Effects of Atmospheric Turbulence and Pointing Errors on Average Capacity of Free-Space Optical Communication Links

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ABSTRACT
In this paper, the average capacity of free-space optical (FSO) links over Gamma-Gamma turbulence channels with pointing errors is investigated. A FSO communication system using intensity modulation/direct detection (IM/DD) with on-off keying (OOK) is considered. Capacity performance is evaluated under various turbulence strengths and misalignment fading conditions. Further, effect of beam width, jitter variable and transmitted optical power on the average capacity is analyzed. Numerical examples are provided to demonstrate adverse effects of atmospheric turbulence and pointing errors on the capacity performance of the FSO links.

Keywords
Atmospheric turbulence, average capacity, free-space optical links, Gamma-Gamma turbulence channel, pointing errors

1. INTRODUCTION
Optical Wireless Communication (OWC) has emerged as an alternative to conventional radio frequency (RF) communication. OWC systems are most suitable for broadband applications due to their inherent wide bandwidth, easy deployment and freedom from licensing requirements. The OWC systems also provide greater security and are more resistant to interferences than RF systems. However, interaction of electromagnetic waves with the atmosphere at optical frequencies is stronger than that corresponding at microwave [1-4].

Apart from extinction due to molecules and aerosols suspended in the air, atmospheric turbulence and pointing errors are two random factors can severely degrade performance of the FSO links. Turbulent mixing of warm air at lower altitudes with cooler air at the higher altitude results in random fluctuations of air’s temperature. As a result of which, the refractive indices of the atmosphere fluctuates randomly [5-6]. Refractive index variation along the transmission path results in random frequency and phase fluctuations (scintillation) of the received signal. Scintillation induced fading may lead to complete loss of a signal [1, 9].

Platform/ building sway due to wind, differential heating and cooling, or motion of ground motion can result in severe misalignment of transmitter and receiver [9]. This phenomenon causes vibrations of the transmitted beam leading to misalignment (pointing errors) between the transmitter and receiver [11].

A. A. Farid and S. Hranilovic have studied outage capacity of the FSO links over atmospheric turbulence with pointing errors in log-normal and Gamma-Gamma turbulence fading channel in [6]. A closed form expression to investigate the error rate performance of FSO links over Gamma-Gamma fading channel is derived in [10]. However, the effect of atmospheric turbulence and pointing errors on the capacity performance of FSO links is not dealt with in [10]. So, in this paper, I investigate capacity performance of FSO links over Gamma-Gamma fading channels with pointing errors.

2. CHANNEL DESCRIPTION
A FSO link using intensity modulation/direct detection (IM/DD) with on-off keying (OOK), a popular technique to deploy commercial communication systems is considered in this paper. The data is modulated onto instantaneous intensity of the optical beam. The modulated beam propagates through Gamma-Gamma turbulence channel with additive white Gaussian noise (AWGN) in the presence of misalignment fading (pointing errors). The received optical beam is converted to electrical signal (photocurrent) at the receiver’s end. The generated photocurrent is function of incident optical power and is related through responsivity (R) of the detector. It is assumed that the receiver integrates photocurrent for each bit period and removes any bias due to background illumination. The received electrical signal is given by [7]

$$y = Rhx + n \tag{1}$$

Where y is the electrical signal output at the receiver, R is responsivity of the detector, h is normalized channel fading coefficient considered to be constant over a large number of transmitted bits, x is the binary transmitted signal and n is AWGN with variance σ_n^2. For unity responsivity, $y = hx + n$. Neglecting deterministic path loss, channel state is the product of random attenuation due to atmospheric turbulence and random attenuation due to pointing errors i.e.

$$h =. h_a h_p \tag{2}$$

Where, h_a is the attenuation due to atmospheric turbulence and h_p is fading induced due to pointing errors. The pdf of irradiance in Gamma-Gamma turbulence channel is given by [1, 15]

$$f_{h_a}(h_a)= \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} h_\alpha^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta h_\alpha})\mathrm{d}h_\alpha \tag{3}$$

Where $K_v(.)$ is the v^{th} order modified Bessel function of the second kind (sometimes called the Macdonald function or Basset function) [11, Eq. (6.106)], and $\Gamma(z) = \int_0^\infty e^{-t}t^{z-1}dt$, $Re(z) > 0$ is the Gamma function [12, Eq. (8.310.1)]. For a plane wave, parameters α and β are given by [1, 15]

$$\alpha = [\exp\left(\frac{0.49\sigma_r^2}{(1+0.18d^2+0.56\sigma_r^{\frac{12}{5}})^{\frac{7}{6}}}\right) - 1]^{-1}$$

$$\beta = \left[\exp\left(\frac{0.51\sigma_r^2}{(1+0.9d^2+0.62d^2\sigma_r^{\frac{12}{5}})^{\frac{5}{6}}}\right) - 1\right]^{-1} \quad (4)$$

Where $\sigma_r^2 = 1.23\, C_n^2 K^{7/6}\ L^{11/6}$ is the Rytov variance, $d = \sqrt{KD^2/4L}$ is the receiver aperture, K is optical wave number and is equal to $2\pi/\lambda$, D is receiver aperture diameter, λ is wavelength, and L is the link length. Here, C_n^2 is called the refractive index structure parameter and is a function of wavelength, atmospheric pressure and temperature. It varies according to local conditions such as terrain type, geographic location, cloud cover, and time of the day. The nominal value of C_n^2 is taken as $10^{-15}\ m^{-2/3}$ [6]. Considering the receiver aperture diameter (D) equal to 0.01 m, 850 nm as the operating wavelength (λ), link length or link span (L) =1000m, the pdf of attenuation due to atmospheric turbulence (h_a) over Gamma-Gamma turbulence channel in weak and strong turbulence is shown in Figure 1. The values of α and β versus log intensity variance depicted in the Figure 2. Figure 3 illustrates the value of Rytov variances in weak turbulence regime, C_n^2 = 6.5× $10^{-15} m^{-2/3}$ and strong turbulence regime, C_n^2 = 6.5× $10^{-14} m^{-2/3}$ for propagation distance up to 1 km.

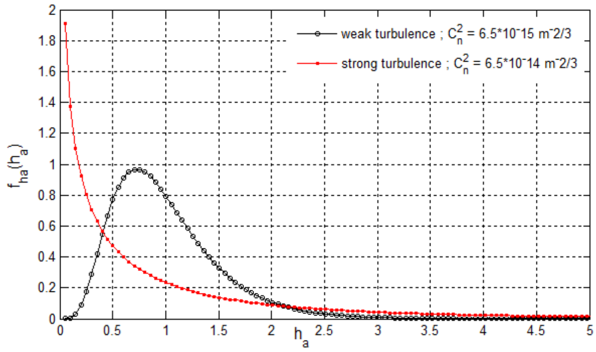

Figure 1: pdf of attenuation in weak and strong turbulence over gamma-gamma fading channel, λ = 850 nm, L = 1000 m, D = 0.01 m

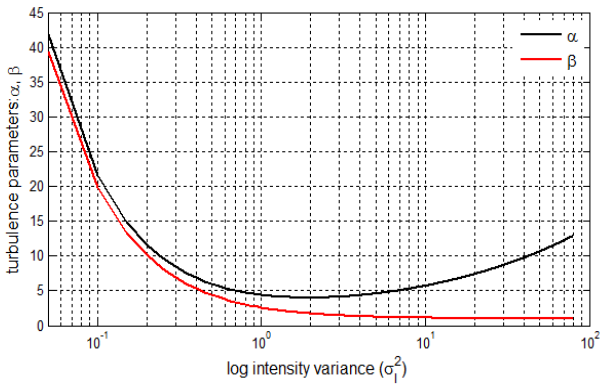

Figure 2: Values of α and β in gamma-gamma model in the case of a plane wave with zero inner scale

3. AVERAGE CAPACITY

A. A. Farid and S. Hranilovic have derived pdf of attenuation due to pointing errors for a Gaussian beam and a circular detection aperture of radius r in [10]

$$f_{h_p}(h_p) = \frac{\gamma^2}{A_o^{\gamma^2}}\ (h_p)^{\gamma^2-1}\ ,\ 0 \le h_p \le A_O \quad (5)$$

Where γ= $W_{zeq}/2\sigma_s$ is the ratio of the equivalent beam radius at the receiver to the pointing error displacement, A_0 = $[\mathrm{erf}(v)]^2$is the fraction of the collected power at r = 0, W_{zeq} is the equivalent beam width with ${W_{zeq}}^2 = W_z^2\sqrt{\pi}\ \mathrm{erf}(v) / [2v\ \exp(-v^2)]$. Where erf(⋅) is the error function, W_z is the beam waist calculated at distance z and $v = \sqrt{\pi}r/\sqrt{2}W_z$.

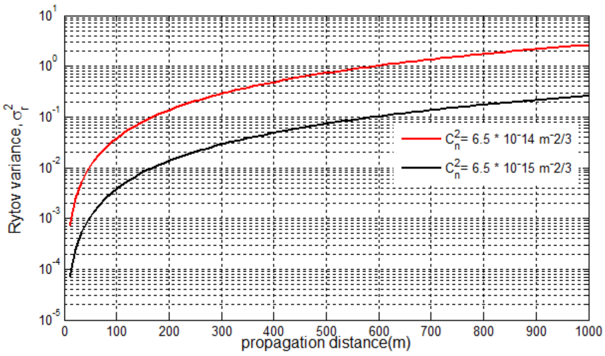

Figure 3: Rytov variances in weak and strong turbulence regime, link span (L) up to 1000 m, λ = 850 nm, D = 0.01m

The pdf for the combined attenuation due to atmospheric turbulence and pointing errors is derived using assumptions and methodology presented in [10]

$$f_h(h) = \frac{2\gamma^2(\alpha\beta)^{(\alpha+\beta)/2)}}{A_o^{\gamma^2}\Gamma(\alpha)\Gamma(\beta)} h^{\gamma^2-1}$$

$$\times \int_{h/A_o}^{\infty} h_a^{((\alpha+\beta)/2)-1-\gamma^2} K_{\alpha-\beta}(2\sqrt{\alpha\beta h_a})dh_a \quad (6)$$

Expressing $K_v(.)$ in the form of Meijer's G function [13, Eq. (14)] and following the methodology presented in [8], the simplified closed form pdf for the combined attenuation due to atmospheric turbulence and pointing errors is obtained as

$$f_h(h) = \frac{(\alpha\beta)\,\gamma^2}{\Gamma(\alpha)\,\Gamma(\beta)\,A_0} G_{1,3}^{3,0}\left[\frac{\alpha\beta h}{A_o}\middle|\begin{matrix}\gamma^2\\ \gamma^2-1,\alpha-1,\beta-1\end{matrix}\right] \quad (7)$$

The average channel capacity is given by [16]

$$\langle C\rangle = \int_0^\infty B\log_2\left[1+SNR(h)\right]f_h(h)\ dh$$

$$= \frac{B}{ln2}\int_0^\infty ln\left[1+SNR(h)\right] f_h(h)\ dh \quad (8)$$

Where B is the bandwidth in Hz and $SNR(h)$ is instantaneous electrical signal to noise ratio per received symbol. For OOK modulation and a slow fading channel, the electrical signal to noise ratio depends on the channel state h through following equation [10]

$$SNR(h) = \frac{2P_t^2 h^2}{\sigma_n^2} \quad (9)$$

Where σ_n^2 is the noise variance, P_t is the transmitted optical power. Expressing $ln(1+x)$ in the form of Meiger's G function [13, Eq. (11)]

$$ln\left[1+\left(\frac{2P_t^2h^2}{\sigma_n^2}\right)\right] = G_{2,2}^{1,2}\left[\frac{2P_t^2h^2}{\sigma_n^2}\middle|\begin{matrix}1 & 1\\ 1 & 0\end{matrix}\right] \quad (10)$$

Substituting (7) and (10) in (8), the average capacity is obtained as

$$\langle C\rangle = \frac{B(\alpha\beta)\,\gamma^2}{(ln\,2)\Gamma(\alpha)\ \Gamma(\beta)A_0} \times \int_0^\infty G_{1,3}^{3,0}\left[\frac{\alpha h}{A_0}\middle|\begin{matrix}\gamma^2\\ -1+\gamma^2,\alpha-1,0\end{matrix}\right]$$

$$G_{2,2}^{1,2}\left[\frac{2P_t^2h^2}{\sigma_n^2}\middle|\begin{matrix}1 & 1\\ 1 & 0\end{matrix}\right] dh \quad (11)$$

Solving the integral in (11) using [13, Eq. (21)], the final closed form expression for the average capacity is obtained as

$$\langle C\rangle = \frac{B2^{\alpha+\beta-2}\gamma^2}{(\pi * \ln 2)\Gamma(\alpha)\Gamma(\beta)}$$

$$G_{8,4}^{1,8}\left[\frac{32P_t^2A_0^2}{\sigma_n^2(\alpha)^2}\middle| \begin{matrix} 1,1,\frac{1-\gamma^2}{2},\frac{2-\gamma^2}{2},\frac{1-\alpha}{2},\frac{2-\alpha}{2},\frac{1-\beta}{2},\frac{2-\beta}{2} \\ 1,0,\frac{-\gamma^2}{2},\frac{1-\gamma^2}{2}\end{matrix}\right] \quad (12)$$

Rewriting (12), the normalized average channel capacity is obtained as

$$\frac{\langle C\rangle}{\mathrm{B}} = \frac{2^{\alpha+\beta-2}\gamma^2}{(\pi * \ln 2)\Gamma(\alpha)\Gamma(\beta)}$$

$$G_{8,4}^{1,8}\left[\frac{32P_t^2A_0^2}{\sigma_n^2(\alpha)^2}\middle| \begin{matrix} 1,1,\frac{1-\gamma^2}{2},\frac{2-\gamma^2}{2},\frac{1-\alpha}{2},\frac{2-\alpha}{2},\frac{1-\beta}{2},\frac{2-\beta}{2} \\ 1,0,\frac{-\gamma^2}{2},\frac{1-\gamma^2}{2}\end{matrix}\right] \quad (13)$$

Where, $G_{p,q}^{m,n}[\mathrm{Z}\,|^{a1, a2, a3, \dots ap}_{b1, b2, b3 \dots bq}]$ is the Meijer's G function. Wolfram Mathematica [14] is used here to compute (13).

4. NUMERICAL RESULTS

The average capacity at various transmitted optical power for a given value of normalized beam width(W_z/r) is calculated using (13). Normalized beam width is taken in steps of six (W_z/r = 6, 8, 10, 12, 14 16). The value of normalized jitter (σ_s/r) is assumed to be 0.1. Considering the value of refractive index structure parameter (C_n^2) as 6.5× 10^{-15}, L = 1000 m, λ = 850 nm, and D = 0.01m, Rytov variance (σ_r^2) is computed to be equal to 0.26. Also, noise standard deviation, σ_n= 10^{-7} A/Hz. Figure 4 shows normalized average channel capacity versus transmitted optical power for various values of normalized beam width in weak atmospheric turbulence.

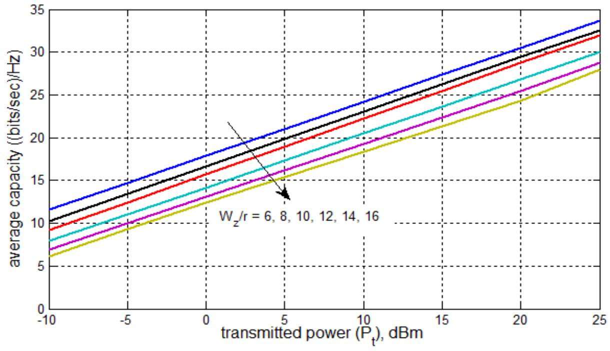

Figure 4: Normalized average channel capacity versus transmitted optical power (P_t, dBm) for various values of normalized beam width(W_z/r) in weak turbulence regime, (C_n^2) = 6.5× $10^{-15}m^{-2/3}$

From the above figure, it is observed that for a given value of normalized jitter, the average capacity in Gamma-Gamma turbulence channel is higher for a narrow beam. Also, the average capacity increases with an increase in the transmitted optical power (P_t, dBm). Even for a broader beam, a higher value of the average capacity can be achieved by increasing the transmitted optical power. However, power friendly systems are desired and by limiting beam width of the light source, higher average capacity can be achieved at expense of much lesser transmitted optical power. For the case of strong atmospheric turbulence i.e. $C_n^2 =$ 6.5× $10^{-14}m^{-2/3}$, the Rytov variance (σ_r^2) is computed to be equal to 2.6. All other parameters are same as taken in the previous case. In strong atmospheric turbulence, the average capacity for W_z/r = (6, 8, 10, 12, 14, 16) at various transmitted optical power is shown in Figure 5.

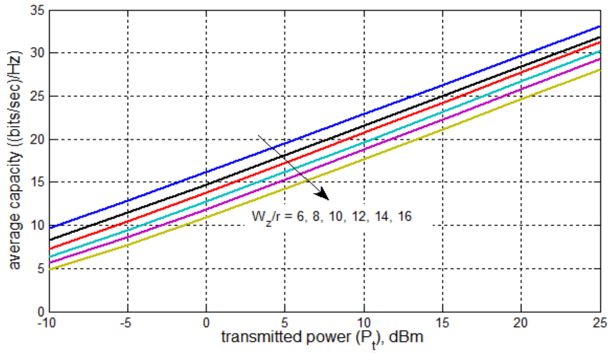

Figure 5: Normalized average channel capacity versus transmitted optical power (P_t, dBm) for various values of normalized beam width(W_z/r) in strong turbulence regime, (C_n^2) = 6.5× $10^{-14}m^{-2/3}$

Figure 6 shows a comparison between the average capacity in weak and strong turbulence. Figure 7 shows the average capacity of the channel versus transmitted optical power in strong atmospheric turbulence for various values of normalized jitter. Normalized beam width is assumed to have a constant value of 10.

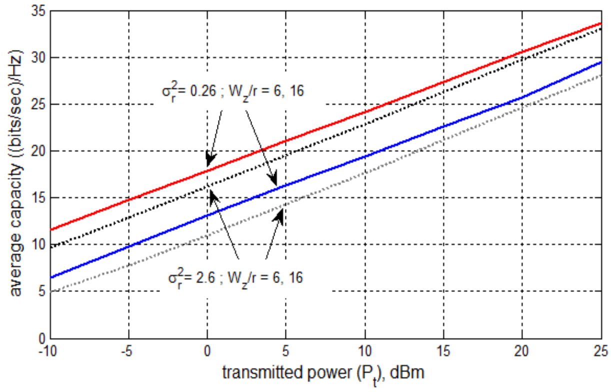

Figure 6: Normalized average channel capacity versus transmitted optical power (P_t, dBm) for various values of normalized beam width(W_z/r) in weak and strong turbulence regimes

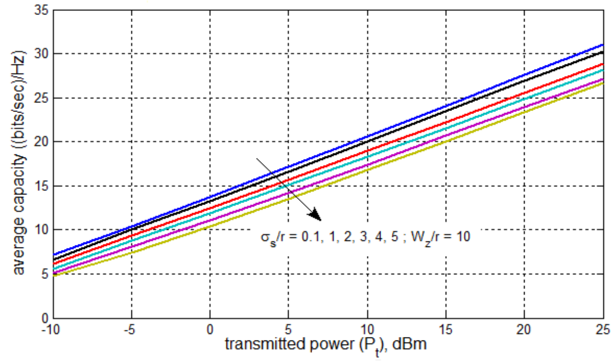

Figure 7: Normalized average channel capacity versus transmitted optical power (P_t, dBm) for various values of normalized jitter(σ_s/r) in strong turbulence regime, (C_n^2) = 6.5× $10^{-14}m^{-2/3}$

Figure 8 shows the average channel capacity in weak turbulence regime, (C_n^2) = 6.5× $10^{-15}m^{-2/3}$ and strong turbulence regime, (C_n^2) = 6.5× $10^{-14}m^{-2/3}$for different values of normalized jitter. Solid, dotted line represents the average capacity in weak and strong turbulence regime respectively.

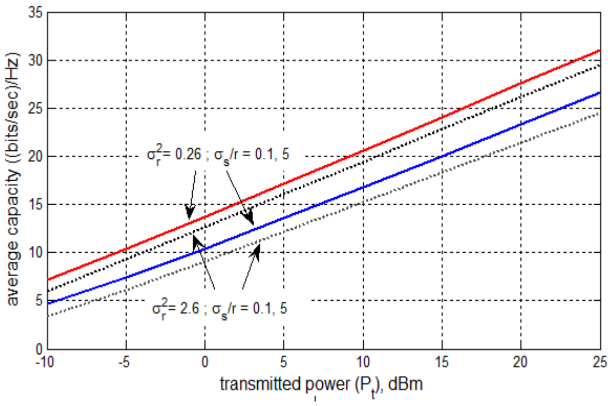

Figure 8: Normalized average channel capacity versus transmitted optical power (P_t, dBm) for various values of normalized jitter (σ_s/r) in weak and strong turbulence regimes

From Figure 3 through Figure 8, it is observed that the atmospheric turbulence and pointing errors degrades the capacity performance of FSO links. However, a desired average channel capacity can be achieved by carefully adjusting the beam width and transmitted optical power level.

5. CONCLUSION

In conclusion, the average capacity of the FSO links over Gamma-Gamma turbulence channel with pointing errors is investigated. For a narrow beam, higher average capacity can be achieved at an expense of much lesser optical power. However, narrow beam makes FSO links more vulnerable to pointing errors. For a broad beam, the average capacity is initially low, but it can be improved by increasing transmitted optical power. Requirement of power friendly communication systems again puts a limit to maximum transmitted optical power level. A tradeoff has to be made in such situation.

6. REFERENCES

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