Growth Rate Analysis of Electromagnetic Waves using Beam Pre-bunching in Cerenkov Free Electron Laser

Anuradha Bhasin
ECE, BPIT

VijayRaj Shokeen
ECE, GPMCE

Bhupesh Bhatia
ECE, GPMCE

Narendra Kumar
CSE,GPMCE

ABSTRACT

The Cerenkov free electron laser (CFEL) model under consideration consists of a pre-bunched relativistic electron beam and a dielectric loaded waveguide. The increase in the growth rate and gain with the change in phase by using a pre-bunched electron beam in a Cerenkov free electron laser (CFEL) has been studied. The growth rate and gain is calculated at experimentally known CFEL and FEL parameters and it is seen that beam pre-bunching on Cerenkov free electron laser (CFEL) offers considerable enhancement in gain and efficiency when the phase of the pre-bunching electron beam is $-\pi/2$, i.e., when the pre-bunched beam is in the retarding zone.

Keywords

Pre-bunched electron beam, Cerenkov free electron laser, Cerenkov emission, efficiency.

1. INTRODUCTION

An electron beam passing through a slow wave structure is unstable to an electromagnetic perturbation whose phase velocity equals the velocity of the beam. This phenomenon of cerenkov emission is the basis of all slow wave devices. A Cerenkov free electron laser generally employs slow wave structures either consisting of a dielectric whose dielectric constant is $|\varepsilon|>1$, that reduces the phase velocity of the radiation below c or a plasma lining having a dielectric constant $\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}$ which can act as a slowing down medium for $\omega_p \gg \omega$ so that $|\varepsilon| \gg 1$(where ω_p is the electron plasma frequency and ω is the radiation frequency). A Cerenkov free electron laser (CFEL)

[1] employs a slow wave medium to slow down the phase velocity of transverse electric (TE) or transverse magnetic (TM) modes to less than c, the velocity of light so that they can be excited by a moderately relativistic electron beam by the process of cerenkov emission. Particularly at very short wavelengths Cerenkov free electron laser (CFEL) is widely used as the source of broad- band, high power microwave generation.

There has been strong motivation to produce coherent high power radiation using Cerenkov free electron laser. A CFEL driven by dense moderately relativistic electron beam, consisting of two dielectrically lined parallel plates has been studied and reported to produce power levels of 10 KW at 400 micrometers and 200 KW at 1mm. As the electron beam density was high and the resonant interaction region was long enough for the beam plasma oscillations to occur, it is expected that CFEL was operating in the collective regime[2-4]. Tripathi and Lui [5] have proposed the operation of a free electron laser in a dielectric loaded waveguide and have found that the main effect of loading dielectrics in free electron lasers is shortening of operating wavelengths by using the same beam energy as in vacuum free electron lasers.

Recently, there has been a great deal of interest in studying free electron laser (FEL) [6-16] using pre-bunched electron beams. Simulation of enhanced pre-bunching in free electron lasers for the generation of high gain radiation at high frequencies has been demonstrated by Freund *et al.* [13]. It is demonstrated that free electron laser with the pre-bunched beam combines the best characteristics of amplifiers and oscillators.

Bhasin and Sharma have examined the effect of beam pre-bunching on gain and efficiency in an FEL: nonlocal theory. They have developed a theoretical model to consider the nonlocal (finite geometry) effects in a pre-bunched FEL and studied gain and efficiency enhancement using pre-bunched electron beam [15]. It is seen that growth rate increases with the increase in the modulation index. Bhasin and Sharma have also studied the gain and efficiency enhancement in a slow wave FEL using pre-bunched electron beam in a dielectric loaded waveguide [16]. They have found that the growth rate and gain of a slow wave FEL increase with the modulation index and is maximum when the pre-bunched beam velocity is comparable to the phase velocity of the radiation wave.

It is the purpose of the present work to present a similar theoretical description of CFEL operation using a pre-bunched electron beam for far infrared wavelength generation. Analytical analysis for the excitation of electromagnetic waves by pre-bunched beam in a slow wave device has been presented. A dielectric loaded waveguide is considered. Boundary effects have been ignored.

The organization of the paper is as follows: We employ fluid theory and follow perturbation techniques to obtain the growth rate and efficiency of CFEL. The increase in growth rate and efficiency with change in phase for constant modulation index in a pre-bunched CFEL has been studied and compared with the pre-bunched FEL calculated at experimentally known FEL parameters. Finally, we have provided the discussion of the results.

2. INSTABILITY ANALYSIS

A axial pre-bunched relativistic electron beam having density n_{b0}, velocity $v_b\hat{z}$, relativistic gamma factor

$$\gamma = 1 + \frac{eV_b}{mc^2}(1+\Delta \sin \omega_0 \tau) \approx \gamma_0 (1+\Delta \sin \omega_0 \tau)$$

propagates through a dielectric loaded waveguide of effective permittivity ε_1 (cf. Fig. 1),where Δ is the modulation index (its value lie from 0 to 1), mc^2 is the rest mass energy of the electrons, e is the electronic charge, $\omega_0\left(\approx k_{z0}v_b\right)$ and k_{z0} are the modulation frequency and wave number of the pre-bunched electron beam, respectively.

In addition, the phase of the pre-bunched electron beam is given by $\psi\left(=\omega_0\tau\right)$.

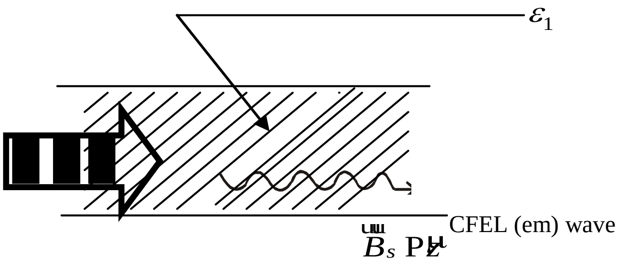

Fig.1. A schematic diagram of the Cerenkov free electron laser

An electromagnetic signal E_1 is also present in the interaction region.

$$\vec{E}_1 = \vec{E}_0 e^{-i\left(\omega_1 t - \vec{k}_1.\vec{x}\right)}, \qquad (1)$$

$$\vec{B}_1 = \frac{c}{\omega_1}\vec{k}_1 \times \vec{E}_1, \qquad (2)$$

where $\vec{E}_0$ and $\vec{k}_1$ lie in the x-z plane, i.e., $\frac{\partial}{\partial y} = ik_y = 0$

The response of the beam electrons to the signal is governed by the relativistic equation of motion

$$\frac{\partial}{\partial t}(\gamma\vec{v}) + \vec{v}.\nabla(\gamma\vec{v}) = -\frac{e}{m}(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}). \qquad (3)$$

Expanding

$$\vec{v} = v_b\hat{z} + \vec{v}_1, \quad \gamma' = \gamma + \gamma^3\frac{\vec{v}_b.\vec{v}_1}{c^2}$$

and linearizing equation (3), we get

$$\gamma\vec{v}_1 + \gamma^3\frac{v_b^2}{c^2}v_{z1}\hat{z} = \frac{e}{im\left(\omega_1 - k_zv_b\right)}\left[\vec{E}_1\left(1 - \frac{k_zv_b}{\omega_1}\right) + \vec{k}\frac{v_bE_z}{\omega_1}\right]. \qquad (4)$$

Velocity components in the x and z directions are given by

$$v_{x1} = \frac{e}{im\gamma(\omega_1 - k_zv_b)}\left[E_{x1} - \frac{k_zv_bE_{x1}}{\omega_1} + \frac{k_{x1}v_bE_{z1}}{\omega_1}\right]. \qquad (5)$$

$$v_{z1} = \frac{eE_{z1}}{im(\omega_1 - k_zv_b)\gamma^3}. \qquad (6)$$

On linearizing and solving equation of continuity, we obtain density perturbation

$$n_1 = n_{b0}\frac{\vec{k}_1.\vec{v}_1}{\left(\omega_1 - k_zv_b\right)}. \qquad (7)$$

Using the value of v_{x1} and v_{z1} from equations (5) and (6) in equation (7), we get

$$n_1 = \frac{en_{b0}}{im\left(\omega_1 - k_zv_b\right)^2}\left[\frac{E_{x1}k_{x1}}{\gamma}(1 - \frac{k_zv_b}{\omega_1}) + \frac{k_{x1}^2v_bE_{z1}}{\gamma\omega_1} + \frac{k_{z1}E_{z1}}{\gamma^3}\right]. \qquad (8)$$

The perturbed current density is given by

$$\vec{J}_1 = -n_{b0}e\vec{v}_1 - n_1e\vec{v}_b. \qquad (9)$$

Substituting the values of $\vec{v}_1$ and n_1 from equations (5), (6) and (8) in equation (9), and keeping the value in the wave equation, we obtain

$$k_1^2\vec{E}_1 - \vec{k}_1\left(\vec{k}_1.\vec{E}_1\right) - \frac{\omega_1^2}{c^2}\varepsilon\vec{E}_1 = \frac{4\pi i\omega_1\vec{J}_1}{c^2} \qquad (10)$$

and writing x and z components of the latter, we obtain

$$(k_{z1}^2 - \frac{\omega_1^2}{c^2}\varepsilon + \frac{\omega_{pb}^2}{\gamma c^2})E_{x1} = (k_{x1}k_z - \frac{\omega_{pb}^2}{\gamma c^2}\frac{k_{x1}v_b}{\left(\omega_1 - k_zv_b\right)})E_{z1}, \qquad (11)$$

where $\omega_{pb}^2 = \frac{4\pi n_{bo}}{m}e^2$.

Equation (11) gives the dispersion relation and can be further rearranged by taking ω_{pb}^2 terms to the right hand side and retaining only those terms which have a resonance denominator $(\omega_1 - k_zv_b)^2$, we get

$$(\omega_1^2 - \frac{k_1^2c^2}{\varepsilon})(\omega_1 - k_zv_b)^2 = \frac{\omega_{pb}^2}{\gamma^3\varepsilon}(\omega_1^2 + k_{x1}^2v_b^2\gamma^2). \qquad (12)$$

The two factors on the left–hand side of equation (12) when equated to zero $\omega_1 - \frac{k_1c}{\sqrt{\varepsilon}} = 0, \omega_1 - k_zv_b = 0,$ give radiation and beam modes, respectively. To determine the growth rate of the CFEL instability, we use the first order perturbation techniques. In the presence of the right hand side terms $\left(i.e., n_{b0} \neq 0\right)$, we assume that the eigen functions are not modified but their eigen value are. We expand ω_1 as

$$\omega_1 = \omega_{1r} + \delta = k_zv_b + \delta = k_{z0}v_b + \delta,$$

where δ is the small frequency mismatch and $\omega_{1r} = \frac{k_1c}{\sqrt{\varepsilon}}$.

On further solving equation (12) we obtain

$$\delta = \left[\frac{\omega_{pb}^2(\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma^2)}{2\omega_{1r}\gamma^3\varepsilon}\right]^{1/3} e^{i\frac{2n\pi}{3}}, n = 0,1,2,3,........ \qquad (13)$$

Hence the growth rate, i.e., the imaginary part of δ is given as

$$\Gamma = \left[\frac{\omega_{pb}^2(\omega_{1r}^2 + k_{x1}^2 v_b^2 \gamma_0^2)}{2\omega_{1r}\gamma^3\varepsilon}\right]^{1/3} \frac{\sqrt{3}}{2} \qquad (14)$$

where $\gamma = \gamma_0(1+\Delta\sin\omega_0\tau)$

for modulation index =0.85, $\gamma = \gamma_0(1+\Delta\sin\omega_0\tau)$.

Using Equation (13) the real part of δ is given as

$$\left|\delta_r\right| = \frac{\Gamma}{\sqrt{3}}, \qquad \text{i.e., } \omega_1 = k_z v_b - \frac{\Gamma}{\sqrt{3}}$$

or $\quad v_b = \frac{\omega_1}{k_z} + \frac{\Gamma}{\sqrt{3}}k_z$ i.e., $v_b > \frac{\omega_1}{k_z}$.

This is the necessary condition for electron bunching and net energy transfer from beam electrons to the radiation wave.

A. Gain

The gain G in dB is defined by

$$G = 10\log(\frac{A}{A_0}) = 10\frac{\Gamma L}{c}, \qquad (15)$$

Where, L is the length of the interaction region, A_0 and A are the amplitudes of the wave at distance z=0 and z=L. From equation (15) we can see that the gain, hence the efficiency of the CFEL device increases with the growth rate Γ.

3. RESULTS AND DISCUSSIONS

In the numerical calculations we have used typical parameters of Cerenkov free electron laser (CFEL). For the comparative study we have also used parameters of free electron maser experiment with a pre-bunched electron beam [1], (e.g., beam energy = 0.07MeV and beam current I_b = 1.0A) and other parameters are same as CFEL. Typical Parameters for CFEL are the following: electron beam energy E_b =1.35MeV, beam current I_b=800A. Values for modulation frequency ω_0 =4.87GHz and effective permittivity ε = 1.7 are the same for CFEL and FEL.

In Fig. 2, we have plotted the growth rate of the CFEL instability (in rad/sec) as a function of the phase of pre-bunched electron beam for the above given parameters and for modulation index Δ = 0.85. From Fig. 2, we can observe that the growth rate reaches maximum when the phase of the pre-bunched beam is $-\pi/2$, i.e., when the beam electrons are in the retarding zone, and the growth rate becomes minimum when the phase of the pre-bunched beam is $+\pi/2$, i.e., when the beam electrons are in the accelerating zone. Moreover, the growth rate is more for FEL than CFEL, for the same modulation index Δ = 0.85.

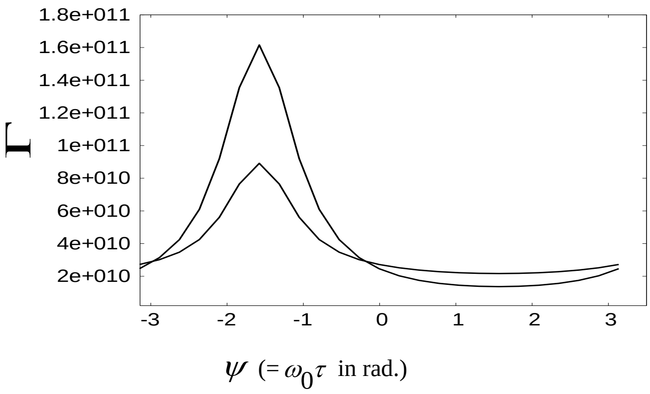

Fig.2. Growth rate Γ (in rad /sec) as a function of phase angle $\psi\left(=\omega_0\tau\right)$ of the pre-bunched electron beam for (a) CFEL parameters for Eb = 1.35MeV, I_b = 900A and for modulation index Δ = 0.85, (b) FEL parameters with E_b = 0.07MeV, I_b = 1.0A and for modulation index Δ = 0.85. For case (b) i.e., with FEL parameters growth rate is higher.

If we introduce plasma in the interaction region of CFEL then further reducing the requirements on beam energy for generating shorter wavelength radiation. We have neglected the radial spread in the beam such as dc and ac space charge effects. We have also neglected the dc self generated magnetic field as we have chosen the beam parameters in such a way that dc self –generated magnetic field may not play an important role.

The growth rate of the pre-bunched CFEL increases when the beam electrons are in the retarding zone and reaches maximum when the phase of the pre-bunched beam is $-\pi/2$. Moreover, the growth rate becomes minimum when the phase of the pre-bunched beam is $+\pi/2$. As the growth rate increases, consequently the gain and efficiency of the device also reaches maximum when the phase of the pre-bunched beam is $-\pi/2$.

4. CONCLUSION

In conclusion, we can say that in pre-bunched CFEL there is considerable enhancement in gain and hence efficiency when the beam electrons are in the retarding zone. Moreover, the growth rate hence the gain and efficiency of pre-bunched CFEL reaches maximum when the phase of the pre-bunched beam is $-\pi/2$. The growth rate of electromagnetic wave is more in FEL than in CFEL. But, as CFELs are simple to design and tune than FELs, pre-bunched CFELs may find application in generation of high frequency radiations. In addition to this it is seen that by using pre-bunched electron beams, requirement for beam energy can be reduced drastically for generating high frequency radiations, for both CFEL and FEL.

The scheme seems to work well at millimeter and sub-millimeter wavelengths.

5. REFERENCES

[1] J. Walsh and B. Johnson and G. Dattoli and A. Renieri, "Undulator and Cerenkov free electron lasers: a preliminary comparison," *Phys. Rev. Lett.*, vol. 53, pp. 779-782, 1984.

[2] B. Johnson and J. Walsh, "A Cerenkov infrared laser," *Nucl. Instr. Methods Phys. Res.*, vol. A237, pp. 239-243, 1985.

[3] E. P. Garate, C. Shaughnessy, J. Walsh and S. Moustaizis, "Cerenkov free-electron laser operation at far-infrared to sub-millimeter wavelengths," *in Proc. 8th Int. Free-Electron Laser Conf.,* Glasgow, Scotland, Sept. 1986.

[4] E. P. Garate, R. Cook, P. Heim, R. Layman, and J. Walsh, "Cerenkov maser operation at lower mm wavelengths," *J. Appl. Phys.,* vol. 58, pp.627-632, 1985.

[5] V. K. Tripathi and C. S. Liu, "A slow wave free–electron laser," *IEEE Trans. on Plasma Science,* vol. 17, pp. 583 –587, 1989.

[6] J. Krall and Y.Y. Lau, "Modulation of an intense beam by an external microwave source: theory and simulation," *Appl. Phys. lett.,* vol. 52, pp. 431-433, 1988.

[7] M. Cohen, A. Kugel, D. Chairman, M. Arbel *et al.*, "Free electron maser experiment with a pre-bunched beam," *Nucl. Instr. and Methods in Phys. Res.,* vol. A358, pp. 82-85, 1995.

[8] Kazuyoshi Saito, Ken Takayama, Toshiyuki Ozaki *et al.*, "X-band prebunched FEL amplifier," *Nucl. Instr. and Methods in Phys. Res.,* vol. A375, pp. 237-240, 1996.

[9] J. Gardelle, J. Labrouche, G. Marchese, J. L. Rullier and D.Villate and J. T. Donohue, "Analysis of the beam bunching produced by a free electron laser," *Phys. Plasmas,* vol. 3, pp. 4197-4206, 1996.

[10] Yukio Shibata, Kimihiro Ishi, Shuichi Ono *et al.*, "Broadband free electron laser by the use of pre-bunched electron beam," *Phys. Rev. Lett.,* vol. 78, pp. 2740-2743, 1997.

[11] A. Doria, R. Bartolini *et* al., "Coherent emission and gain from a bunched electron beam," *IEEE J. Quantum Electron*, QE-29, pp. 1428-1436, 1993.

[12] F. Ciocci, R. Bartolini, A. Doria, G. P. Gallerano *et al.*, "Operation of a compact free-electron laser in the millimeter–wave region with a bunched electron beam," *Phys. Rev. Lett.,* vol. 70, pp. 928-931, 1993.

[13] H. P. Freund, P. G. O'Shea, J. Neumann, "Simulation of pre-bunching in free-electron lasers," *Nucl. Instr. and Methods in Phys. Res.,* vol. A507, pp. 400-403, 2003.

[14] Vivek Beniwal, Suresh C. Sharma and M. K. Sharma, "Effect of beam pre-modulation on gain and efficiency in a free electron laser," *Phys. Plasmas,* vol. 11, pp. 5716-5722, 2004.

[15] Anuradha Bhasin and Suresh C. Sharma, "Effect of beam pre-bunching on gain and efficiency in a free electron laser: nonlocal theory," *Phys. Plasmas,* vol. 14, pp. 73102 - 4, 2007

[16] Anuradha Bhasin and Suresh C. Sharma, "Effect of beam pre-modulation and guide magnetic fields on slow wave free electron laser: Nonlocal Theory," *IEEE J. Quantum Electron*, QE-45, pp. 1129, 2009.

.