# Particle/Kalman Filter for Efficient Robot Localization 

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#### Abstract

This paper presents a comparison of different fitters namely: Extended Kalman Filter (EKF), Particle Filter (PF) and a proposed Enhanced Particle / Kalman Filter (EPKF) used in robot localization. These filters are implemented in matlab environment and their performances are evaluated in terms of computational time and error from ground truth and the results are reported. The considered robot localizer uses radio beacons that provide the ability to measure range only. Since EKF and its variants are not capable to efficiently solve the global localization problem, we propose the Enhanced Particle / Kalman Filter (EPKF) which provide the required initial location to address this drawback of EKF. We propose using PF as Initialization phase to coarsely predict the initial location and numerous sets of data are experimented to get robust conclusion. The results showed that the proposed localization approach which adopts the particle filter as initialization step to EKF achieves higher accuracy localization while, the computational cost is kept almost as EKF alone.


## General Terms

Algorithms and robotics

## Keywords

Particle Filter, Extended Kalman Filter, Robot Localization

## 1. INTRODUCTION

The problem of robot localization is known as answering the question Where am I or determining the place of the robot. This means that the robot is trying to locate it in comparison to the surrounding environment. When we research for location, pose, or position we mean the x and y coordinates and heading direction of a robot in a global coordinate system.
The mobile robot localization problem comes in many multiple flavors. Position tracking is the simplest localization problem where the initial robot pose is known and the problem is to compensate incremental errors in a robot's odometry. Global localization problem [1] is another challenging one, where a robot is not told its initial pose but instead has to determine it from scratch.

Several methods to support the robot localization problem showed in [2.3]. The Kalman filter applied often to the problem of localization of the robot. It works frequently, and it does not require a history of previous states of the robot. This results in a simplified algorithm that can run on the online in real-time systems. Unfortunately, the absolute rang measurements is a set of nonlinear (as in our case), which require the use of the Kalman Filter Extended (EKF), which must be linearized the measurements around the current state estimate. These results in a weakness common to all linear methods which means that the kalman filter will not converge when the initial state is not sufficiently accurate [4]

Recently Particle Filter (PF) becomes public approach used for treatment of this problem. This is due to its ability to deal with the problem of non-linear non-Gaussian problem, typical
features of the problem of localization [5.6] this. Several applications of PF in [7-9].In this paper, the particle filter is introduced to initialize kalman filter to overcome the initial state problem of original kalman filter. Different filters namely Kalman filter (KF), Particle Filter (PF) and a proposed Enhanced Particle/Kalman Filter (EPKF) implemented in Matlab environment and their performance are evaluated in terms of computational complexity and amount of error from ground truth .The obtained results are reported and compared.

This paper is organized as follows: section 2 presents overview of an Enhanced Particle/Kalman filter and their implementation algorithms, section 3 studied the effect in Robot Localization by using different filters, section 4 shows the discussion of the obtained results, and finally section 5 is devoted to conclusion.

## 2. OVERVIEW OF AN ENHANCED PARTICLE / KALMAN FILTER

The used absolute range measurements are non-linear, requiring the use of an Extended Kalman Filter (EKF), The kalman filter there is modified to filter known as extended kalman filter.

### 2.1 Extended Kalman Filter (EKF) for Localization

## [1] The motion model [10]

If the robot pose (position and heading) at time k is represented by the state vector $q_{k}=\left[x_{k}, y_{k}, \theta_{k}\right]^{T}$ then the motion model of the wheeled robot used in this experiment are completely-modeled by the following non-linear equations:
$q_{k+1}=\left[\begin{array}{l}X_{k}+\Delta D_{k} \cos \left(\theta_{k}\right) \\ y_{k}+\Delta D_{k} \sin \left(\theta_{k}\right) \\ \theta_{k}+\Delta \theta_{k}\end{array}\right]+v_{k}$
Where: vk is a noise vector. Here, $\Delta \mathrm{Dk}$ point at the center of the robot's front axle, obtained by averaging the distances measured by the left and right wheel encoders. The incremental orientation change $\Delta \theta \mathrm{k}$ is obtained by the onboard gyro. These dead reckoning measurements forming the control input vector $u_{k}=\left[\Delta D_{k}, \Delta \theta_{k}\right]^{T}$

The system matrix A (k) is represented by the Jacobian:

$$
A(k+1)=\left.\frac{\partial f}{\partial q_{k}}\right|_{q-\hat{q}_{k}}=\left[\begin{array}{ccc}
1 & 0 & -\Delta D_{k} \sin \left(\theta_{k}\right)  \tag{2}\\
0 & 1 & \Delta D_{k} \cos \left(\theta_{k}\right) \\
1 & 0 & 1
\end{array}\right]
$$

The input gain matrix $B(k)$ is was built similarly:

$$
B(k+1)=\left.\frac{\partial f}{\partial q_{k}}\right|_{q-\hat{q}_{k}}=\left[\begin{array}{cc}
\cos \left(\theta_{k}\right) & 0  \tag{3}\\
\sin \left(\theta_{k}\right) & 0 \\
0 & 1
\end{array}\right]
$$

[2] The measurement model [10]:
At time $k+1$ the range from a beacon located at $(x b, y b)$ to the robot with state vector $\mathrm{q}_{\mathrm{k}+1}$ can be written as:

$$
\begin{equation*}
h\left(q_{k+1,}\left[x_{b}, y_{b}\right]^{T}\right)=\sqrt{\left(x_{k+1}-x_{b}\right)^{2}+\left(y_{k+1}-y_{b}\right)^{2}} \tag{4}
\end{equation*}
$$

## [3] Time Propagation [11]

When a new control input vector $u(k)=\left[\Delta D_{k}, \Delta \theta_{k}\right]$ is received, the robot's state is updated according to the process model equation. Using the standard equations of kalman filtering, the covariance matrix maintaining our uncertainty about the current state is propagated in time:

$$
\begin{equation*}
p_{k}^{-}=A(k) p_{k-1}^{+} A(k)^{T}+B(k) \Gamma B(k)^{T}+Q(k) \tag{5}
\end{equation*}
$$

So, the state maintained during the time propagation step indicates the pose of the robot at the robot reference point.
[4] The measurement update: [12]
When a measurement is obtained, using the method of the update step is broken up as follows:

* Shift the robot reference point's coordinates to get the coordinates of the current antenna, (xa, ya)).
*Expecting the current measurement onto the xy plane of the robot.
* Using (xa, ya) and the known beacon location (xb, yb), compute Hk.
* Find the variance $\mathrm{k}_{\mathrm{R}}$ and the mean k y associated with the current measurement from its previous stored PDF.
* Using the measurement model, compute the expected range rk to the beacon. Let $v(\mathrm{k})=\mathrm{y}-\mathrm{r}$ be the innovation.
* Compute $S_{k}=H_{k} p_{k}^{-} H_{k}^{T}+R_{k}$
* Compute the Kalman gain $K_{k}=p_{k}^{-} H_{k}^{T} S_{k k}^{-1}$
* Compute the normalized innovation squared and tests the measurement against the chi square
* If the measurement passes the gating test, update the state by letting $\hat{q}_{k}^{+}=\hat{q}_{k}^{-}+K_{k} v(k)$ and update the covariance matrix by letting $p_{k}^{+}=p_{k}^{-}-K_{k} S_{k} K_{k}^{T}$.
* Now, employ this updated estimate of the pose at the antenna which reported the current measurement, shift back in $x$ and $y$ to get the updated pose estimate at the robot reference point.


### 2.2 Particle Filter Algorithm

The PFs are formulated on the concepts of the Bayesian theory and the sequential importance-sampling which are very effective in dealing with non-Gaussian and non-linear problems [13-16]

The PF approximates recursively the posterior distribution using a finite set of weighted samples. The idea is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on these samples and weights. PF uses the probabilistic system transition model $\mathrm{p}\left(\mathrm{X}_{\mathrm{t}} \mid \mathrm{X}_{\mathrm{t}-1}\right)$, (which describes the transition for state vector $X_{t}$ ) to predict the posterior at time $t$ as:

$$
\begin{equation*}
p\left(X_{t} \mid Z_{\mathrm{l}: \mathrm{t}-1}\right)=\int p\left(X_{t} \mid X_{t-1}\right) p\left(X_{t-1} \mid Z_{\mathrm{l}: \mathrm{t}-1}\right) d X_{\mathrm{t}-1} \tag{6}
\end{equation*}
$$

Where $Z_{1}$ : ${ }_{t-1}=\left\{Z_{1}, Z_{2}, \ldots . Z_{t-1}\right\}$ are available observations at times $1,2, \ldots ., \mathrm{t}-1, \quad \mathrm{p}\left(\mathrm{X}_{\mathrm{t}} \mid \mathrm{X}_{\mathrm{t}-1}\right)$ expresses the motion model, $\mathrm{p}\left(\mathrm{X}_{\mathrm{t}-1} \mid \mathrm{Z}_{1: \mathrm{t}-1}\right)$ is posterior probability density function at time $\mathrm{t}-1$ and $\mathrm{p}\left(\mathrm{X}_{\mathrm{t}} \mid \mathrm{Z}_{1: \mathrm{t}-1}\right)$ is the prior Probability Density Function (PDF) at time $t$. At time $t$, the observation $Z_{t}$ is available, then the state can be updated using Bayes's rule as:

$$
\begin{equation*}
p\left(X_{t} \mid Z_{1: t}\right)=\frac{p\left(Z_{t} \mid X_{t}\right) p\left(X_{t} \mid Z_{1: t-1}\right)}{p\left(Z_{t} \mid Z_{1: t-1}\right)} \tag{7}
\end{equation*}
$$

Where $p\left(Z_{t} \mid X_{t}\right)$ is described by the observation equation. The posterior PDF $\mathrm{p}\left(X_{t-1} \mid Z_{t-1}\right)$ is approximated recursively as a set of $N$ weighted samples $\left\{X_{t-1}^{(s)}, W_{t-1}^{(s)}\right\}_{s=1}^{N}$, and $W_{t-1}^{(s)}$ is the weight for particle $X_{t-1}^{(s)}$. Using a Monte Carlo approximation of the integral, we get:

$$
\begin{equation*}
p\left(X_{t} \mid Z_{t}\right)=p\left(Z_{t} \mid X_{t}\right) \sum_{s=1}^{N} W_{t-1}^{(s)} p\left(X_{t} \mid X_{t-1}^{(s)}\right) \tag{8}
\end{equation*}
$$

The $N$ samples $X_{t}^{(s)}$ are drawn from the proposal distribution:

$$
\begin{equation*}
q(X)=\sum_{s=1}^{N} W_{t-1}^{(s)} p\left(X_{t} \mid X_{t-1}^{(s)}\right) \tag{9}
\end{equation*}
$$

Then it is weighted by the likelihood.

$$
\begin{equation*}
W_{t}^{(s)}=p\left(Z_{t} \mid X_{t}^{(s)}\right) \tag{10}
\end{equation*}
$$

This produces a weighted particle approximation
$\left\{X_{t-1}^{(s)}, W_{t-1}^{(s)}\right\}_{s=1}^{N}$ for the posterior $\operatorname{PDF} \mathrm{p}\left(X_{t} \mid Z_{t}\right)$ at time $t$.

## 3. IMPLEMENTATION AND RESULTS OF ROBOT LOCALIZATION ALGORITHM USING STUDIED FILTERS

We studied a localization system which uses radio beacons that provide the ability to measure range only [18]. Obtaining range from radio beacons has the advantage that line of sight between the beacons and the transponder is not required, and the data association problem can be completely avoided. In this work from 7-10 radio beacons are distributed over the areas of robot movement. Robot is programmed to move in a
repeating path. All studied filters approaches are used to fuse range data with dead reckoning data collected from a real system which integrates proprioceptive measurements from wheel encoders, gyros, and accelerometers to localize the robot. Mat lab environment is used for experimenting with localization process.

Fig [1] shows the dead reckoning path, ground truth path and tag locations for all paths dataset [A1, A2, E1, E2, E4, B1, and B2] from reference [18].Therefore different filters approaches [17-19] are introduced to improve this performance.


Fig .1: The ground truth path, tag locations and dead reckoning of $\mathbf{B} 2$ datastet.

We notice from these figures that the dead reckoning tends to move away from the true path with the passage of time. This is due to increasing errors in odometry. This can be a good localization method for a short distance, but it provides no means of recovering from error that accumulates in nature.

### 3.1 The Results Of The Different

## Approaches

In this paper we have used numerous carefully-collected datasets and processed them with an extended kalman filter, a particle filter, and enhanced particle / kalman filter. Our implementation of particle filter in matlab environment requires no initial estimate of the robot's position. In all experiments, the robot's travel is clipped from results plot, giving the filter time to converge. Figs (2-4) show the results of studied filters. Table [1-7] summarizes the results of these figures concerning the error in the estimates of the studied filters.






E3




Fig.2: Particle filters localization performance on all datasets



E1



B1



Fig.3: Extended kalman filter localization performance on all datasets



E1



E3


B1



Fig. 4: Enhanced particle / kalman filter localization performance on all data sets

Table 1: Results of Error Calculation Using Different Filters in the Dataset (A1)

| Error/in meter | PF | EKF | EPKF |
| :---: | ---: | ---: | ---: |
| XTE_abs_avg | 3.5883 | 0.8787 | 0.8841 |
| XTE_abs_max | 23.4162 | 2.5697 | 2.5673 |
| XTE_abs_std | 3.622 | 0.5946 | 0.5983 |
| ATE_abs_avg | 5.3397 | 1.1241 | 1.1386 |
| ATE_abs_max | 32.7119 | 3.5205 | 3.5204 |
| ATE_abs_std | 5.0402 | 0.792 | 0.7908 |
| Cartesian_abs_avg | 7.0857 | 1.5502 | 1.5697 |
| Cartesian_abs_max | 33.3154 | 3.5315 | 3.5314 |
| Cartesian_abs_std | 5.4501 | 0.7748 | 0.7662 |

Table 2: Results of Error Calculation Using Different
Filters in the Dataset (A2)

| Error/in meter | PF | EKF | EPKF |
| :---: | ---: | ---: | :--- |
| XTE_abs_avg | 7.2004 | 0.6052 | 0.6119 |
| XTE_abs_max | 36.9777 | 1.8401 | 1.7059 |
| XTE_abs_std | 7.3529 | 0.3987 | 0.3952 |
| ATE_abs_avg | 8.768 | 0.5405 | 0.5368 |
| ATE_abs_max | 37.3803 | 1.6589 | 1.7392 |
| ATE_abs_std | 8.3345 | 0.3644 | 0.3603 |
| Cartesian_abs_avg | 12.5861 | 0.8862 | 0.8882 |
| Cartesian_abs_max | 40.9376 | 1.8673 | 1.8283 |
| Cartesian_abs_std | 9.6871 | 0.402 | 0.3996 |

Table3: Results of Error Calculation Using Different Filters in the Dataset (E1)

| Error/in meter | PF | EKF | EPKF |
| :---: | :--- | :--- | :--- |
| XTE_abs_avg | 3.6754 | 1.326 | 1.2913 |
| XTE_abs_max | 16.68 | 4.4408 | 4.4397 |
| XTE_abs_std | 3.4236 | 1.0372 | 1.0291 |
| ATE_abs_avg | 4.0507 | 1.2324 | 1.2153 |
| ATE_abs_max | 19.5827 | 5.0258 | 5.0247 |
| ATE_abs_std | 3.5535 | 1.0471 | 1.0429 |
| cartesian_abs_avg | 6.0728 | 2.0735 | 2.0494 |
| cartesian_abs_max | 19.7418 | 5.0439 | 11.7332 |
| cartesian_abs_std | 4.1693 | 1.061 | 1.0886 |

Table 4: Results of Error Calculation Using Different Filters in the Dataset (E2)

| Error/in meter | PF | EKF | EPKF |
| :---: | ---: | ---: | ---: |
| XTE_abs_avg | 1.9758 | 1.2717 | 1.0347 |
| XTE_abs_max | 5.7412 | 3.3569 | 3.3434 |
| XTE_abs_std | 1.2317 | 0.7432 | 0.6941 |
| ATE_abs_avg | 2.4837 | 1.4564 | 1.2113 |
| ATE_abs_max | 6.6696 | 3.74 | 3.7284 |
| ATE_abs_std | 1.735 | 0.958 | 0.8559 |
| Cartesian_abs_avg | 3.4695 | 2.1113 | 1.7586 |
| Cartesian_abs_max | 7.0373 | 3.7708 | 6.4165 |
| Cartesian_abs_std | 1.6 | 0.8338 | 0.8315 |

Table 5: Results of Error Calculation Using Different Filters in the Dataset (E3)

| Error/in meter | PF | EKF | EPKF |
| :---: | :--- | :--- | ---: |
| XTE_abs_avg | 2.185 | 0.996 | 0.951 |
| XTE_abs_max | 7.3308 | 2.8536 | 2.6634 |
| XTE_abs_std | 1.7212 | 0.6584 | 0.6414 |
| ATE_abs_avg | 2.4189 | 1.2685 | 1.1976 |
| ATE_abs_max | 8.321 | 3.6665 | 3.6664 |
| ATE_abs_std | 1.7934 | 0.8606 | 0.8544 |
| Cartesian_abs_avg | 3.7085 | 1.7548 | 1.6702 |
| Cartesian_abs_max | 8.3238 | 3.6802 | 9.632 |
| Cartesian_abs_std | 1.7466 | 0.82 | 0.8409 |

Table 6: Results of Error Calculation Using Different Filters in the Sixth Path Data (B1)

| Error/in meter | PF | EKF | EPKF |
| :---: | ---: | :---: | ---: |
| XTE_abs_avg | 3.2344 | 1.4218 | 1.8224 |
| XTE_abs_max | 8.7061 | 6.8547 | 5.0735 |
| XTE_abs_std | 2.5025 | 1.3133 | 1.2318 |
| ATE_abs_avg | 4.6406 | 2.0149 | 2.375 |
| ATE_abs_max | 8.7007 | 6.9627 | 5.4277 |
| ATE_abs_std | 2.4882 | 1.5216 | 1.222 |
| Cartesian_abs_avg | 6.3948 | 2.7671 | 3.3788 |
| Cartesian_abs_max | 9.405 | 7.0529 | 5.4761 |
| Cartesian_abs_std | 1.8826 | 1.5554 | 0.7362 |

Table 7: Results of Error Calculation Using Different Filters in the Seventh Path Data (B2)

| Error/in meter | PF | EKF | EPKF |
| :---: | ---: | ---: | ---: |
| XTE_abs_avg | 2.6273 | 2.2597 | 1.9056 |
| XTE_abs_max | 7.4856 | 6.4665 | 4.6961 |
| XTE_abs_std | 1.7841 | 1.5033 | 1.2534 |
| ATE_abs_avg | 2.1962 | 1.9174 | 1.6403 |
| ATE_abs_max | 8.2077 | 6.8337 | 4.888 |
| ATE_abs_std | 1.4938 | 1.4273 | 1.134 |
| Cartesian_abs_avg | 3.8918 | 3.3646 | 2.8993 |
| Cartesian_abs_max | 8.4053 | 6.845 | 4.9148 |
| Cartesian_abs_std | 1.4112 | 1.3065 | 0.8582 |

XTE:Cross Track Error, How far left or right of the true position our estimation is, Orthogonal to the true heading, ATE: Along Track Error, Tangential component of the position error ,Cartesian error: Total Euclidean distance error

## 4. DISCUSSION OF RESULTS

The results are summarized graphically using bar chart in Fig [4]. Chiefly, we consider the cross-track error (abbreviated XTE), which gives the component of position error that is orthogonal to the robot's path. We also present the along-track error (abbreviated ATE), which measures the tangential component of position error.


ATE, XTE and Cartesian errors A2


ATE, XTE and Cartesian errors E1



XTE,ATE and Cartesian errors E3



ATE, XTE and Cartesian errors B2


Fig.5: The EKF, the Enhanced EPKF and PF along-track error, the cross-track error and Cartesian errors

From Table [1-7] and fig [5] comparable results we notice the slight difference in calculated error among extended kalman filter and the proposed enhanced particle / kalman filter while the particle filter posses excessive error.
Considering computational complexity and time consumed in a Matlab run, Fig [6] shows the time consumed by each filter in the same environmental Conditions. There is a slight increase in time for the propose EPKF compared with EKF while the PF consumes higher time. Therefore, the proposed filter achieves the same results of EKF while keeping the computational cost reasonable and in the same time solving the problem inherent of all Kalman filters which require a defined initial state.

Table. 8 Summarizes the Average Time each Algorithm Requires to Incorporate an Incoming Range Measurement into the Robot Position Estimate

| Running Times | seconds per <br> measurement update |
| :--- | :--- |
| Particle Filter | 0.142494 |
| EKF | 0.007988 |
| EPKF: PF estimate an <br> initial state which to <br> seed the EKF. | 0.014385 |



Fig.6: Comparing the time required to update the robot pose estimate after a range measurement is taken.

## 5. CONCLUSION

This paper presents a study for the effect of several filter approaches in the behavior of robot localizer using radio beacons that provide the ability to measure range only. Different filters namely Extended Kalman Filter (EKF), Particle Filter (PF) and a proposed Enhanced Particle/Kalman Filter (EPKF) are implemented in Matlab environment and their behavior are evaluated. The Enhanced Particle/ Kalman Filter (EPKF) provide the required initial location while there is no significant change in the computational cost compared with Extended Kalman Filter (EKF). Moreover in some data sets, the performance of the proposed filter approach is superior in terms of localization errors.

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