

# Design Optimal PID Controller for Quad Rotor System

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## ABSTRACT

Quad rotor aerial vehicles are one of the most flexible and adaptable platforms for undertaking aerial research. Quad rotor in simplicity is rotorcraft that has four lift-generation propellers (four rotors), two rotors rotate in clockwise and the other two rotate anticlockwise, the speed and direction of Quad rotor can be controlled by varying the speed of these rotors. This paper describes the PID controller has been used for controlling attitude, Roll, Pitch and Yaw direction. In addition, optimal PID controller has been achieving by using particle swarm optimization (PSO), Bacterial Foraging optimization (BFO) and the BF-PSO optimization. Finally, the Quad rotor model has been simulating for several scenarios of testing.

## Keywords

Quad rotor, PID controller, particle swarm (PSO), Bacterial Foraging optimization (BFO) and the BF-PSO optimization.

## 1. INTRODUCTION

In recent years, the using of a Quad rotor for different applications has been increase rapidly. The Quad rotor has clear advantages such as higher manoeuvrability, low cost, decreased radar signature, strength, as well as decreased risk of human life. These advantages lead the use of these vehicles more often in various applications such as surveillance, reconnaissance, inspection for natural disasters, as a remote sensor for atmospheric measurements, inspection of pipelines or power lines, or for aerial photography [1-2].

The Quad rotor control system design challenges are difficulties in modelling, complexity, expensive, power consumption, and selection suitable control method. The PID controller is a simple control method, lower software complexity, and it requires low speed processing with low memory storage. Therefore, the PID can be implementing by simple and small size microcontroller.

The reason of this status is for its simple structure, which can be easy to understood and implemented. In addition, it presents robust performance within a large range of operating conditions [3]. Despite the popularity, the tuning aspect of PID parameters is not easy for researchers and plant operators. Several methods have been developed in literatures for determining the PID parameters, which is first found by Ziegler Nichols tuning [3]. In general, it is often hard to determine optimal PID parameters with the Ziegler-Nichols formula in many industrial plants [4]. Artificial intelligence (AI) techniques such as neural network, fuzzy system, and neural-fuzzy logic have been widely applied to proper tuning of PID controller parameters. Many random search methods, such as genetic algorithm (GA), simulated annealing (SA) and Chaotic algorithm have recently received much attention for achieving high efficiency and searching global optimal solution in problem space.[5]

Particle swarm optimization (PSO) as one of the modern heuristic algorithms, was developed through simulation of a simplified social system, and has been found to be robust in

solving continuous nonlinear optimization problems. The PSO technique can generate a high-quality solution within shorter calculation time and stable convergence characteristic than other stochastic methods [4, 6]. Bacterial Foraging Optimization (BFO) is a population-based numerical optimization algorithm. Until date, BFO has been applied successfully to some engineering problems, such as optimal control, harmonic estimation, transmission loss reduction and machine learning [7,8]. However, experimentation with complex optimization problems reveals that the original BFO algorithm possesses a poor convergence behaviour compared to other nature-inspired algorithms and its performance also heavily decrease with the growth of the search space dimensionality[8]. BF-PSO algorithm combines both BFO and PSO. The aim is to make PSO ability to exchange social information and BF ability in finding new solution by elimination and dispersal, a unit length direction of tumble behaviour is randomly generated.

In this paper, Quad rotor mathematical model will be achieve, and then the PID controller will be design to control the Quad rotor system. Optimal PID controller has been achieve by PSO, BFO, and BF-PSO optimization algorithms. Several tested scenarios have been achieve to simulate the Quad rotor model and measure the performance of Quad rotor PID controller. In section 2 explain the structure of Quad rotor ,in section 3 show the dynamic model of Quad rotor ,section 4 applied at control part of Quad rotor ,section 5 explain methods to find optimal PID controller of Quad rotor

## 2. STRUCTURE OF QUAD ROTOR

The Quad rotor model is consisting of four rotors powered by electrical motors and fixed at each corner of the + frame as shown in Fig. 1. The motors (M1 and M3) are rotating in the same direction (e.g.. Clockwise) while motors (M2 and M4) are rotating in the other direction (counter clockwise). The motion speed and direction of Quad rotor can be controlled by controlling the speed of these motors [2, 4 and 6].

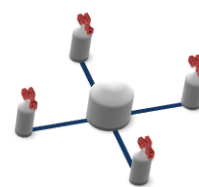


Fig.1: Top view of Quad rotor

However, the dynamics of Quad rotor and specifically it was low damping rate can make the vehicle difficult to control. The modern Quad rotor has a brushless DC motor (BLDC) because the BLDC motor has high power and energy to weight ratio [4]

### 3. DYNAMIC MODEL OF QUAD ROTOR

Dynamic model of Quad rotor is divided into two parts; the first part called dynamics equations, which the position of a Quad rotor has been evaluated by these equations, and another called rotational equations (Roll, Pitch, and Yaw rotation) which used to evaluate the direction of motion. According to Newton low, the full Quad rotor dynamic model for rotation and translation motion.

Linear acceleration in X-axis direction:

$$\ddot{x} = \frac{u_1(\sin \varphi \sin \theta + \cos \varphi \sin \theta \cos \phi)}{m_o} - k_1 \dot{x} \quad (1)$$

Linear acceleration in Y-axis direction:

$$\ddot{y} = \frac{u_1(\sin \varphi \sin \theta \cos \phi - \sin \varphi \sin \phi)}{m_o} - k_2 \dot{y} \quad (2)$$

Linear acceleration in Z-axis direction:

$$\ddot{z} = \frac{u_1 \sin \varphi \sin \theta}{m_o} - g - k_3 \dot{z} \quad (3)$$

Rolling angular acceleration in the X- directions:

$$\ddot{\phi} = \dot{\theta} \dot{\phi} I_x - \frac{J_r}{I_{xx}} \dot{\theta} \omega + \frac{1}{I_{xx}} u_2 - k_4 \dot{\phi} \quad (4)$$

Pitching angular acceleration in the X-directions:

$$\ddot{\theta} = \dot{\phi} \dot{I}_y + \frac{J_r}{I_{yy}} \dot{\phi} \omega + \frac{1}{I_{yy}} u_3 - k_5 \dot{\theta} \quad (5)$$

Yawing angular acceleration in the X- directions:

$$\ddot{\phi} = \dot{\theta} \dot{I}_z + \frac{1}{I_{zz}} u_4 - k_6 \dot{\phi} \quad (6)$$

Where  $J_r$  is moment of inertia of Quad rotor,  $I_{xx}$ ,  $I_{yy}$  and  $I_{zz}$  are moment inertia of a Quad rotor for directions (X, Y and Z) [8, 10]. The moment inertia of Quad rotor can be found by

$$J_r = 2 \left[ \frac{ml^2}{4} + \frac{mh^2}{12} \right] + \frac{m_o R^2}{4} + \frac{m_o H^2}{12} \quad (7)$$

The moment inertia about X, Y and Z-axis are approximated by equations below:

$$I_{xx} = \frac{m\rho^2}{2} + \frac{mh^2}{2} + 2ml^2 + \frac{m_o R^2}{4} + \frac{m_o H^2}{12} \quad (8)$$

$$I_{yy} = \frac{m\rho^2}{2} + \frac{mh^2}{2} + 2ml^2 + \frac{m_o R^2}{4} + \frac{m_o H^2}{12} \quad (9)$$

$$I_{zz} = 4ml^2 + \frac{m_o R^2}{2} \quad (10)$$

Where  $m, h, \rho$  is mass, height and radius of the motors, and  $m_o, H, R$  is mass, height and radius of Quad rotor,  $l$  is radius of rotation. In addition,  $I_x, I_y$  and  $I_z$  are described by

$$I_x = \frac{I_{yy} - I_{zz}}{I_{xx}} \quad (11)$$

$$I_y = \frac{I_{zz} - I_{xx}}{I_{yy}} \quad (12)$$

$$I_z = \frac{I_{xx} - I_{yy}}{I_{zz}} \quad (13)$$

And  $u_1, u_2, u_3,$  and  $u_4$  are inputs of Quad rotor and  $\omega$  is a disturbance in speed of Quad -rotor that can be found by [1-3]

$$u_1 = b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_4^2) \quad (14)$$

$$u_2 = b(\omega_4^2 - \omega_2^2) \quad (15)$$

$$u_3 = b(\omega_3^2 - \omega_1^2) \quad (16)$$

$$u_4 = d(\omega_2^2 + \omega_4^2 - \omega_1^2 + \omega_3^2) \quad (17)$$

$$\omega = \omega_2 + \omega_4 - \omega_1 - \omega_3 \quad (18)$$

Where  $\omega_1, \omega_2, \omega_3$  and  $\omega_4$  are the speeds of motors of Quad rotor, and  $\omega$  is a disturbance in the speed of motors.

### 4. PID CONTROLLER STAGE

A method used to control the attitude, Roll angle, Pitch angle and Yaw angle has been presented in this section. The PID controllers are widely used in different application due to their simplicity; this controller has the advantage of an easy implementation and proven stability while taking nonlinearity of the parameters. The based control laws of PID controller for height and other directions using the following form [3, 4]

$$u_1 = k_{patt}(z_{ref} - z) + k_{datt}(z_{ref} - z) \frac{s}{0.1s+1} + k_{iatt} \frac{1}{s} (z_{ref} - z) \quad (19)$$

$$u_2 = k_{p\phi}(\phi_{ref} - \phi) + k_{d\phi}(\phi_{ref} - \phi) \frac{s}{0.1s+1} + k_{i\phi} \frac{1}{s} (\phi_{ref} - \phi) \quad (20)$$

$$u_3 = k_{p\theta}(\theta_{ref} - \theta) + k_{d\theta}(\theta_{ref} - \theta) \frac{s}{0.1s+1} + k_{i\theta} \frac{1}{s} (\theta_{ref} - \theta) \quad (21)$$

$$u_4 = k_{p\varphi}(\varphi_{ref} - \varphi) + k_{d\varphi}(\varphi_{ref} - \varphi) \frac{s}{0.1s+1} + k_{i\varphi} \frac{1}{s} (\varphi_{ref} - \varphi) \quad (22)$$

### 5. CALIBRATION QUAD ROTOR MODEL

The simulation model developed so far is an un-calibrated model. For calibration, set of real values of various parameters is needed, which is taken from practical system. Table 1 shows the list of various quantities used for calibrate Quad rotor model. The UFO mini aircraft Quad rotor has been used for model calibration.

**Table1: Calibration Data for the Quad rotor**

Parameter Name	Symbol	Numerical Value	Unit
Rotational inertia along x-axis, y-axis	$I_{xx}, I_{yy}$	0.0019	Kg.m <sup>2</sup>
Rotational inertia along z-axis	$I_{zz}$	0.0033	Kg.m <sup>2</sup>
Rotor inertia	$J_r$	0.0099	Kg.m <sup>2</sup>
Motor mass	$m$	30	G
Total mass	$m_o$	0.625	Kg
Pro area	$A$	0.3	m <sup>2</sup>
Prop radius	$R_p$	0.15	M
Arm length	$L$	0.96	M
Air density	$\rho_a$	1.1	Kgm <sup>-3</sup>
Height motor	$h$	0.02	m
Height Quad rotor	$H$	0.10	m
Drag constant	$D$	7.5E-7	N.s <sup>2</sup>
Acceleration due to gravity	$g$	9.81	m.s <sup>-2</sup>

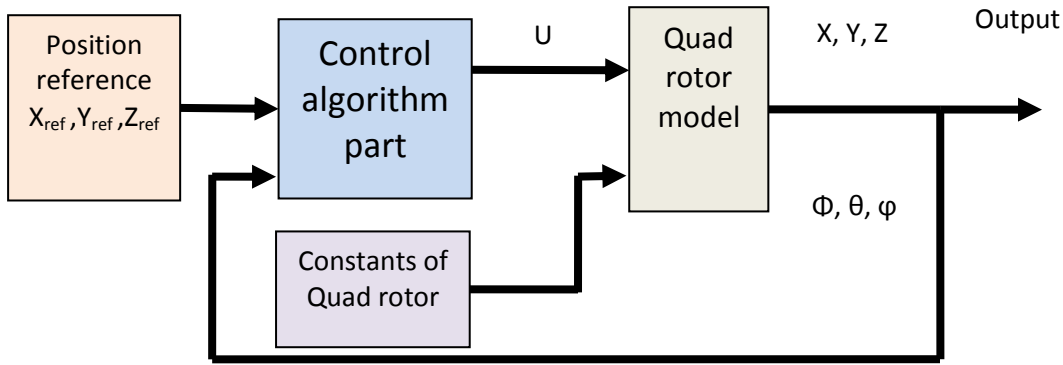


Fig.2 Block diagram of Quad rotor

In paper , a several scenarios taken for path of Quad rotor using different methods of PID control and show the difference between them and discuss the best of them.

### 5.1 Simulation at trial PID controller

The first scenario is the QR travel from ground to 5m height under trial PID controller. The figure (a) appear the height and error of height of movement of the Quad rotor, Figure(b) show the angles of Quad rotor at movement , figure(c) show the path of Quad rotor move in XY plan , figure (d) show path of Quad rotor in 3d dimensions.

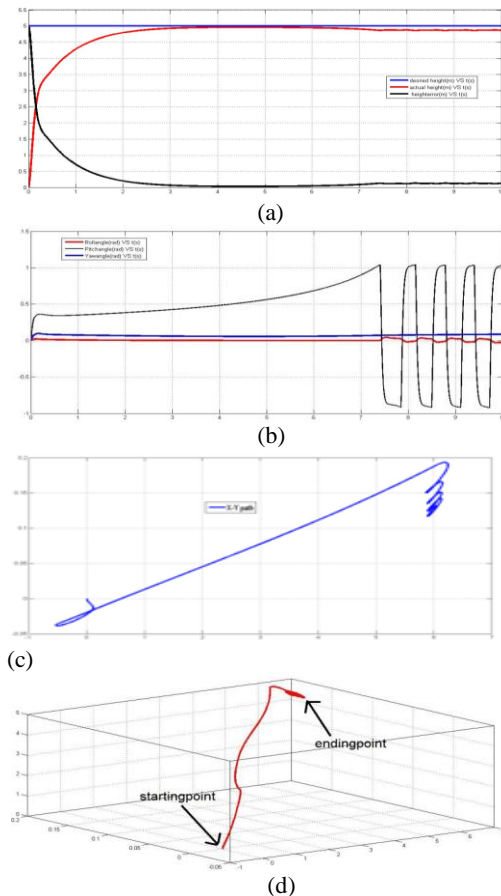


Figure (3): First Scenario of Qr

#### A. Particle Swarm Optimization

Particle Swarm Optimization is a technique used to find the parameters required to optimize a particular objective. This technique, first described by James Kennedy and Russell C. Eberhart in 1995 [9], originates from two separate concepts:

the idea of swarm intelligence based on the observation of swarming habits by certain kinds of animals (such as birds and fish), and the field of evolutionary computation.

The PSO algorithm works by simultaneously maintaining several candidate solutions in the explore space. In each iteration of the algorithm, each candidate solution that can be thought of as a particle flies through the search space to find the maximum or minimum of the cost function (fitness value) [9]. PID controller of the Quad rotor is optimization using PSO algorithm is developed to improved the step response of Quad rotor. The PSO algorithm is mainly utilized to determine  $k_p$ ,  $k_i$  and  $k_d$  for attitude , Roll ,Pitch and Yaw optimal parameters.

The searching produce of implemented PSO-PID controller in flowchart below.

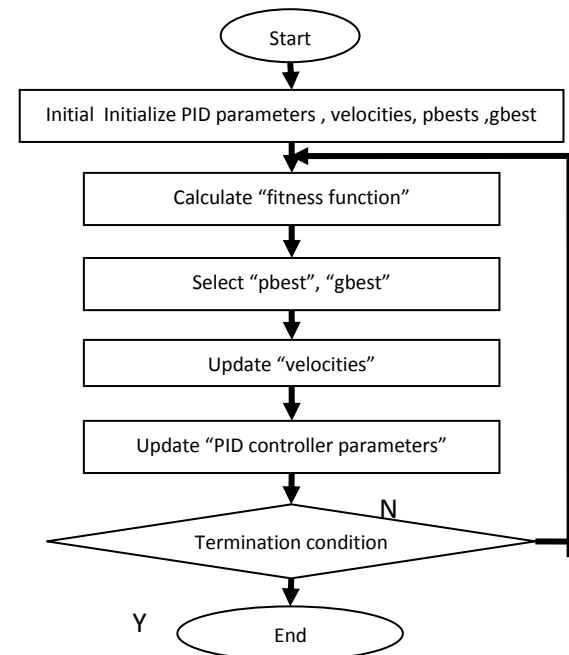


Figure (4): The PSO-PID flowchart

The update of member velocity and PID controller parameters as follow:

$$v_{j,g}(t+1) = \omega \cdot v_{j,g}(t) + c_1 r_{11} [pbest_{i,g}(t) - K_{j,g}(t)] + c_2 r_{12} [gbest_{i,g}(t) - K_{j,g}(t)] \quad (23)$$

$$K_{j,g}(t+1) = K_{j,g}(t) + v_{j,g}(t+1) \quad (24)$$

Where  $j=1,2,...,n$  and  $g=1,2,...,m$ , and  $K=PID$  parameters vector.

The second scenario is the QR travel from ground to 5m height under PSO-PID controller. The figure (a) appear the height and error of height of movement of the Quad rotor, Figure(b) show the angles of Quad rotor at movement, figure(c) show the path of Quad rotor move in XY plan, figure (d) show path of Quad rotor in 3d dimensions.

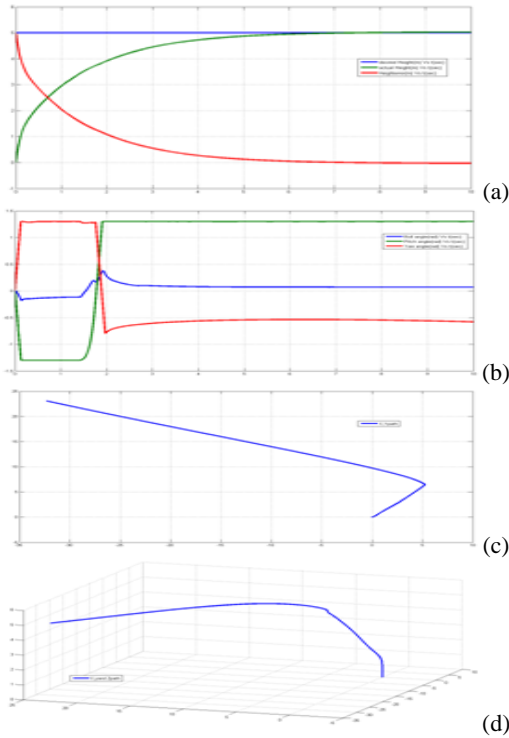


Figure (5): Second scenario of quad rotor.

### B. Bacterial Foraging Optimization

Bacterial foraging optimization (BFO) has emerged as a powerful technique for the solving optimization problems. BFO mimics the foraging delineation of *E. coli* bacteria, which attempt to maximize the energy intake per unit time. From the very early days, it has drawn attention of researchers due to its effectiveness in the optimization domain. To improve its performance, a large number of modifications have already been undertaken. The bacterial foraging system consists of four principal mechanisms, namely chemotaxis, swarming, reproduction and elimination-dispersal. A brief description of each of these processes along with the pseudo-code of the complete algorithm is described below.[8]

The update PID controller parameters follow

$$K^i(j+1, k, l) = K^i(j, k, l) + c(i) \frac{\Delta(i)}{\sqrt{\Delta^i(i)\Delta(i)}} \quad (25)$$

$$J_{cc}(K, P(j, k, l)) = \sum_{i=1}^s J_{cc}(K, K^i(j, k, l)) \quad (26)$$

Where  $K$  is PID controller parameters vector, where  $J_{cc}(K, P(j, k, l))$  is the objective function value to be added to the actual objective function (to be minimized) to present a time varying objective function. The fitness function is over shoot and error in height.

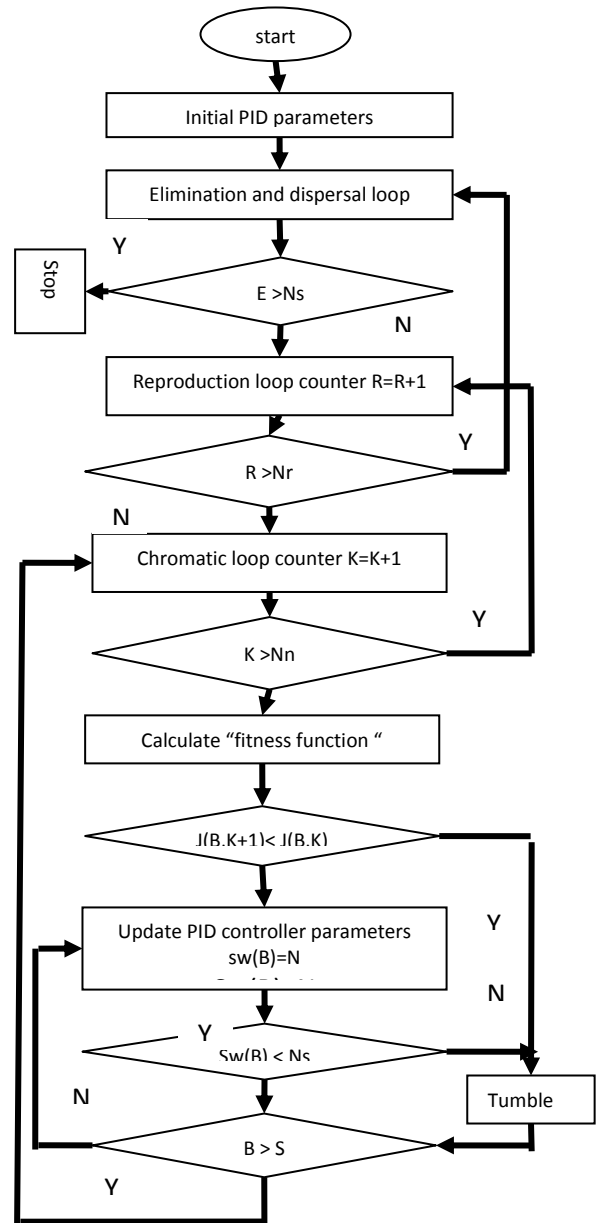
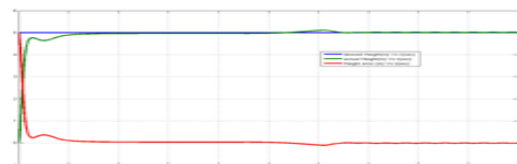


Figure (6): Flow chart of BFO

The third scenario is the QR travel from ground to 5m height under PFO-PID controller. The figure (a) appear the height and error of height of movement of the Quad rotor, Figure(b) show the angles of Quad rotor at movement, figure(c) show the path of Quad rotor move in XY plan, figure (d) show path of Quad rotor in 3d dimensions.



(a)

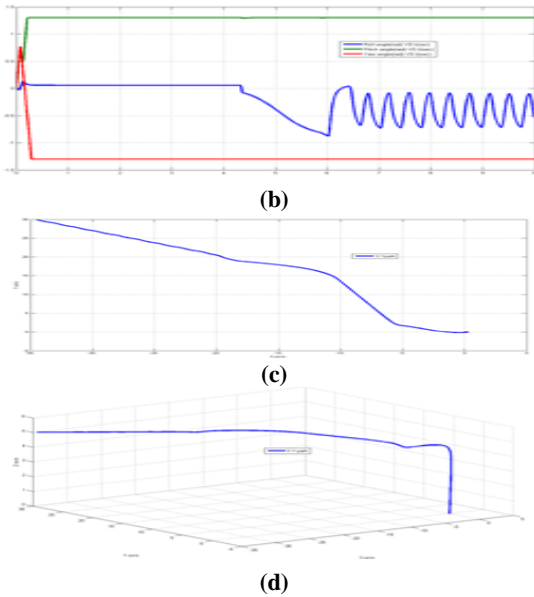


Figure (7): Third scenario of Quad rotor.

**C. Mixture between PSO and BFO BF-PSO Optimization**  
BF-PSO algorithm combines both BFO and PSO. The aim is to make PSO ability to reciprocity social information and BF ability in finding new solution by elimination and dispersal, a unit length direction of tumble behavior is randomly generated. Random direction may lead to delay in reaching the global solution. In "BF-PSO" algorithm the unit length random direction of tumble behavior can be decided by the global best position and the best position of each bacterium. During the chemotaxis loop tumble direction is updated by:

$$K(j+1) = \omega K(j) + c1 \text{rand}(pbest - pcurrent) + c2 * \text{rand}(gbest - pcurrent) \quad (27)$$

Where pbest is the best position of each bacterium and gbest is the global best bacterium, K is PID controller parameters. And the flow chart same of BFO in figure (6).

The fourth scenario is the Quad rotor travel from ground to 5m height under BF-PSO-PID controller. The figure (a) appear the height and error of height of movement of the Quad rotor, Figure(b) show the angles of Quad rotor at movement, figure(c) show the path of Quad rotor move in XY plan, figure (d) show path of Quad rotor in 3d dimensions.

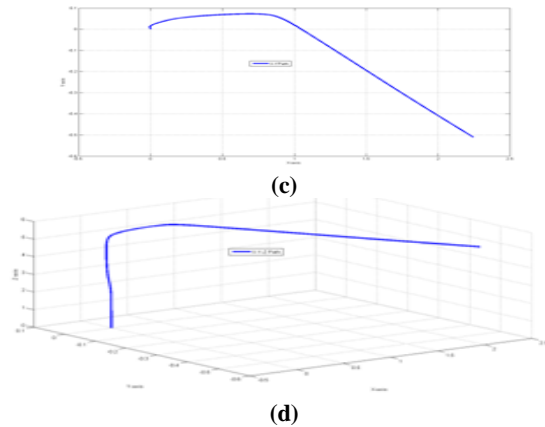
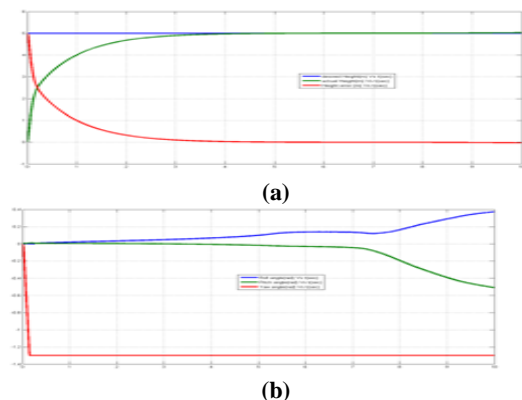


Figure (8): fourth scenario of Quad rotor

After optimization PID controller of Quad rotor parameters, the properties of the Quad rotor when it travelled in step response of 5m height put in table below, from figures above the table is:

Table 2 Properties of Quad rotor

	Trial-PID	PSO-PID	BFO-PID	BF-PSO-PID
Rise time(s)	4.49	7.05	5.13	3.12
Settling time(s)	0.5	0.75	0.6	0.23
Settling time Min(s)	4	6.3	4.5	2.85
Settling time Max(s)	4.5	7.1	5.1	3.13
Over shoot(m)	0.08	0	0.15	0
Under shoot(m)	0	0	0.2	0
Peak value(m)	5.08	5	5.15	5
Peak time(s)	6	7.25	5.5	3.2
Time of operation(H)	-	9	21	30
$k_{patt}$	10	55.800	15.079	18.002
$K_{iatt}$	1.9	16.578	10.532	5.9001
$K_{datt}$	4.65	69.738	5.5955	10.300
$k_{pRoll}$	5.98	27.944	10.711	1.0809
$K_{iRoll}$	2.43	3.4485	0.32	0.0001
$K_{dRoll}$	3.98	4.0247	1.4415	-0.0195
$k_{pPitch}$	0.23	2.4953	6.3014	-1.5675
$K_{iPitch}$	3.56	15.818	2.5735	-0.0050
$K_{dPitch}$	2.13	-9.1253	0.5999	-0.0012
$k_{pYaw}$	0.98	-5.0796	10.969	0.350
$K_{iYaw}$	0.64	2.5795	0.345	-0.003
$K_{dYaw}$	2.75	25.37	1.229	-0.008

## 6. CONCLUSIONS

In this paper, the architecture of a QR is described and then the dynamic model of aircraft is analyzed. Several scenarios have been achieved in order to emulate the motion of QR and then the results have been recorded. In the case of the model PID controller the flying to the specific height has been achieved with a good result as shown in Figures above.

The trial and error has small steady state error in height, but in PSO it delete the steady state error but the response is too slow. In PFO the response is fast but the height has error in height and over shoot. In PSO-BFO the response is good, no over shoot and no steady state error in height, as shown in figures above.

The parameters of QR using trial and error take more than one week to find them, and in PSO take nearly 9 hours, and in BFO take 21 hours and PSO-BFO it take 30 hours.

The best response is using BF-PSO. The proposed approach with new defined time-domain cost function, has a very easy implementation, stable convergence characteristic and ability of fast tuning of optimum PID controller parameters that requires fewer number of iterations.

Due to the simplicity of PID controller, this type of controller can be achieved by the small size of processors like microcontrollers, ARM, FPGA, Etc. So it can be used in small size QR system.

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