

# Positive Deviation Method for Decision Making using Intuitionistic Fuzzy Sets

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## ABSTRACT

A new technique named as Intuitionistic fuzzy positive deviation method for decision making is the selection of the professional students based on their skills by the recruiters using Intuitionistic Fuzzy sets. Sanchez's approach for decision making is studied and the concept is generalized by the application of Intuitionistic Fuzzy Set (IFS) theory. Through a survey the relations between the skills and the opportunities are discussed. The proposed method is compared with max-min composition method.

## Key words

Fuzzy sets, Intuitionistic fuzzy sets (IFS), Intuitionistic fuzzy relations (IFRs), Intuitionistic fuzzy Max-min composition method.

## 1. INTRODUCTION

The present paper deals with the study of Intuitionistic fuzzy positive deviation method for decision making based on Sanchez's method [5,6] using the notion of IFS theory. The degrees of membership and non-memberships are the single value between 0 and 1. However, in reality, it may not always be certain that the sum of the degrees is just one. Atanassov, [1,2] and Biswas [3]. Fuzzy set theory has a number of properties that make it suitable for decision making. There are two popular techniques for decision making using Sanchez's approach. One is the method that uses the max-min composition rule. The other is the method that uses the distance measure between fuzzy sets for decision making.

Intuitionistic fuzzy sets [IFSs] as a generalization of fuzzy sets was introduced by Zadeh [12] and K. Atanassov [1,2] which contains three functions namely, membership non membership and hesitancy. The hesitancy plays an important role in making decisions in medical diagnosis[7,8], sales analysis, Enterprise planning, Network Security analysis and marketing services, etc. there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object.

In real life, most of the existing mathematical tools for formal modeling, reasoning and computing were crisp deterministic and precise in nature. The classical crisp mathematical tools are not capable of dealing with problems involving uncertainty and imprecision. There are many mathematical tools available for modeling complex systems such as probability theory, fuzzy set theory, interval mathematics etc. Probability theory is applicable only for a stochastically stable system. Interval mathematics is not sufficiently adaptable for

problems with different uncertainties. Intuitionistic fuzzy set is used as a tool for reasoning in the presence of imperfect facts and precise knowledge

In this paper Intuitionistic fuzzy sets is used as a tool for reasoning in the presence of imperfect facts and precise knowledge. Using max-min composite relation method Supriya Kumar et al [7] Edward Samuel and .Balamurugan [4] made an attempt to provide a formal model of the process to identify the diseases of the patients based on the symptoms of four main types of diseases. Szmidt et.al [8,9] discussed Medical diagnosis via distances for Intuitionistic fuzzy sets. In this paper, a new technique named as Intuitionistic fuzzy positive deviation method for decision making is proposed to study the selection of the professional students based on their skills by the recruiters using Intuitionistic Fuzzy sets. The results are compared with the results of max-min composition method [7,10,11]. The techniques summarize the simulation results to compare the outcomes of the decision making techniques by using IFS theory and implement it in the form of field suggestion system. This is the system by which the recruiters use his knowledge to infer the opportunities from the skills, based on his test results.

## 2. PRELIMINARIES

The basic definitions of Intuitionistic fuzzy set theory that are useful for subsequent discussions are given.

**Definition 2.1.**[1] A set  $E$  be fixed. An IFS  $A$  in  $E$  is an object having the form  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle | x \in E \}$ , where the functions  $\mu_A : E \rightarrow [0,1]$  and  $\nu_A : E \rightarrow [0,1]$  define the degree of membership and degree of non-membership respectively of the element  $x \in E$  to the set  $A$ , which is a subset of  $E$ , and for every  $x \in E$ ,  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ .

The amount  $\pi_A(x) = 1 - (\mu_A(x) + \nu_A(x))$  is called the hesitation part, which may cater to either membership value or non-membership value or both.

**Definition 2.2.**[10] If  $A$  and  $B$  are two Intuitionistic fuzzy sets of the set  $E$ , then

$$A \subset B \text{ iff } \forall x \in E, \mu_A(x) \leq \mu_B(x) \quad \text{and} \quad \nu_A(x) \geq \nu_B(x),$$

$$A \subset B \text{ iff } B \subset A,$$

$$A = B \text{ iff } \forall x \in E, \mu_A(x) = \mu_B(x) \text{ and } \nu_A(x) = \nu_B(x),$$

$$\bar{A} = \{ \langle x, \nu_A(x), \mu_A(x) \rangle \mid x \in E \},$$

$$A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) \rangle \mid x \in E \},$$

$$A \cup B = \{ \langle x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) \rangle \mid x \in E \},$$

Clearly every fuzzy set has the form  $\{ \langle x, \mu_A(x), \mu_{A^c}(x) \rangle \mid x \in E \}$

**Definition 2.3** [3] Let  $X$  and  $Y$  be two sets. An Intuitionistic Fuzzy relation (IFR)  $R$  from  $X$  to  $Y$  is an IFS of  $X \times Y$  characterized by the membership function  $\mu_R$  and non-membership function  $\nu_R$ . An IFR  $R$  from  $X$  to  $Y$  will be denoted by  $R(X \rightarrow Y)$

**Definition 2.4** [3,4] Let  $Q(X \rightarrow Y)$  and  $R(Y \rightarrow Z)$  be two IFRs. The max-min- composition  $R \circ Q$  is the Intuitionistic fuzzy relation from  $X$  to  $Z$ , described by the membership function

$$\mu_{R \circ Q}(x, z) = \bigvee_y [\mu_Q(x, y) \wedge \mu_R(y, z)] \text{ and the non-}$$

membership function

$$\nu_{R \circ Q}(x, z) = \bigwedge_y [\nu_Q(x, y) \vee \nu_R(y, z)]$$

$$\forall (x, z) \in X \times Z \text{ and } \forall Y \in Y. (\text{Where } \vee = \max, \wedge = \min)$$

**Definition 2.5** [10] Let  $Q(X \rightarrow Y)$  and  $R(Y \rightarrow Z)$  be two IFRs. The max-min-average composition  $R \circ Q$  is the Intuitionistic fuzzy relation from  $X$  to  $Z$ , described by the membership function

$$\mu_T(p_i, d) = \bigvee_y \left\{ \frac{1}{2} [\mu_Q(p_i, s) + \mu_R(s, d)] \right\}$$

The non-membership function is

$$\nu_T(p_i, d) = \bigwedge_y \left\{ \frac{1}{2} [\nu_Q(p_i, s) + \nu_R(s, d)] \right\}$$

$$\forall p_i \in p \text{ and } d \in D.$$

**Definition 2.6** The positive deviation  $D(A, B)$  between two IFSs  $A$  and  $B$  is defined as

$$D(A, B) = \frac{1}{2n} \sum_{i=1}^n \sum_{j=1}^n \left\{ \begin{array}{l} \text{Positive values of } [\mu_A(x_{ij}) - \mu_B(x_{ji})] \\ + \text{Positive values of } [\nu_B(x_{ji}) - \nu_A(x_{ij})] \\ + \text{Positive values of } [\pi_A(x_{ij}) - \pi_B(x_{ji})] \end{array} \right\}$$

### 3. POSITIVE DEVIATION METHOD FOR DECISION MAKING

In this section an application of Intuitionistic Fuzzy set theory using positive deviation method for decision making is presented.

In a given set of system, let  $P = \{x_1, x_2, \dots, x_n\}$  be the set of students and  $S = \{y_1, y_2, \dots, y_n\}$  be the set of skills and  $D = \{z_1, z_2, \dots, z_n\}$  be the set of opportunities. Using composition relation in mathematical Analysis, the Intuitionistic Fuzzy relation  $R$  from the set of skills to the set of opportunities  $D$  is formed. This relation reveals the degree of association and the degree of non-association between the skills and opportunities.

The proposed method is based on the following three steps.

- (i) Determination of skills
- (ii) Formulation of Intuitionistic Fuzzy relation
- (iii) Classification of opportunities on the basis of definition 2.6 and composition of Intuitionistic Fuzzy relations

An Intuitionistic Fuzzy relation  $Q$  is given from the set of students  $X$  to the set of skills  $Y$  and another Fuzzy relation  $R$  is given from the set of skills  $Y$  to the set of opportunities  $Z$ . the composite function  $T$  from the Intuitionistic Fuzzy relation  $R$  and  $Q$ .

### 4. ALGORITHM

**Step 1:** Form the Intuitionistic Fuzzy relation

$$Q(P \rightarrow S)$$

**Step 2:** Take the Intuitionistic fuzzy relation  $R(S \rightarrow D)$  (hypothetical)

**Step 3:** Find the positive deviation  $D(A, B)$  that describes the state of students  $P$  in terms of the selection as an IFR from (students) to  $D$  (opportunities) given by the membership, the non-membership and the hesitation function using Definition 2.6.

**Step 4:** Identify the minimum deviation for each student  $P_i$

### 5. CASE STUDY

The test results of four students Arun, John, Peter, and Ram are considered for the case study. In the discrimination analysis, the skills are ranked according to the grades of each opportunity by particular skills.

Let  $X = \{\text{Arun, John, Peter, Ram}\}$  be the set of four students,  $Y = \{\text{Technical skill, Analytical skill, Presentation skill, Communication skill}\}$  be the set of skills and  $Z = \{\text{Hardware, Software, higher studies, Others}\}$  be the set of opportunities.

**Table 1: Determination of skills by using Intuitionistic fuzzy relation  $Q(P \rightarrow S)$  entries as per the survey is in the form of IFS  $(\mu_A, \nu_A)$**

Q	Technical	Analytical	Presentat ion	Communica tion
Arun	(0.8, 0.1)	(0.6, 0.1)	(0.4, 0.6)	(0.6, 0.1)
John	(0.3, 0.6)	(0.4, 0.4)	(0.4, 0.5)	(0.7, 0.2)
Peter	(0.4, 0.5)	(0.5, 0.4)	(0.8, 0.2)	(0.6, 0.3)
Ram	(0.6, 0.1)	(0.8, 0.1)	(0.3, 0.4)	(0.7, 0.2)

**Table 2: Intuitionistic fuzzy relation  $R (S \rightarrow D)$  gives a determination of field by using Intuitionistic fuzzy relation as per selection of candidate by field experts are in the form of IFS  $(\mu_A, \nu_A)$**

R	Hardware	Software	Others
Technical	(0.7,0.2)	(0.5, 0.4)	(0.4, 0.5)
Analytical	(0.8, 0.1)	(0.4, 0.5)	(0.6, 0.3)
Presentation	(0.4, 0.3)	(0.8, 0.1)	(0.4, 0.4)
Communication	(0.5, 0.3)	(0.6, 0.2)	(0.7, 0.1)

**Table 3: Determination of opportunities by using the positive deviation method D (A,B) using Definition 2.6.**

S <sub>T</sub>	Hardware	Software	Others
Arun	<b>0.1000</b>	0.1500	0.1625
John	0.0500	0.0375	<b>0.0125</b>
Peter	0.0750	<b>0.0250</b>	0.0750
Ram	<b>0.0750</b>	0.2125	0.1625

$$a_{11} = \frac{1}{2(4)} \left\{ \begin{array}{l} (0.8 - 0.7) + (0.6 - 0.5) \\ + (0.2 - 0.1) + (0.3 - 0.1) \\ + (0.3 - 0.1) + (0.3 - 0.2) \end{array} \right\} = 0.1000$$

$$a_{11} = \frac{1}{2(4)} \left\{ \begin{array}{l} (0.8 - 0.5) + (0.6 - 0.4) \\ + (0.4 - 0.1) + (0.5 - 0.1) + (0.2 - 0.1) \\ + (0.3 - 0.1) + (0.3 - 0.2) \end{array} \right\} = 0.1500.$$

Similarly the other values are shown in the following table

**Table 4: Determination of opportunities by using the max-min composition method  $T = R \circ Q$  for the data in Table 1 and 2**

T	Hardware	Software	Others
Arun	<b>(0.7, 0.1)</b>	(0.6, 0.2)	(0.6, 0.1)
John	(0.5, 0.3)	(0.6, 0.2)	<b>(0.7, 0.2)</b>
Peter	(0.5, 0.3)	<b>(0.8, 0.2)</b>	(0.6, 0.3)
Ram	<b>(0.8, 0.1)</b>	(0.6, 0.2)	(0.7, 0.1)

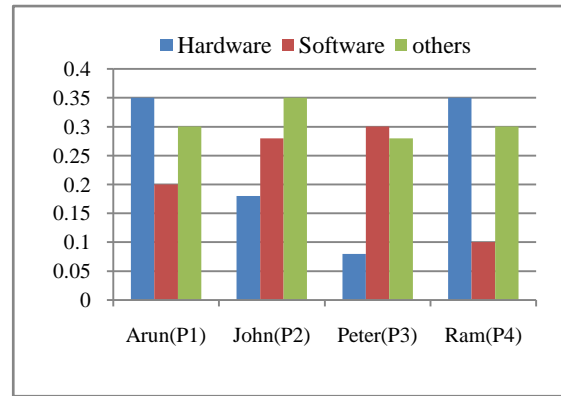
**Table 5: Score value for max-min composition method**

$$ST = \mu_T - V_T \times \pi_T \text{ using Table-4.}$$

S <sub>T</sub>	Hardware	Software	Others
Amity	<b>0.68</b>	0.56	0.57
John	0.44	0.56	<b>0.68</b>
Peter	0.44	<b>0.80</b>	0.57
Ram	<b>0.79</b>	0.56	0.68

## 6. CONCLUSION

From Table 3 it is seen that the minimum deviation value of Arun and Ram is 0.1000, 0.0750 respectively, the Interviewer agrees that Arun and Ram are suitable for hardware profession, the minimum deviation value of Peter is 0.0250 and he is suitable for software profession whereas the minimum deviation value of John is 0.0125, who is suitable for the profession other than Hardware and Software. From Table 5 it is clear that the result obtained by max-min composition method is same as that of the result by the proposed method. The proposed approach makes it possible to introduce weights for all skills and reduces the confusion about the possibility of two opportunities in a student and also it is an efficient tool for decision making problem. This is represented by the chart diagram as follows:



**Fig. 1 Result by Positive Deviation Method**

## 7. ACKNOWLEDGMENT

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