Two Layered Mathematical Model for Blood Flow through Tapering Asymmetric Stenosed Artery with Velocity Slip at the Interface under the Effect of Transverse Magnetic Field

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ABSTRACT
The paper considers a mathematical model for two-layered blood flow through a tapered artery with the growth of a asymmetric mild stenosis and velocity slip at the interface. The model consists of a core region of red blood cell suspension in the middle layer and the peripheral plasma layer (PPL) in the outer region. It is assumed that both the core and the peripheral plasma layer are represented by a Newtonian fluid with different viscosities. In this model, the flow is assumed to be steady, laminar and unidirectional and analytical expressions are obtained for axial velocity, flow rate and wall stresses. Their variations with different flow parameters are plotted graphically and the behaviour of these flow variables in this constricted region has been discussed. It is observed that fluid velocity, flow rate as well as wall shear stress decreases with the introduction of the magnetic field and when its intensity is increased. Also it is seen that fluid velocity, flow rate and shear stress increases with the increase of Reynolds number.

General Terms
Magnetic field effect on stenosis in a tapered artery

Keywords
Tapered artery, Two layered model, Blood Flow, Newtonian fluid, Asymmetric Mild Stenosis, Slip velocity. Magnetic Field.

1. INTRODUCTION
Stenosis refers to localized narrowing of an artery and is caused mainly due to intravascular atherosclerotic plaque which develops at the arterial wall and protrudes into the lumen of the vessel. An artery which is affected by this abnormal growth can lead to serious consequences such as blockage of the artery, stroke and many other arterial diseases. It has been realized that various hydrodynamic effects (e.g. pressure distribution and wall shear, etc.) play important role in the development and progression of this disease [1]. Thus the study of blood flow through stenotic arterial region plays an important role in the diagnostic and fundamental understanding of cardiovascular diseases [2-6].

To describe the flow of blood inside an artery, researchers have proposed various models from different perspectives (Biswas [7], Ponalgusamy [8], Sankar and Lee [9], Halder [10], Misra and Chakravarty[11]) treating blood as a Newtonian fluid. High wall shear stress causes the innermost membrane of an artery or a vein thickening but also activate platelets aggregation. There is no doubt that height of the stenosis is a more important factor influencing blood flow than tapering of an artery[12].

Different mathematical models have been studied by some researchers to explore the various aspects of blood flow in stenosed artery by representing blood as a single-layered model. Bugliarello and Sevilla [13] have shown experimentally that the blood flowing through narrow tubes can be well represented by a two-layered model instead of one [14]. In this type of models there is a peripheral layer of plasma and a core region of suspension of red blood cells. Shukla *et al*. [3] have taken two-layered model to analyze the peripheral layer viscosity. Ponalagusamy [8] focused on slip velocity, thickness of peripheral layer and core layer viscosity at the vessel wall. Srivastava [15] studied analytically and numerically effects of mild stenosis on blood flow characteristics in a two-fluid model. Haldar and Andersson[16] discussed two layered model of blood flow through stenosed arteries.

Many researchers studied the pulsatile flow of blood in stenosed artery (Biswas and Chakraborty [17], Chaturani and Ponalagusamy [18], Chaturani and Ponalagusamy [19], Chaturani and Biswas [20], Verma and Parihar [21], Bhuyan and Hazarika[22]) have considered the behaviour of blood in presence of magnetic effect. The idea of electromagnetic fields in medical research was firstly given by Kolin and later Korchevskii *et al.* discussed the possibility of regulating the movement of blood in human system by applying magnetic field [23].

The study of blood flow through tapered tubes (Verma and Parihar [21], [24]) is important not only for an understanding of the flow behaviour of the marvelous body fluid blood in arteries, but also for the design of prosthetic blood vessels (How and Black[25]). In the blood flow modelling, a number of studies of blood flow in particular both theoretical (Bloch [26] and Number[26]) and experimental (Bugliarello and Hayden [27]; Bennett[28]) have reported the presence of slip at the flow boundaries. It seems that consideration of a velocity slip at the vessel wall be quite rational in blood flow modelling and it seems to be appropriate if a slip velocity at the interface of fluids is considered. Biswas and Barbhuiya [12] studied the effects of the stenosis on the axial velocity, flow rate and wall shear stress for Newtonian fluid through a tapered artery in a two-layered blood flow with velocity slip at the interface. An attempt is made in this analysis to study the flow of blood through a tapered artery in presence of stenosis with the effect of slip at the interface in presence of transversed magnetic field in a two layered blood flow model.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

2.1 Flow Geometry

We consider the steady, laminar, axisymmetric flow of blood through a tapered artery with an axially non symmetrical (but radially symmetrical) stenosis. The artery length is assumed to be large enough as compared to its radius so that the entrance, exit and special wall effects can be neglected.

The model basically consists of a core of red blood cell suspension in the middle layer and the peripheral plasma layer in the outer layer (as shown in fig. 1). It is assumed that both the core region and the peripheral plasma layer are represented by a Newtonian fluid with different viscosities μ_1 and μ_2 respectively.

The flow geometry of blood flow through the tapered artery with non-symmetric stenosis can be expressed mathematically (Mekheimer and Kothari [29]) in dimensionless form as-

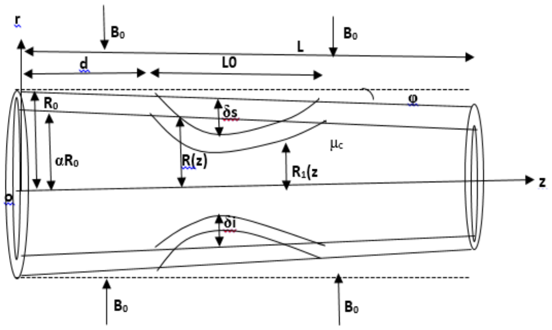

Fig 1. Flow Geometry of a tapered asymmetric stenosed artery in a two-layered model.

For peripheral layer (PPL): $R_1(z) \le r \le R(z)$

$$R(z) = \begin{cases} (1-mz)\left[1 - A\delta_s\left\{L_0^{\,n-1}(z-d) - (z-d)^n\right\}\right], & d \le z \le d + L_0 \\ (1-mz), & otherwise \end{cases} \tag{1}$$

and for core region: $0 \le r \le R_1(z)$

$$R_1(z) = \begin{cases} (\alpha-mz)\left[1 - A\delta_i\left\{L_0^{\,n-1}(z-d) - (z-d)^n\right\}\right], & d \le z \le d + L_0 \\ (\alpha-mz), & otherwise \end{cases} \tag{2}$$

Where

R_0 : The radius of the non-stenotic region.

$R(z)$: Radius of the stenotic region (radius of the peripheral layer)

$R_1(z)$: Radius of the central layer in stenotic region (radius of the core region).

L: The length of the artery.

L_0: The length of the asymmetric stenosis.

d: Location of the stenosis.

δ_s: Instantaneous maximum height of the stenosis in the peripheral region.

δ_i: Maximum bulging of interface.

m: The slope of the extended vessel.

φ: The angle of tapering.

α: Ratio of the central core radius to the tube radius.

Also $\delta_s = h\, sec\varphi$, $\delta_i = h_1\, sec\varphi$, $m = \frac{tan\ \varphi}{R_0}$ and $A = \frac{n^{\frac{n}{n-1}}}{R_0 L_0 (n-1)}$.

2.2 Flow Analysis and Coordinate System

We consider an axially non-symmetric, steady, laminar and fully developed flow of blood, in a stenotic tapered artery as shown in the fig.1. Blood has been assumed as a homogenous Newtonian fluid and the flow is one- dimensional, in axial direction under the influence of externally applied magnetic field. Fluid velocity has the form $\bar{u} = \big(0,0,w(r)\big)$in cylindrical polar coordinate system (r,θ,z) respectively the radial, circumferential and axial coordinate respectively. The equations of motion governing the fluid flow in (r,θ,z)coordinate system (Schlichting [30]) under the effect of magnetic field are written as-

$$\frac{\partial \bar{p}}{\partial \bar{r}} = 0 \tag{3}$$

$$\frac{\partial \bar{p}}{\partial \bar{\theta}} = 0 \tag{4}$$

$$\frac{\partial \bar{p}}{\partial \bar{z}} = \bar{\mu}\frac{\partial^2 \bar{w}}{\partial \bar{r}^2} + \frac{\bar{\mu}}{\bar{r}}\frac{\partial \bar{w}}{\partial \bar{r}} - \sigma {B_0}^2 \bar{w} \tag{5}$$

Where $\bar{w} = \bar{w}(\bar{r})$represents the axial velocity; $\bar{p}$, blood pressure; $\frac{\partial \bar{p}}{\partial \bar{z}}$, pressure gradient; B_0 , the external magnetic field along the radial direction; $\bar{\mu}$ is the viscosity of the blood and σ is the conductivity of the blood.

2.3 Boundary Conditions

The boundary conditions are

(i) $\bar{w}_p = 0$ at $\bar{r} = \bar{R}(\bar{z})$(no slip at the peripheral layer)

(ii)$\bar{w}_c - \bar{w}_p = \bar{w}_s$ at $\bar{r} = \bar{R}_1(\bar{z})$ (slip at the interface)

(iii) $\frac{\partial \bar{w}_c}{\partial \bar{r}} = 0$ at $\bar{r} = 0$ (symmetry condition) (6)

(iv) $\frac{\partial \bar{w}_p}{\partial \bar{r}} = 0$ at $\bar{r} = 0$ (symmetry condition)

where $\bar{w}_s$ is the slip velocity at the interface of the stenotic region and $\bar{w}_c, \bar{w}_p$ are velocities in the core region and the peripheral layer respectively.

3. SOLUTION PROCEDURE

Introducing the following non- dimensional variables-

$$z = \frac{\bar{z}}{\bar{R}_0},\ d = \frac{\bar{d}}{\bar{R}_0},\ r = \frac{\bar{r}}{\bar{R}_0},\ R(z) = \frac{\bar{R}(\bar{z})}{\bar{R}_0},\ w_s = \frac{\bar{w}_s}{\bar{w}_0},\ w_p = \frac{\bar{w}_p}{\bar{w}_0}$$

$$w_c = \frac{\bar{w}_c}{\bar{w}_0},\ w = \frac{\bar{w}}{w_0},\ \bar{w}_0 = \frac{\bar{C}{\bar{R}_0}^2}{4\bar{\mu}_2},\ R_1(z) = \frac{\bar{R}_1(\bar{z})}{\bar{R}_0},\ p = \frac{\bar{p}}{\bar{\rho}{\bar{w}_0}^2}\ .$$

In the equation (5), the equation reduces to-

$$\frac{\partial p}{\partial z} = \frac{1}{Re}\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - M^2 w\right) \tag{7}$$

where $Re = \frac{\bar{\rho}\bar{R}_0\bar{w}_0}{\bar{\mu}}$ (Reynolds number)

$M = \sqrt{\frac{\sigma}{\bar{\mu}}} B_0 \bar{R}_0$ (Hartmann number)

and $w = w(r)$ i.e. axial velocity is a function of 'r' only.

It is seen that from the above equations (3-5), the pressure gradient is the function of 'z' only i. e. $p = p(z)$, which causes the motion of the flow in the z- direction only, therefore $\frac{\partial p}{\partial z}$ can be written

as- $\frac{\partial p}{\partial z} = \frac{dp}{dz}$

We take $C = -\frac{dp}{dz}$

So the equation (7) reduces to the ordinary differential equation as-

$$\frac{dp}{dz} = \frac{1}{Re}\left(\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} - M^2 w\right) \quad (8)$$

The boundary conditions (6) (in dimensionless form) are becomes-

$w_p = 0$ at $r = R(z)$ (no slip at the peripheral layer)

$w_c - w_p = w_s$ at $r = R_1(z)$ (slip at the interface)

$$\left.\begin{array}{l} \frac{dw_c}{dr} = 0 \text{ at } r = 0 \text{ (symmetry condition)} \\ \frac{dw_p}{dr} = 0 \text{ at } r = 0 \text{ (symmetry condition)} \end{array}\right\} \quad (9)$$

The total flow rate $\bar{Q}(\bar{z})$ is determined as-

$$\bar{Q} = \bar{Q}_c + \bar{Q}_p$$

where $\bar{Q}_c$ and $\bar{Q}_p$ are flow rates corresponding to core and peripheral layers respectively, given by-

$$\bar{Q}_c = 2\pi \int_0^{\bar{R}_1/\bar{R}_0} \bar{r}\bar{w}_c d\bar{r} \quad \text{and} \quad \bar{Q}_p = 2\pi \int_{\bar{R}_1/\bar{R}_0}^{\bar{R}/\bar{R}_0} \bar{r}\bar{w}_p d\bar{r}$$

The dimensionless volumetric flow rate $Q(z)$ is given by the formula

$$Q = 4\left[\int_0^{R_1(z)} r w_c dr + \int_{R_1(z)}^{R(z)} r w_p dr\right] \quad (10)$$

where $Q = \frac{\bar{Q}(\bar{z})}{\bar{Q}_0}$ and $\bar{Q}_0 = \frac{\pi \bar{C} \bar{R}_0^{\;4}}{8\bar{\mu}_2}$.

The wall shear stresses of the flow at the vessel wall and the interface of the fluid is given by

$$\bar{\tau}_{\bar{R}(\bar{z})/\bar{R}_0} = \left(-\bar{\mu}_2 \frac{\partial \bar{w}_p}{\partial \bar{r}}\right)_{\bar{r}=\bar{R}(\bar{z})}, \quad \bar{\tau}_{\bar{R}_1(\bar{z})/\bar{R}_0} = \left(-\bar{\mu}_1 \frac{\partial \bar{w}_c}{\partial \bar{r}}\right)_{\bar{r}=\bar{R}_1(\bar{z})}$$

The dimensionless form of the wall shear stresses of the fluid are

$$\left.\begin{array}{l} \tau_{R(z)} = \left(-\frac{1}{2}\frac{\partial w_p}{\partial r}\right)_{r=R(z)} \text{ (at the vessel wall)} \\ \tau_{R_1(z)} = \left(-\frac{1}{2}\mu_1 \frac{\partial w_c}{\partial r}\right)_{r=R_1(z)} \text{ (at the interface)} \end{array}\right\} \quad (11)$$

where $\tau_{R(z)} = \frac{\bar{\tau}_{\bar{R}(\bar{z})}}{\bar{\tau}_0}$, $\tau_{R_1(z)} = \frac{\bar{\tau}_{\bar{R}_1(\bar{z})}}{\bar{\tau}_0}$ and $\bar{\tau}_0 = \frac{\bar{C}\bar{R}_0}{2}$

The equation (8) for velocities of flow are solved numerically using Shooting method. Similarly the volumetric flow rate and wall shear stresses are also evaluated numerically with the help of that method for various parameters.

4. RESULTS AND DISCUSSIONS

In our cardiovascular system, among different flow geometries, there exists tapering vessels. Also it is reported that blood while flowing through narrow blood vessel, leaves a cell poor region near the wall. Therefore, a two-layered blood flow with peripheral plasma layer (ppl) and a core of red cell suspension may be realistic here. Also, it may be physically important to investigate blood flow through a tapered artery [12].

The problem under consideration is reduced to a boundary value problem given by (8). This problem is solved numerically using Shooting method. Numerical calculations have been done for various combinations of parameters i.e. the magnetic parameter (Hartmann number, M) and Reynolds number, Re with slip at the interface. The axial velocity profiles, flow rate and wall shear stresses are computed for the various parameters.

Numerical results are shown graphically by using the following parameter values- α=0.8, L_0=10, d=5, δi=.1, δs=.15, C= .51, R_0 =1, R_1=.01.Values of Hartmann number, M=0, 1, 2, 3,4; Reynolds number, Re= 0,1,2,3,4,5,6,7,8,9,10 and n= 2, 3 are used in this analysis. It has been observed that the effect of magnetic parameter M and Reynolds number Re on the velocity field, flow rate as well as wall shear stress is very prominent.

In order to analyze the flow field insensibly, figures (2-3) exhibits the velocity profile for different values of Hartmann number at n=2, 3 respectively and Re=10. It is seen that as the Hartmann number (Magnetic parameter) increases, the velocity of the fluid decreases in both the layers of the flow. It can also be seen that in the absence of the external magnetic field (M=0), the fluid's velocity is higher than in its presence (M > 0). Therefore, the presence of an external magnetic field reduces blood's velocity and as the intensity of the magnetic field is increased it is further reduced [31]. This occurs due to the opposing Lorentz's force that is introduced when the magnetic field is applied. Hence, magnetic fields can be used to control blood flow. Also it is seen that as the value of 'n' increases from n=2 to n=3 the fluid velocity decreases (with slip). It is noticed that the maximum velocity attains at the tube axis and minimum at the vessel wall and also for the increasing the heights of the stenosis, the velocity becomes lower, with slip conditions at the interface.

As axial coordinate 'z' at the stenotic region increases from either end to the throat of asymmetric stenosis, velocity decreases from its greatest value at the maximum tubular area to a minimum at the minimum constricted area, and so, velocity is maximum at the initiation of the stenosis and minimum value at the throat of the stenosis.

The effect of Reynolds number Re on the velocity field is shown in figures (4) and (5) at M=1. It is observed that as the Reynolds number increases the fluid velocity increases in both core and peripheral layers of the flow with slip at the interface (w_s =0.05). Here also it is seen that for the increase of the value of the parameter 'n' of asymmetric stenosis, the fluid velocity decreases and the maximum velocity attains at the tube axis and minimum at the vessel wall.

In figure (6), flow rate is analyzed with magnetic field M for different values of Reynolds number Re. It is seen that as the Reynolds number increases the flow rate is also increases. The flow rate is more in the beginning but gradually it decreases with the increase of the magnetic field intensity M.
The effect of magnetic field on flow rate is shown in figure (7). The flow rate is higher in the absence of the magnetic field than in its presence. As the intensity of the magnetic field increases (i.e. M increases) the flow rate decreases. Also as the Reynolds number Re, increases, the flow rate decreases both in the absence and presence of the magnetic field.
The presence of an external magnetic field also affects the flow rate. In figure (8) flow rate is observed that as Hartmann number, M, increases, the flow rate Q decreases and that the flow rate is higher in the absence of the magnetic field than its presence.
Figure (9) shows the flow rate along axial distance for various values of Reynolds number. It is observed that the flow rate increases with the increase of Reynolds number. Also maximum flow rate at the initiation of the stenosis and minimum at the throat of the stenosis is obtained in this case.
It could also be noticed that the flow rate is more influenced by tapering, the flow rate increases with divergent tapering ($\varphi > 0$) (figure (10) and (11)). But for converging tapering ($\varphi < 0$), the flow rate decreases, the flow rate is higher at the initial of stenosis (at location 'd') than that of final end of stenosis (at location'$d+L_0$'). It is also seen that with higher slip velocity ($w_s = 0.5$), the flow rate is higher along the length of the stenosis.

The wall shear stress is observed in both of the interface as well as the boundary of the stenosis with magnetic field M for different values of Reynolds number in figure (12) and (13) respectively. It is observed that as the Reynolds number increases, the shear stress increases in both the cases. Also we have seen that for a particular value of Re as the magnetic field intensity increases the shear stress increases. So the presence of an external magnetic field as well as Reynolds number affects the wall shear stress. So the shear stress is higher in the presence of the magnetic field than in its absence (M=0).

In figures (14) and (15) wall shear stresses are observed with stenosis height δs for different values of the Reynolds numbers at M=3. It is seen that shear stress is higher for higher values of Reynolds number. Also shear stresses increases as the height of the stenosis increases at both the interface as well as boundary of the vessel walls.

5. CONCLUSION

In this analysis of blood flow in small vessels in the presence of a magnetic field where a asymmetric stenosis in a tapered artery is manifested, a two layered model is used to model blood. The two layered are a Newtonian peripheral plasma layer and a core region of erythrocytes suspended in plasma (i.e. particle-fluid mixture). The fluid velocity, flow rate and wall shear stresses are examined numerically.

1. Hartmann number retards the fluid velocity in the stenotic region. The fluid velocity is reduced as the magnetic field is introduced and as its intensity is increased.
2. Reynolds number accelerates the fluid velocity in the stenotic region.
3. Flow rate increases with the increase of Reynolds number and decreases with the increase of magnetic field intensity.
4. Flow rate is maximum at the initiation of the stenosis and minimum at the throat of the stenosis.
5. The flow rate is influenced by tapering. It increases with divergent tapering and decreases with convergent tapering.
6. Wall shear stress shows higher values with the increase of Reynolds number and it also increases with the increase of the values of the Magnetic parameter M.
7. Wall shear stress increases with the increase of the stenosis height.

Thus the mathematical expressions may help medical practitioners to control the blood flow of a patient by applying a suitable magnetic field.

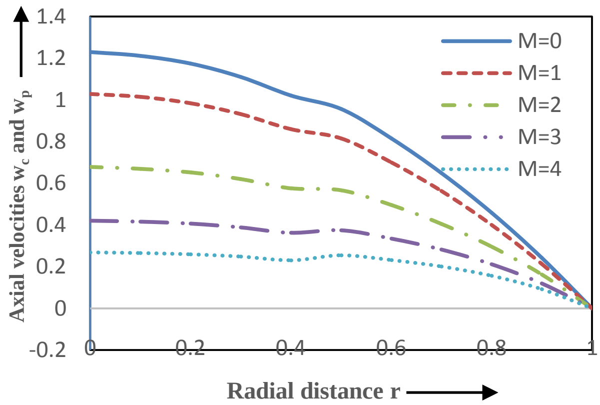

Fig.2 Variation of axial velocities for different values of Hartmann number M at n= 2

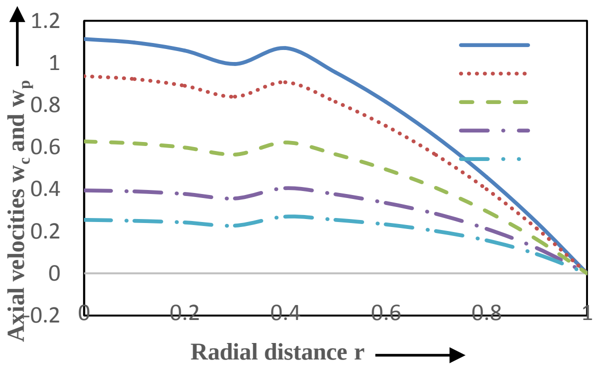

Fig.3 Variation of axial velocities for different values of Hartmann number M at n= 3.

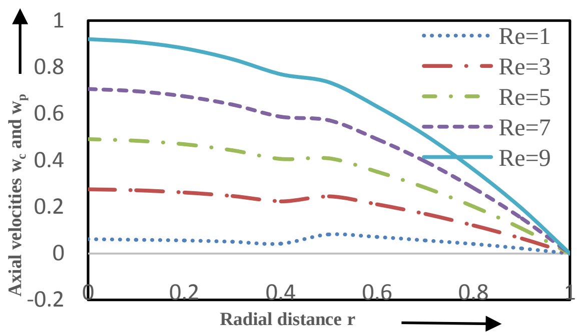

Fig.4Variation of axial velocities for different values of Reynolds number at n= 2

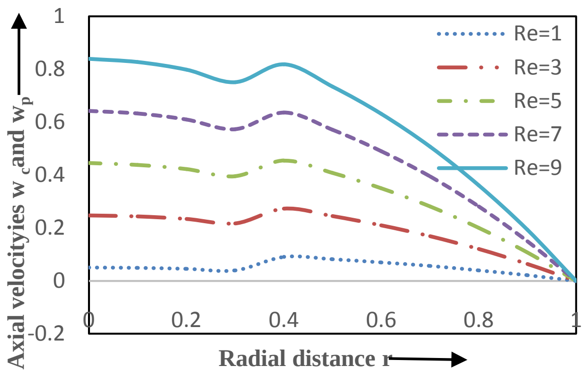

Fig.5 Variation of axial velocities for different values of Reynolds number at n= 3

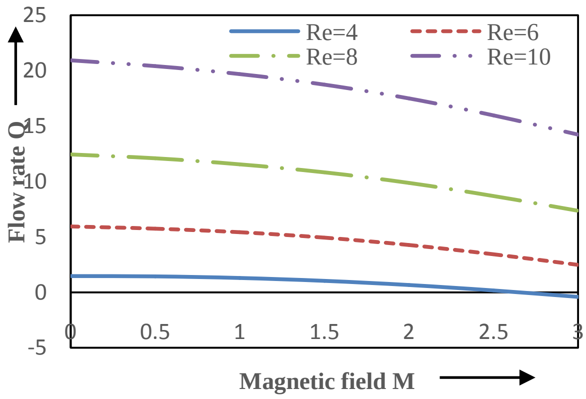

Fig.6 Variation of volumetric flow rate Q with magnetic field for different values of Reynolds number

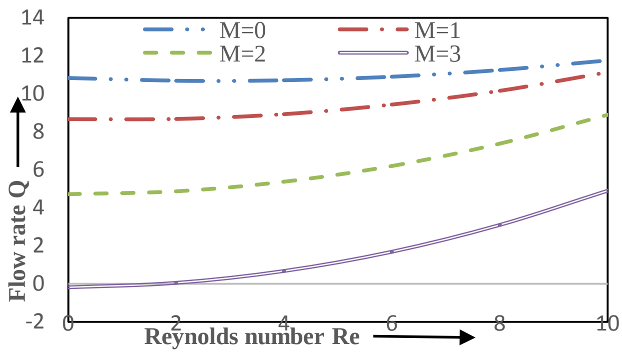

Fig.7 Variation of volumetric flow rate Q with Reynolds number Re for different values of Hartmann number M.

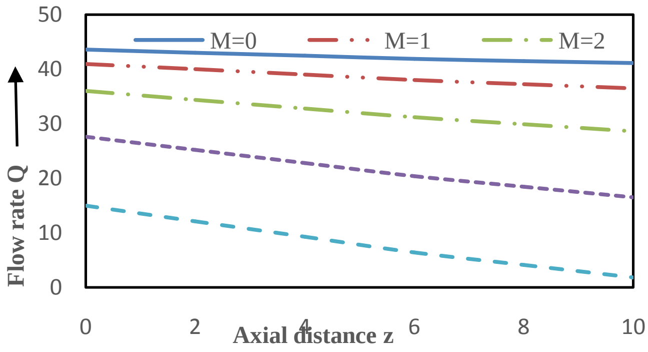

Fig.8 Variation of volumetric flow rate Q against axial distance with different values of Hartmann number M

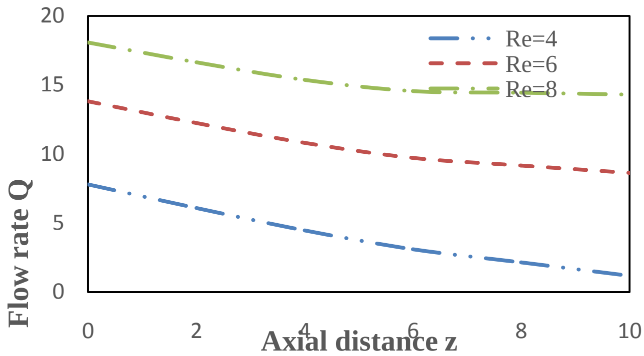

Fig.9 Variation of volumetric flow rate Q against axial distance with different values of Reynolds number Re.

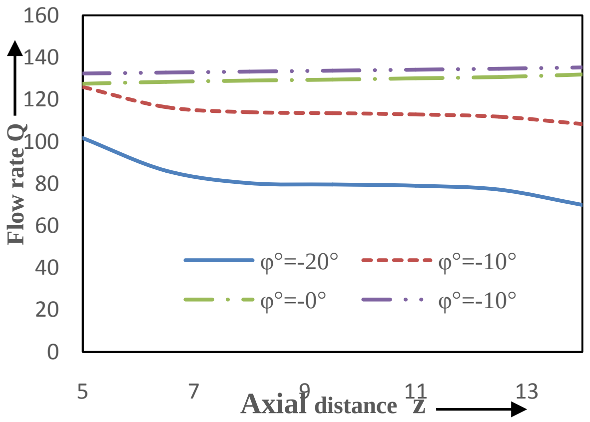

Fig.10Variation of volumetric flow rate Q against axial distance with different values of tapering angles for n=2 (ws=.05).

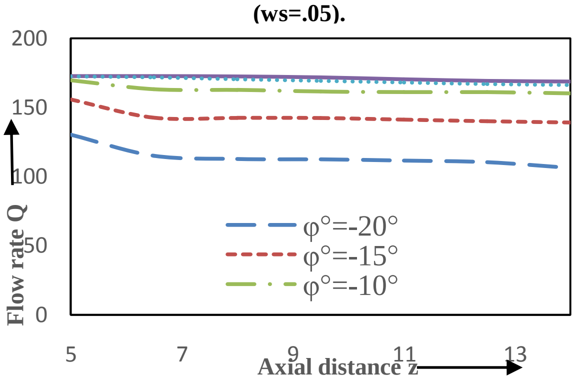

Fig.11. Variation of volumetric flow rate Q against axial distance with different values of tapering angles for n=2 (ws=0.5)

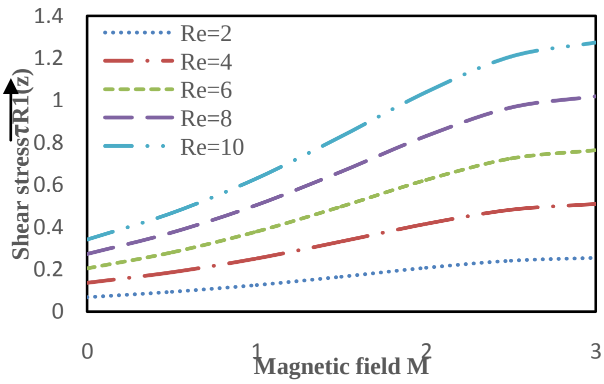

Fig.12. Variation of shear stress at the interface with magnetic field for different values of Reynolds number Re.

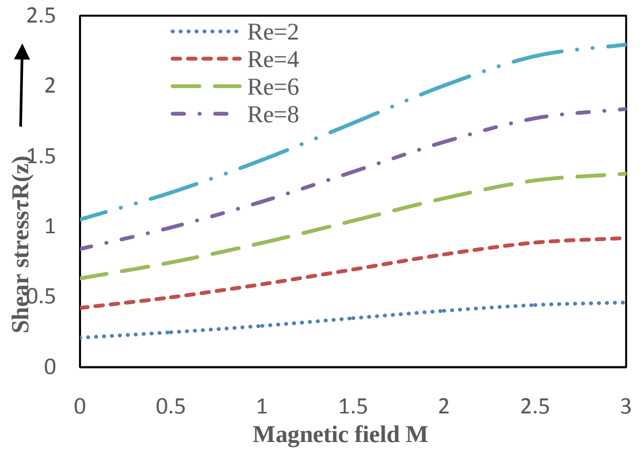

Fig.13. Variation of shear stress at the boundary of the artery with stenosis height δs for different values of Reynolds number Re.

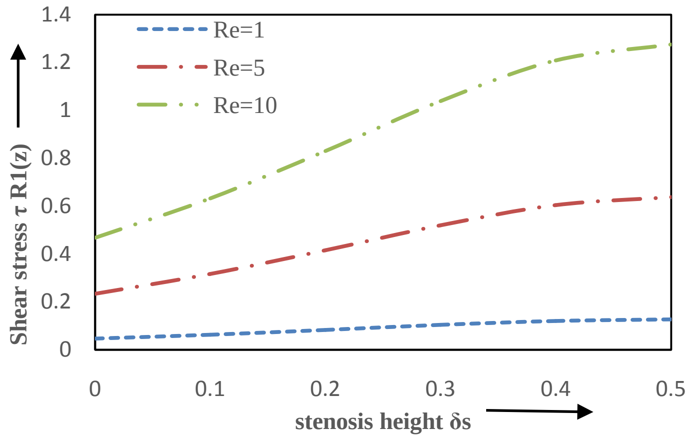

Fig.14. Variation of shear stress at the interface with stenosis height δs for different values of Reynolds number Re.

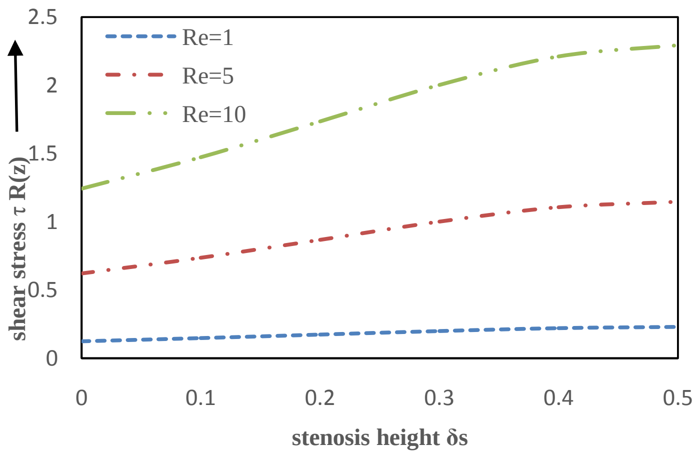

Fig.15. Variation of shear stress at the boundary of the artery with stenosis height δs for different values of Reynolds number Re.

6. ACKNOWLEDGMENTS

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