Stochastic Analysis of Static and Fatigue Failures with Fluctuating Manpower and Business

Sundar Viswanathan
Senior Testing Engineer, ANSYS Inc.,
275 Technology Drive, Canonsburg, PA 15317

ABSTRACT
This paper considers two stochastic models with static and fatigue failures under various situations of availability and slackness of manpower and business. An operating system is exposed to a shock process which causes static or fatigue failures. The static failures are repaired and fatigue failures call for replacement of the entire system when they occur. Considering a continuous time Markov chain approach, the backlog level probabilities of the occurred static failures, steady state fatigue failure and various other measures are obtained for Model (A) and Model (B) under the assumption that the backlog accumulates to any arbitrarily large level in the first model and in the second model the operating system is replaced when the backlog exceeds a limit.

General Terms
Manpower, Business, Availability and Slackness, Markov Chain

Keywords
Static and Fatigue failures, Stationary probability, infinitesimal generator and Matrix approach

1. INTRODUCTION
The causes of failure of an operating system may be due to factors such as complex stress cycles, engineering design, manufacturing defects, defects inadvertently introduced in various stages of production line and so on. These types of static and dynamic (fatigue) failures may result in the break down or collapse of entire operation. For an analysis of static and fatigue failure, one may refer Aher and Sonawane [1], Akano and Fakinlede [2] and Murer and Leguilion [3] where several research models in various situations are studied and it has been pointed out that the repeated loading of the mechanism cause fluctuating stresses and cause fatigue failure. Bhaumik, Sujata and Venkateswamy [4] have studied Fatigue failure of aircraft components. Dutton, Clarke and Bonnet [5] have presented models of static and fatigue failure in wind turbine blades Fatigue failure can occur at stress level that is significantly lower than those that cause static failure. In fact a small unseen crack may be sufficient for a major or sudden failure and total collapse of an operating system. Unlike static failures, fatigue failures are difficult to predict and repair. Static failures can be attended and repaired during the system operation without stopping any primary system and the interconnected systems. Also fatigue failure may cause the replacement of the operating system whereas static failures permit engineers/technicians to attend them locally without affecting the production or operation. Premature or unexpected failure of a system reduces customer confidence in the finished products. It is also noticed that whenever a shock occurs to an operating system, immediate fatigue failure or a static failure occurs. Parzen [6] has listed various models for the probability of fatigue of a structure and failure distributions. Since failures queue up for repair, there is a close connection with queueing systems. For matrix- geometric solutions in stochastic models and queueing systems, one may refer Neuts, [7]. Chakravarthy and Neuts [8] have discussed in depth a multi-server queueing model. Aissani.A and Artalejo.J.R [9] and Ayyappan, Subramanian and Gopal Sekar [10] have analyzed retrial queueing system. In this paper static and fatigue probabilistic failure models are studied under various assumptions of seasonal availability of manpower and business. A study on such models are very relevant and much required because the failure of an operating system may reduce profitability of the operations and at times may motivate manpower and experts available to depart causing unnecessary hardship to organizations. A static and fatigue failures model with stochastic assumptions of failure and repairs or replacement have not been treated as a continuous Markov chain at any depth so far. Two models with static and fatigue failures are presented here where both failures are probabilistic in nature and the fatigue failure causes replacement of the operating system. In Model A, the static failures are repaired and in Model B, the static failure calls for replacement of the system at a finite number of accumulated failure level. Using matrix partitioning method of the infinitesimal generator introduced by Neuts [7], results obtained and numerical results are presented in support of the same. Section II and section III treat the Model A and the Model B. Section IV considers examples.

2. MODEL (A) REPLACEMENT FOR FATIGUE FAILURE

2.1 Assumptions
i. Shocks occur to an operating system with exponential inter occurrence time distribution whose parameter is c.

ii. Manpower is available (in level 1) for a random time whose distribution is exponential with parameter λ and after which it becomes unavailable (level 0) for a random time whose distribution is exponential with parameter μ. It is thus available and unavailable states (level 1 and level 0) alternately.

iii. The operating system generates business (demand) for its operation. Business alternates between peak-level (available level or level 1) and sluggish-level (unavailable level or level 0). The holding time distributions of them are exponential with parameters a, b respectively.

iv. The occurrence of a shock causes static failure with probability α_i, and fatigue failure with probability β_i, $\alpha_i + \beta_i = 1$, for i = 1, 2, 3, 4 according as (i) both the manpower and the business are in unavailable state or (ii) the manpower is unavailable but the business is in peak level or (iii) the manpower is available but the business is in sluggish level or

(iv) both the manpower and the machine are available and peak levels respectively in that order.

v. The operating system fails when fatigue failure occurs. The replacement time distribution of the operating system is exponential with parameter δ.

vi. When a static failure occurs it is repaired while the system is in operation. The repair time distribution of static failure has exponential distribution with parameter di, for i = 1, 2, 3, 4 in various levels as stated above for failure probabilities. At each repair time, N number of static failures are repaired with probability P (N=i) =pi, for 1≤ I ≤ m where $\sum_1^m p_i$ =1 where m is the maximum number of repairs at a time. When n static failures (n < m) are to be repaired, then P (N=i) =p_i, for 1≤ i ≤ n-1 and n static failures are repaired with probability$\sum_n^m p_i$, (since the existing static failure size is n). When the failure of the operating system occurs due to a fatigue failure, all static failures are given up.

It is natural to assume different probability values for fatigue failures depending on whether both manpower and business levels are (1, 1), (0, 1), (1, 0) and (0, 0). When the manpower is not available and the business is in peak level, the stress effect to cause fatigue failure may be naturally more compared to the case when manpower is available and business is sluggish.

2.2. Analysis

For studying the above model, the various states of the continuous time Markov chain X (t) are defined as follows. X (t) = {(R, i, j): i = 0, 1; j = 0, 1}∪{(n, i, j): 0 ≤ n < ∞; for i = 0, 1; for j = 0, 1}. (1)

The two co-ordinates of a state, the second and the third of (X, i, j) represent respectively the level of the manpower system is i and the level of the business is j for i, j =0, 1 where 0 indicates the unavailable state and the level 1 indicates the available state of the manpower and business as explained earlier. The first co-ordinate X= R when the operating system is under replacement and X =n, when n static failures are to be repaired for n=0, 1, 2, 3…… Let the probability generating function of N, the number of static failures repaired in a repair time be given by

$$\varphi(r) = \sum_{i=1}^{m} p_i\ r^i \tag{2}$$

Consider the survivor probability

$$P(N > i) = P_i = 1 - \sum_{j=1}^{i} p_j \quad \text{for } i=1, 2..., m-1 \tag{3}$$

Its generating function Ø(r) is

$$Ø(r) = \sum_{i=1}^{m-1} P_i\, r^i \tag{4}$$

The relation between them is

$$Ø(r) = (r/1-r)\ \varphi(r) \tag{5}$$

The continuous time Markov chain describing model has the infinitesimal generator Q of infinite order which can be presented in block partitioned form with each block is of order 4. The infinitesimal generator of the model is given below in equation (6). The states of the matrices are listed lexicographically as

$\underline{R}, \underline{0}, \underline{1}, \underline{2}, \underline{3}, \ldots . \underline{n}, \ldots$ where

$\underline{X}$ = ((X,0,0), (X, 0, 1), (X,1, 0), (X, 1, 1)), where X is R or n, for n =1,2,…. The block matrices are all of order 4. The matrices C_1, B_1 and A_1 have negative diagonal elements and their off diagonal elements are non- negative. The matrices $A_0, A_2, A_3, \ldots, A_{m+1}, B_2, B_3, \ldots, B_{m+2}, C_0$ and C_2 have non-negative elements and are diagonal matrices. They are given below.

$$Q = \begin{bmatrix} C_1 & C_0 & 0 & 0 & . & . & . & . & . & \cdots \\ C_2 & B_1 & A_0 & 0 & . & . & . & . & . & \cdots \\ C_2 & B_2 & A_1 & A_0 & 0 & . & . & . & . & \cdots \\ C_2 & B_3 & A_2 & A_1 & A_0 & . & . & . & . & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_2 & B_m & A_{m-1} & A_{m-2} & . & 0 & . & . & . & \cdots \\ C_2 & B_{m+1} & A_m & A_{m-1} & . & A_0 & 0 & . & . & \cdots \\ C_2 & 0 & A_{m+1} & A_m & . & A_1 & A_0 & 0 & . & \cdots \\ C_2 & 0 & 0 & A_{m+1} & . & A_2 & A_1 & A_0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \end{bmatrix} \tag{6}$$

Consider a matrix Q’ of order 4,

$$Q' = \begin{bmatrix} -(\mu+b) & b & \mu & 0 \\ a & -(\mu+a) & 0 & \mu \\ \lambda & 0 & -(\lambda+b) & b \\ 0 & \lambda & a & -(\lambda+a) \end{bmatrix} \tag{7}$$

Then C_1 = Q’- diag (δ, δ, δ, δ); B_1 = Q’ – diag (c, c, c, c) and
A_1 = Q’ – diag ((c +d_1), (c +d_2), (c +d_3), (c +d_4));
C_0 = diag (δ, δ, δ, δ); C_2 = diag (cβ_1, cβ_2, cβ_3, cβ_4);

B_2= diag (d_1, d_2, d_3, d_4); B_j = diag $[d_1 P_{j-2}, d_2 P_{j-2}, d_3 P_{j-2}, d_4 P_{j-2}]$, for 3 ≤ j≤ m+1
A_0= diag (cα_1, cα_2, cα_3, cα_4) and A_j = diag $[d_1 p_{j-1}, d_2 p_{j-1}, d_3 p_{j-1}, d_4 p_{j-1}]$, for 2 ≤ j≤ m+1. (8)

The basic system generator given in (7)
Q’ = $\sum_{i=0}^{m+1} A_i + C_2$. Its probability vector w satisfies wQ' =0; w.e =1 and w = $\frac{1}{(a+b)(\lambda+\mu)}$ (a λ, b λ, a μ, b μ). (9)

The stability condition for the existence of a stationary distribution for continuous time Markov chain given by Q in (6) is, (see Neuts [9])

$$wA_0 e < w\left[\sum_{j=2}^{m+1} (j-1) A_j\right] e \tag{10}$$

This gives, c w.$\underline{\alpha}$ < E (N) $w.\underline{d}$ (11)

where $\underline{\alpha} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4)^t$, $\underline{d} = (d_1, d_2, d_3, d_4)^t$ and E (N) is the expected number of static failures repaired during a repair time. Using the generator Q’ of (7) and (9), the expected or effective static failure probability is E (α) =w.$\underline{\alpha}$. The expected or effective fatigue failure probability is, E (β) = w. $\underline{\beta}$ and E (α) + E (β) = 1. The expected parameter of static repair time distribution is E (d) = $w.\underline{d}$. This gives the stability condition (11) as, c E (α) < E (N) E (d). When the inequality given by (11) is satisfied, the stationary distribution exists, Neuts [7].

Let π (X, i, j), for X=R or n, for n = 0, 1, 2, 3 ….; i =0,1; j = 0,1 be the stationary probability of the states listed in (1) and let π_X be the vector of type 1 x 4 given by π_X= (π(X,0,0), π(X,0,1), π(X,1,0), π(X,1,1)), for X = R, or n , where n = 1,2,…… (12)

The stationary probability vector $\pi = (\pi_R, \pi_0, \pi_1, \pi_2 \ldots\ldots)$. satisfies the equations. πQ = 0, and πe = 1 (13)

From (13), the following are seen

$$\pi_R C_1 + \sum_{i=0}^{\infty} \pi_i\, C_2 = 0 \tag{14}$$

$\pi_R C_0 + \sum_{i=0}^{m} \pi_i B_{i+1} = 0$ (15)

$\pi_{n-1}A_0+\pi_n A_1+\pi_{n+1}A_2+\pi_{n+2}A_3+\ldots+\pi_{n+m}A_{m+1} = 0$, for n ≥ 1 (16)

Introducing the rate matrix R as the minimal non-negative solution of the non-linear matrix equation $A_0+R A_1+R^2A_2+ R^3A_3 + \ldots + R^{m+1}A_{m+1} = 0$, (17)
it can be proved that (Neuts [9]) that π_n satisfies for n =1, 2, 3……, the equation. $\pi_n = \pi_0 R^n$ for n ≥ 1. (18)

From (18) and (14), $\sum_0^\infty \pi_i = \pi_0 (I-R)^{-1}$
$\pi_R = \pi_0 (I-R)^{-1} C_2 (-C_1)^{-1}$ (19)

Substituting π_R in (15), it can be seen
$\pi_0 \left((I-R)^{-1} C_2 (-C_1)^{-1} C_0 + \sum_{j=0}^{m} R^j B_{j+1} \right) = 0$ (20)

The equation (20) gives π_0 up to a multiplicative constant since the coefficient matrix of π_0 in (20) is a generator whose row sum is zero. This can be seen as follows by taking a common factor $(I-R)^{-1}$,

$((I-R)^{-1}C_2(-C_1)^{-1}C_0 e + \sum_{j=0}^{m} R^j B_{j+1})e$
$= (I-R)^{-1}[C_2 \left(-C_1\right)^{-1} C_0 e + \left(\sum_{j=0}^{m} R^j B_{j+1} - \sum_{j=0}^{m} R^{j+1} B_{j+1}\right)\Big) e]$
$=(I-R)^{-1}[C_2\left(-C_1\right)^{-1}C_0 e + B_1 e + \left(\sum_{j=1}^{m} R^j (B_{j+1} - B_j\right)e - R^{m+1}B_{m+1}e\big)$
$=(I-R)^{-1}[C_2\left(-C_1\right)^{-1}C_0 e + B_1 e + \left(\sum_{j=1}^{m} R^j (-A_j\right)e - R^{m+1}A_{m+1}e\big)$ by (8). Using (17) we get this as
$= (I-R)^{-1}[C_2(-C_1)^{-1}C_0 e + B_1 e + A_0 e)]$
$= (I-R)^{-1}[C_2(-C_1)^{-1}C_0 e - C_2 e)$]
$= (I-R)^{-1}C_2(-C_1)^{-1}[C_0 e + C_1 e] = 0$ (21)

The normalizing constant can be found using (13). Using (19)

$\pi_0 (I-R)^{-1} C_2 (-C_1)^{-1} e + \pi_0 (I-R)^{-1} e = 1$ (22)

From the equations (22), (20), (19) and (18) π_R, π_0 and π_i, for $i \geq 1$. The matrix R is computed by substitutions in the recurrence relation starting with R (0) = 0 and using, R (n+ 1) = $-A_0 A_1^{-1} - \sum_{j=2}^{m+1} R^j(n) A_j A_1^{-1}$, n ≥0. The iteration may be terminated to get a solution of R, at an approximate level when ‖ R (n+1)-R (n) ‖< ε, where ε is a given small number. (23)

The generating function of the probabilities can be given by $\Phi(s) = \pi_R e + \sum_0^\infty \pi_i s^i R^i e = \pi_R e + \pi_0 (I-sR)^{-1} e$ (24)
and the expected number of static failures is E (S) = $\pi_0 (I-R)^{-2} Re$. The probability of operating system is in failed state $=\pi_R$. The probability of no static failure for repair is = π_0 .

3. MODEL. (B) SYSTEM REPLACEMENT FOR FATIGUE AND STATIC FAILURES

In this model only a finite number of static failures that can wait for repair at a time instead of any arbitrarily long length is permitted. This is also the case when more than certain numbers of static failures are waiting for repair; the management may go for condemning the operating system and may opt for replacement of the same instead of repairing for economic considerations. Such policy decisions are very common when repair cost is more compared to replacement cost. The significance of this model can also be seen in another angle. In the Model (A), the fatigue failure is considered purely as random in nature and the fatigue causes system failure immediately on the occurrence of a shock with some probability. When the system failure does not occur due to a shock, then it has been assumed that the shock gives a repairable damage (static failure). But accumulation aspect of such damages for causing a system failure has not been considered there. Here the critical level is M for the static damages beyond which any static damage also causes a system failure. Model (B) has all the assumptions of Model A and has one additional assumption.

3.1 Assumption

vii. When M or more, static failures are waiting for repair, the management orders for replacement of the operating system. The replacement time distribution of the operation system is same as in Model (A) with independent exponential with parameter δ.

3.2. Analysis

The various states of the continuous time Markov chain X (t) are defined as follows with 4(M+2) states for this model. X (t) = {(R, i, j): i = 0, 1; j = 0, 1}∪{(n, i, j): 0 ≤ M; for i = 0, 1; for j = 0, 1}. (25)
Using the same definition given for Model A for the various co- ordinates of the states of the system, the continuous time Markov chain describing model has the infinitesimal generator Q” of finite order 4(M+2) which can be presented in block partitioned form with each block is of order 4. The infinitesimal generator Q” of the model is given below in (29). The matrices given below are its blocks namely $A_0, A_1, A_2, A_3, \ldots, A_{m+1}, B_1 B_2, B_3, \ldots, B_{m+2}, C_0, C_1$ and C_2. they are same as presented for model A in equation (8)

The matrices C_3 and D_1 are of order 4. They are C_3 = diag (c, c, c, c) and $D_1 = C_1 - C_3$. (26)
Let π (X, i, j), for X=R or n, for n = 0, 1, 2, 3 …M; i =0,1; j = 0,1 be the stationary probability of the states listed in (25) and let π_X be the vector of type 1 x 4 given by π_X= (π(X,0,0), π(X,0,1), π(X,1,0), π(X,1,1)), for X = R, or n , where n = 1,2,……M (27)
The stationary probability vector $\pi = (\pi_R, \pi_0, \pi_1, \pi_2 \ldots \ldots \pi_M)$ satisfies the equations πQ” = 0, and πe = 1. (28)

$$Q'' = \begin{bmatrix} C_1 & C_0 & 0 & \cdots & . & . & . & . & . \\ C_2 & B_1 & A_0 & \cdots & . & . & .. & . & . \\ C_2 & B_2 & A_1 & \cdots & . & . & . & . & . \\ C_2 & B_3 & A_2 & \cdots & . & . & . & . & . \\ C_2 & B_4 & A_3 & \cdots & . & . & . & . & . \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ C_2 & B_m & A_{m-1} & \cdots & 0 & . & . & . & . \\ C_2 & B_{m+1} & A_m & \cdots & A_0 & 0 & . & . & . \\ C_2 & 0 & A_{m+1} & \cdots & A_1 & A_0 & 0 & . & . \\ C_2 & 0 & 0 & \cdots & A_2 & A_1 & A_0 & 0 & . \\ C_2 & 0 & 0 & \cdots & A_3 & A_2 & A_1 & A_0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ C_2 & 0 & 0 & \cdots & A_{m+1} & \cdots & A_2 & A_1 & A_0 \\ C_3 & C_0 & 0 & \cdots & 0 & \cdots & 0 & 0 & D_1 \end{bmatrix} \quad (29)$$

This being a finite system, it can be solved using matrix inverse method. From (28), $\pi_R e + \sum_{i=0}^{N} \pi_i e = 1$. Let the matrix obtained by replacing the first column of Q” by vector e be Ψ. Then since πΨ= (1, 0, 0,…..,0) and π= (1,0,0,…0)Ψ^{-1} (30)
Then the probability of the operating system is in failed state

$\pi_R = \pi$ x $(\underline{1}, \underline{0}, \underline{0}, \underline{0}, \ldots.. \underline{0})'$;
the probability of zero static damages waiting for repair is
$\pi_0 = \pi$ x $(\underline{0}, \underline{1}, \underline{0}, \underline{0}, \ldots.. \underline{0})'$

and the expected number of static failures waiting to be repaired E (S) = π x $(\underline{0}, \underline{0}, \underline{1}, \underline{2}, \ldots.. \underline{M})'$ (31)

4. NUMERICAL EXAMPLES FOR MODELS

Six numerical examples three each for Models A and B are presented. The same set of parameter values are used for both the models so that the effect of stopping at backlog level of static failures can be seen to understand the impact of stopping.

4.1 Model (A):

The fatigue failure probabilities are allowed to vary here and other parameters are fixed. The expected number of static failures waiting for repair E (S), the probabilities of number of static failures waiting for repairs and probability of operating system is in fatigue failed state, π_i *and* π_R for i = 0, 1, 2, 3,and $\pi_* = \sum_{k=3}^{\infty} \pi_k$ are calculated. The manpower status parameters namely λ and μ, business status parameters namely a and b, the occurrence parameter of the shock process c, the repair time parameters of static failure namely vector d_i for i = 1, 2,3, 4, in various manpower and business states, the probabilities P (N= i) $=p_i$of i static failures repaired in a repair time for i =1, 2, 3…. , and the parameter δ of replacement of operating system in down state are fixed as follows; λ = .1, μ = 1; a = .2, b = 2; c = 20;$d_1 = 7, d_2 = 8, d_3 = 9, d_4 = 10$; $p_1 = .5, p_2 .25, p_3 = .15, p_4 = .1,\ p_i = 0, for\ i > 4$; and δ = 30. The steady state probability vector of the matrix Q' of order four is seen as w = (w_1, w_2, w_3, w_4) = (0.008264463, 0.082644628, 0.082644628, 0.826446281), with E (N) = 1.85 and the effective static repair rate of inequality (11), E (N) $w.\underline{d}$ = 17.995455. Three examples are studied fixing the fatigue probabilities, $\beta_1 = .1, \beta_2 = .1, \beta_4 = .15$ *and varying* β_3= .05, .1 and .15. $(\alpha_i = 1 - \beta_i, i = 1,2,3, and\ 4)$. The iteration for the rate matrix R is stopped at the iteration 12 when the difference-norm is of order E-05. Using the rate matrix R, various measures are calculated and tabulated below.

Table 1 Calculated values for Model (A)

	Example1 $\beta_3 = .05$	Example 2 $\beta_3 = .1$	Example 3 $\beta_3 = .15$
E(β)	0.137190083	0.141322314	0.145454545
πR	0.083493316	0.085970876	0.08832241
π0	0.262431771	0.267741243	0.272121332
π1	0.185431338	0.18828718	0.190140201
π2	0.131541078	0.13270953	0.133059445
π3	0.093683976	0.093746837	0.093262246
π*	0.24341852	0.231544335	0.223094366
E(S)	2.36556159	2.248039318	2.174626564

4.2. Model (B)

Here the above three examples are studied. The same values of the parameters of Examples 1, 2, 3 are considered for Examples 4,5, 6 with the stopping level for static damages M = 10 respectively in that order so that the effect of stopping at a level can be seen. The results obtained are listed below in a table.

Table 2 Calculated values for Model (B)

	Example 4 $\beta_2 = .05$	Example 5 $\beta_2 = .1$	Example 6 $\beta_2 = .15$
E(β)	0.137190083	0.141322314	0.145454545
πR	0.085206113	0.087522741	0.08976062
π0	0.277089768	0.280772065	0.284180347
π1	0.19583635	0.197358833	0.198457795
π2	0.13877192	0.138878017	0.138660071
π3	0.098486623	0.097735979	0.096842883
π*	0.204609226	0.197732363	0.192098283
E(S)	1.901214905	1.85724019	1.820659673

4.3. Combined Chart for Models (A) and (B)

The following figures give the values of various quantities calculated. They give the variations seen in changing the fatigue probabilities. The increase in fatigue effective value E (β) decreases the various measures. For set of the parameter values under consideration π_* and E(S) are more for Model (A), compared to Model (B) indicating the advantage of stopping at a level. The numerical method may be used when the parameter values and M are known to find whether Model A or Model B is advantageous The figures 1 and 2 show the lower level probabilities are more and E(S) values are less for Model (B) compared to Model (A) indicating the stopping level plays a role. The combined chart of all measures and all probabilities also show significant variations as the expected fatigue level increases and the stopping level plays a role in increasing lower level probabilities and decreasing the expected static damage levels. The selection of M, the stopping level, also has relation with future cost of maintenance of the entire system and fixing the same may have to be optimal in line with budget estimates in addition to probabilistic consideration.

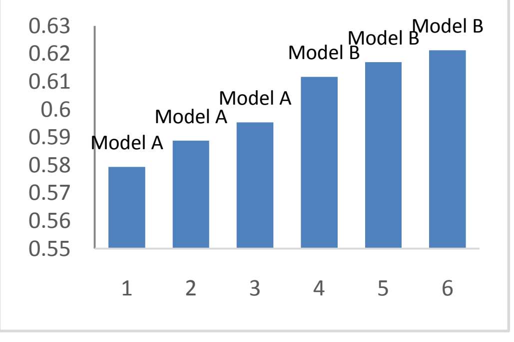

Figure 1 Probabilities of static damages less than 3 for Models (A) and (B) in the examples 1 to 6

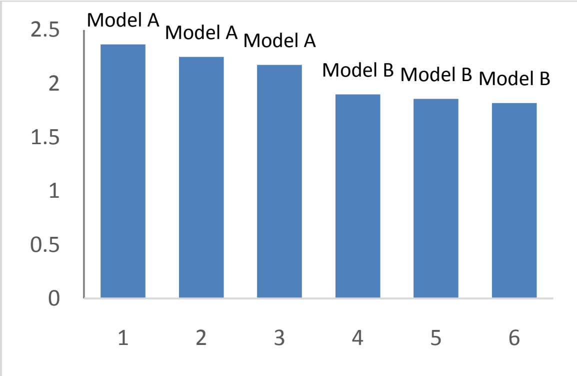

Figure 2 Expected Static damages E(S) for Models (A) and (B)

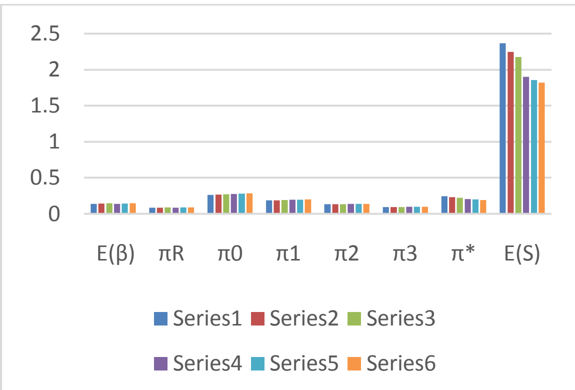

Figure 3 Combined chart for measure of Models (A) and (B)

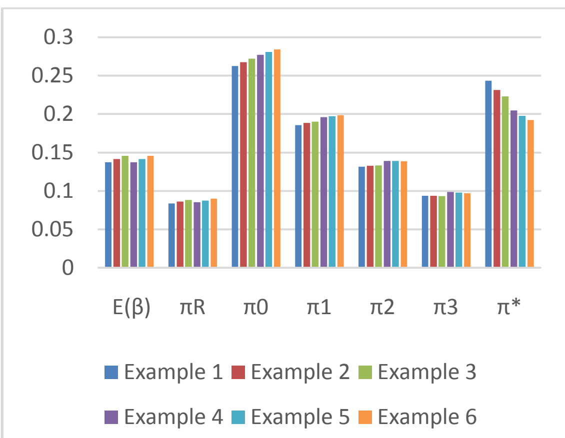

Figure 4 Probabilities of static damage levels for Models

5. CONCLUSION.

Considering a continuous time Markov chain approach, the backlog level probabilities of occurred static failures, fatigue failures and various other measures are obtained. From this numerical studies, it is found that the replacement of the system at a suitable static failure level (Model B) is advantageous because the probabilities of lower level static damages are more and E(S) values are less when compared to not replacing the system for periodic static failures (Model A). Cost of replacement and cost of repair of static failures if introduced in some form for fixing the optimal stopping level M is another area of vital interest for future studies in this field.

6. ACKNOWLEDGMENT:

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