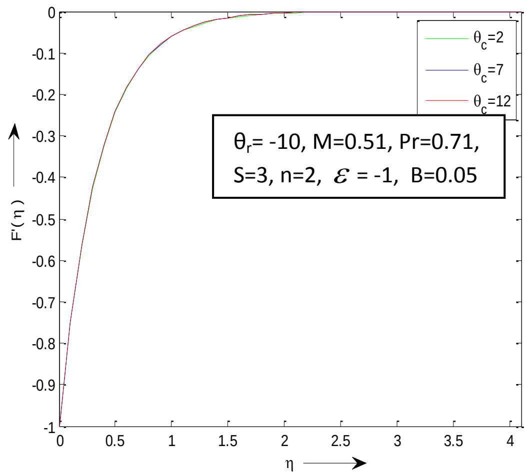

Fig. 13: Longitudinal velocity profiles for different θ_c

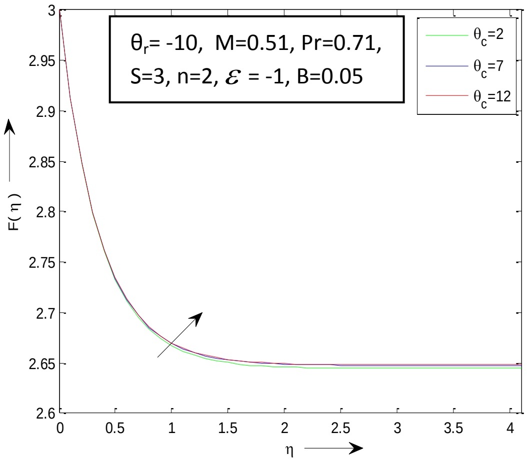

Fig. 14: Transverse velocity profiles for different θ_c

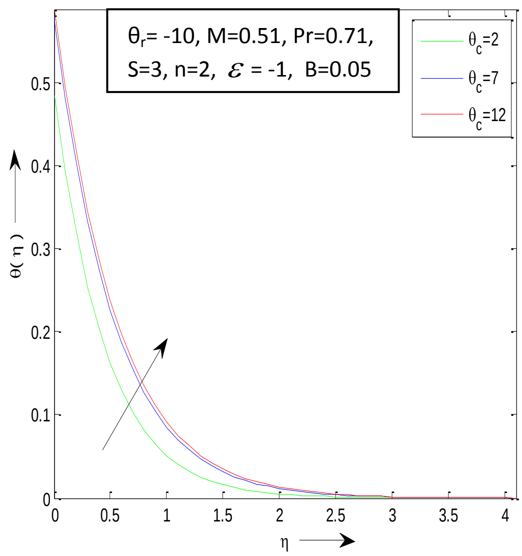

Fig.15: Temperature profile for different θ_c

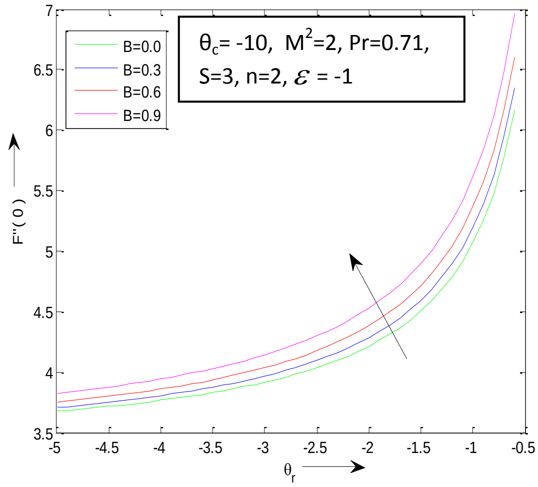

Fig. 16: Variation of $f''(0)$ for different *B*

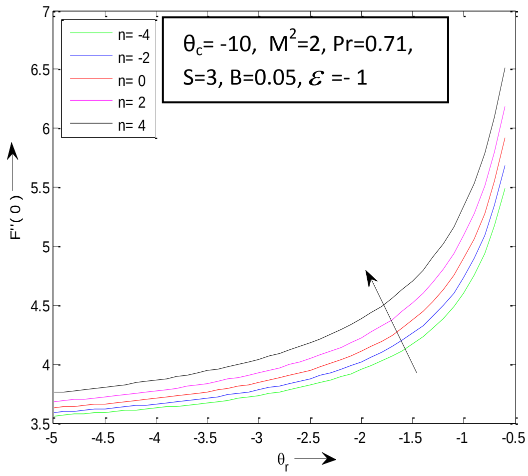

Fig. 17: Variation of $f''(0)$ for different n

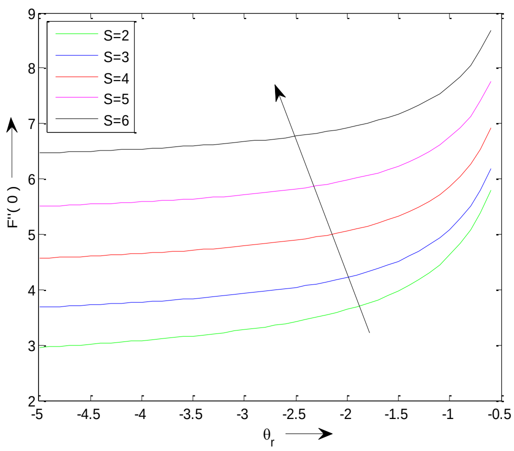

Fig. 18: Variation of $f''(0)$ for different *S* when θ_c= -10, M^2=2, Pr=0.71, B=0.05, n=2, ε = -1

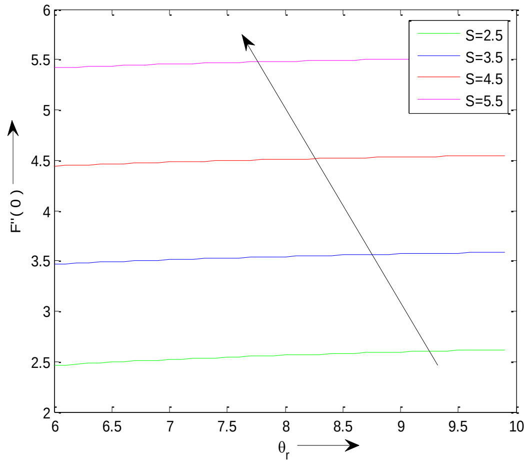

Fig. 19: Variation of $f''(0)$ for different S when θ_c= -10, M^2=2, Pr=0.71, B=0.05, n=2, ε = -1

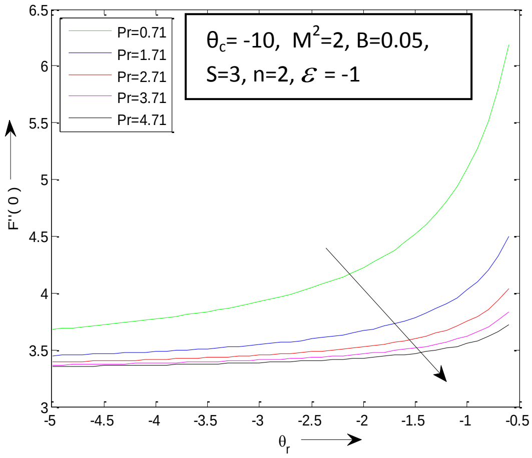

Fig. 20: Variation of skin friction for different Pr

5. ACKNOWLEDGMENTS

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6. CONCLUSIONS

In general, the flow field and temperature distribution are affected by the physical parameters.

In order to assure the accuracy of the applied numerical scheme the computed values of skin friction coefficient are compared with the available results of *Anjali Devi and Raj*[24] for two dimensional case and viscosity and thermal conductivity are taken as constant.

From the above results and discussion, the following conclusion are arrived:

- Viscosity parameter enhances the velocity profiles in a shrinking sheet where retards it in a stretching.
- Both viscosity parameter and thermal conductivity parameter enhance the skin friction coefficient.
- Viscosity parameter retards the wall temperature while that of thermal conductivity parameter enhances it.

- Magnetic parameter increases the wall shear stress whereas wall temperature decreases significantly i.e., the temperature of the fluid decreases with an increase in values of magnetic parameter.
- Stretching/shrinking parameter retards both the wall shear stress and wall temperature.
- The heat source parameter, heat flux parameter and suction parameter increases the wall shear stress whereas it is decreased significantly by the Prandtl number.
- It is believed that the results of the present work finds application in production engineering to upgrade the quality of the final product.

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