Hyper Chaos Control using Fuzzy Sliding Mode Controller with Application to a Satellite Motion

S.M.Hamidzadeh
Department of Electrical Engineering, Mashhad Branch
Islamic Azad University, Mashhad, Iran

Arman Zarringhalam
Department of Electrical Engineering, Mashhad Branch
Islamic Azad University, Mashhad, Iran

Mahdi Yaghoobi
Department of Electrical Engineering, Mashhad Branch
Islamic Azad University, Mashhad, Iran

ABSTRACT
In this paper, the problem of hyper-chaos stabilization was discussed via Fuzzy Sliding Controller. The equation of a satellite is a six-Dimensional nonlinear system which includes some types of nonlinear behavior such as periodic trajectory, chaotic dynamics. A Fuzzy Sliding Controller is applied to regulate the state trajectory hyper-chaos satellite to the unstable equilibrium points. Using Lyapunov theory, the stability control system is proven. Simulation results show that the proposed controller can be chaotic satellite attitude in the presence of disturbance inputs and uncertainties will converge to the unstable equilibrium points.

Keywords
Hyper-chaos, Fuzzy, Satellite, Dynamic Error

1. INTRODUCTION
Chaos is a common phenomenon in the natural. In 1963, Lorenz discovered the first chaotic system when he studied atmospheric convection [1]. A chaotic system dynamical behavior, such as depending sensitively on tiny variation of initial conditions, is having bounded trajectories in the phase space, etc. This complex behavior could be undesirable in many engineering applications. For this reason, it is often necessary to design control mechanisms that will force system to exhibit a desired dynamic, even intrinsically chaotic. In 1979, Rossler reported the first hyper-chaotic system with two positive Lyapunov exponents [2]. A hyper-chaotic attractor is characterized as a chaotic attractor with more than on positive Lyapunov exponent. In recent years, the study of hyper-chaotic systems has grown up in many field such as laser [3,4], nonlinear circuit[5], communication[6] and aerospace[7,8]. Especially recent decades of years, many techniques and methods have developed to control hyper-chaotic dynamic, such as adaptive feedback control [9], Fuzzy [10], Sliding mode Control [11], feedback controller [12]. The research in [13, 14] was proven to be chaotic attitude motion satellite. Recently, various researches and publications introduced the chaotic dynamics of the satellites. Methods that have been introduced thus far include predictive control [15], impulsive control [16], and neural networks [17].

The control of the satellite, on the other hand, is a stabilization problem. Due to the large dimensions and complexity of the satellite equations, requires a controller that can act quickly and appropriately. So in this paper, we investigate the stability of hyper-chaotic satellite that is including angular velocities and attitude angles, using the Fuzzy Sliding Mode (FSM) controller. The paper is organized as follows, after this introduction, the section 2 described satellite system with hyper-chaotic dynamic. section3, describe FSM controller design technique and section 4 presents simulation results to demonstrate the effectiveness of the proposed control method for stabilization hyper-chaotic satellite dynamics. Finally, section 5 draws conclusion.

2. SATELLITE SYSTEM AND HYPER-CHAOTIC DYNAMICS
In this section, the satellite system and the hyper-chaotic dynamics are studied. The kinematic equation of a satellite or spacecraft can be written as:

$$\begin{aligned}\dot{\phi} &= \omega_x + \sin\phi\tan\theta\omega_y + \cos\phi\tan\theta\omega_z \\ \dot{\theta} &= \cos\phi\omega_y - \sin\phi\omega_z \\ \dot{\psi} &= \sin\phi\sec\theta\omega_y + \cos\phi\sec\theta\omega_z\end{aligned} \tag{1}$$

The rotational motion for general rigid spacecraft acting under the influence of external torques is given by [14]. The dynamical equation of a satellite, similar to a rigid body can be expressed as:

$$I\dot{\omega} = -\Omega I\omega + H + U \tag{3}$$

Where I is the moment of inertia tensor, ω is the angular velocity vector, U is the control torque, and H contains any external disturbance torques. The dynamical equations of a satellite are:

$$\begin{aligned}I_x\dot{\omega}_x &= \omega_y\omega_z(I_y - I_z) + H_x \\ I_y\dot{\omega}_y &= \omega_x\omega_z(I_z - I_x) + H_y \\ I_z\dot{\omega}_z &= \omega_y\omega_y(I_x - I_y) + H_z\end{aligned} \tag{4}$$

Where I_x, I_y and I_z are the principal moments of inertia, ω_x, ω_y and ω_z are the angular velocities of the satellite, H_x, H_y and H_z are perturbing torques. Principal moments of inertia and perturbing torques such as:

$$M = \begin{bmatrix} -1.2 & 0 & \frac{\sqrt{6}}{2} \\ 0 & 0.35 & 0 \\ -\sqrt{6} & 0 & -0.4 \end{bmatrix}\begin{bmatrix}\omega_x \\ \omega_y \\ \omega_z\end{bmatrix}, \begin{cases} I_x = 3Kg.M^2 \\ I_y = 2Kg.M^2 \\ I_z = 1Kg.M^2 \end{cases} \tag{5}$$

This torques is chosen so as to force the satellite into chaotic motion. By changing the elements value of system matrices, many various dynamical behaviors could be observed. We consider the system (1) and (4) under perturbing torques (5) such as:

$$\dot{\phi} = \omega_x + \sin\phi \tan\theta \omega_y + \cos\phi \tan\theta \omega_z$$
$$\dot{\theta} = \cos\phi \omega_y - \sin\phi \omega_z$$
$$\dot{\psi} = \sin\phi \sec\theta \omega_y + \cos\phi \sec\theta \omega_z$$
$$\dot{\omega}_x = \omega_y \omega_z \frac{(I_y - I_z)}{I_x} - \frac{1.2}{I_x}\omega_x + \frac{\sqrt{6}}{2I_x}\omega_z \qquad (6)$$
$$\dot{\omega}_y = \omega_x \omega_z \frac{(I_z - I_x)}{I_y} + \frac{0.35}{I_y}\omega_y$$
$$\dot{\omega}_z = \omega_y \omega_x \frac{(I_x - I_y)}{I_z} - \frac{\sqrt{6}}{I_z}\omega_x - \frac{0.4}{I_z}\omega_z$$

In equation (6) it is seen that the three coupled nonlinear relationship exists between the satellite dynamics. Thus, it can be seen in the attitude of the satellite is the most complex chaotic dynamics.

The control problem hyper-chaotic is to suppress the chaos and regulate the state trajectory of this system to a desire fixed point or around the equilibrium point is unstable or:

$$\begin{bmatrix}\phi^* & \theta^* & \psi^* & \omega_x^* & \omega_y^* & \omega_z^*\end{bmatrix}^T = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$$

Hence, the proposed controller will be described in the next section. By changing the equation (6) to form:

$$\begin{bmatrix}\phi & \theta & \psi & \omega_x & \omega_y & \omega_z\end{bmatrix}^T = \begin{bmatrix}x_1 & x_2 & x_3 & x_4 & x_5 & x_6\end{bmatrix}^T$$

. Equation (6) with unknown input can be rewritten such as:

$$\dot{X}_1 = x_4 + (\sin x_1 \tan x_2)x_5 + (\cos x_1 \tan x_2)x_6 - U_1$$
$$\dot{X}_2 = \cos x_1 . x_5 - \sin x_1 . x_6 - U_2$$
$$\dot{X}_3 = (\sin x_1 \sec x_2).x_5 + (\cos x_1 \sec x_2).x_6 - U_3$$
$$\dot{X}_4 = \zeta_x x_5 x_6 - \frac{1.2}{I_x}x_4 + \frac{\sqrt{6}}{2I_x}x_6 - U_4 \qquad (7)$$
$$\dot{X}_5 = \zeta_y x_4 x_6 + \frac{0.35}{I_y}x_5 - U_5$$
$$\dot{X}_6 = \zeta_z x_4 x_5 - \frac{\sqrt{6}}{I_z}x_4 - \frac{0.4}{I_z}x_6 - U_6$$

Where $U_i, \mathrm{i} = 1,2,3,4,5,6$, control inputs should be designed, and ζ_i is $\zeta_x = \frac{I_y - I_z}{I_x}$, $\zeta_y = \frac{I_z - I_x}{I_y}$, $\zeta_z = \frac{I_x - I_y}{I_z}$.

The error states are defined as $e_i = x_i - x_i^*$. The state error is of the form:

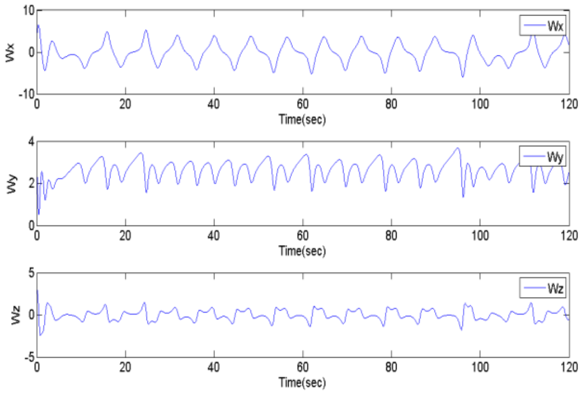

Fig.1 Angular Velocities

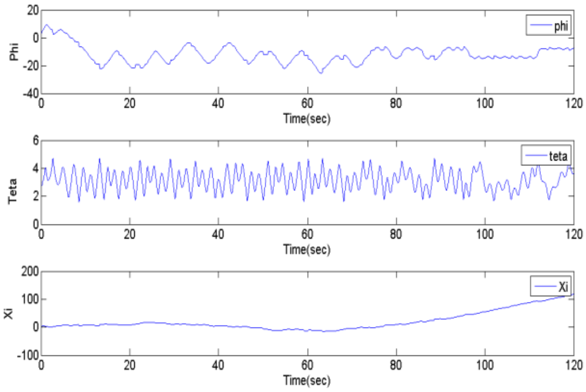

Fig.2 Attitude Angles

$$\dot{e}_1 = \dot{X}_1 - \dot{X}_1^*$$
$$\dot{e}_2 = \dot{X}_2 - \dot{X}_2^*$$
$$\dot{e}_3 = \dot{X}_3 - \dot{X}_3^*$$
$$\dot{e}_4 = \dot{X}_4 - \dot{X}_4^* \qquad (8)$$
$$\dot{e}_5 = \dot{X}_5 - \dot{X}_5^*$$
$$\dot{e}_6 = \dot{X}_6 - \dot{X}_6^*$$

In order to control the system (7) to the unstable equilibrium point $\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T$,

3. FSM CONTROLLER DESIGN

The control input of the system (7) determined using the Fuzzy Sliding Mode controller; the sliding surface can be defined as,

$$S_i = e_i + \int_0^t \lambda_i e_i d(t) \qquad i = 1,2,\ldots,6 \qquad (9)$$

Where $\lambda_i \in \Re$ is constant. The existence of the sliding-mode requires the following conditions to be satisfied [18],

$$S_i = e_i + \int_0^t \lambda_i e_i d(t) = 0 \quad i = 1,2,\ldots,6 \qquad (10)$$

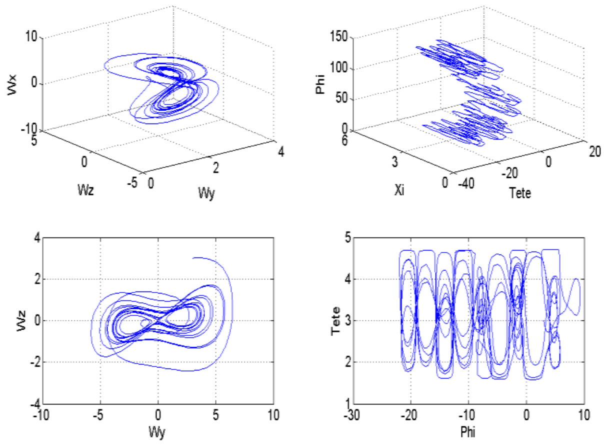

Fig.3 Phase Portraits of Hyper-Chaotic Satellite

And

$\dot{S}_i = \dot{e}_i + \lambda_i e_i = 0 \quad i = 1, 2, ..., 6$
(11)

The controls $U_i, \mathrm{i} = 1, 2, ..., 6$ of the dynamic system (7) are designed to ensure that the error states the sliding surfaces.

Therefore:

$$
\begin{aligned}
U_1 &= \dot{e}_1 + \lambda_1 e_1 k_1 FSM(\mathrm{S}, \dot{\mathrm{S}}) \\
U_2 &= \dot{e}_2 + \lambda_2 e_2 k_2 FSM(\mathrm{S}, \dot{\mathrm{S}}) \\
U_3 &= \dot{e}_3 + \lambda_3 e_3 k_3 FSM(\mathrm{S}, \dot{\mathrm{S}}) \\
U_4 &= \dot{e}_4 + \lambda_4 e_4 k_4 FSM(\mathrm{S}, \dot{\mathrm{S}}) \\
U_5 &= \dot{e}_5 + \lambda_5 e_5 k_5 FSM(\mathrm{S}, \dot{\mathrm{S}}) \\
U_6 &= \dot{e}_6 + \lambda_6 e_6 k_6 FSM(\mathrm{S}, \dot{\mathrm{S}})
\end{aligned} \quad (12)
$$

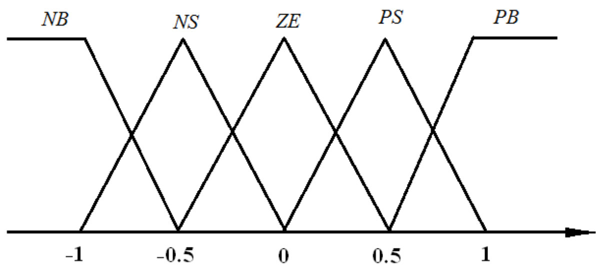

Fig.4 Inputs Membership Function ($S, \dot{S}$)

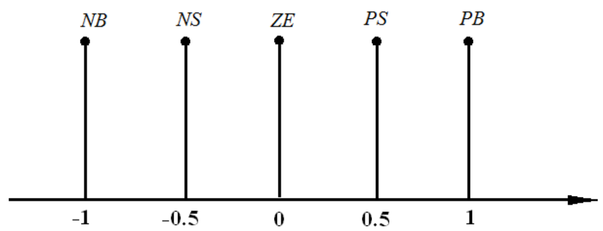

Fig.5 Output Membership Function

Where $k_i, (\mathrm{i} = 1, 2, ..., 6)$ are positive constant and $FSM(\mathrm{S}, \dot{\mathrm{S}})$ represents the functional characteristics of the fuzzy logic decision scheme. Fig.4 shows the membership functions of input. They are decomposed into five fuzzy partitions, which are negative big (NB), negative small (NS), zero (ZE), positive small (PS), and positive big (PB). The fuzzy rules of FSM are:

$If\ S, \dot{S}\ is\ PB, Then\ FSM\ is\ PB$

$If\ S, \dot{S}\ is\ PS, Then\ FSM\ is\ PS$

$If\ S, \dot{S}\ is\ ZE, Then\ FSM\ is\ ZE$

$If\ S, \dot{S}\ is\ NS, Then\ FSM\ is\ NS$

$If\ S, \dot{S}\ is\ NB, Then\ FSM\ is\ NB$

The crisp FSM is calculated using a weighted average defuzzification method:

$$FSM = \frac{\sum_{i=1}^{6} \mu_i u_i}{\sum_{i=1}^{6} u_i} \quad (13)$$

Where μ_i the premise membership is function value of the ith-rule and u_i is the singleton control vector in the ith-rule.

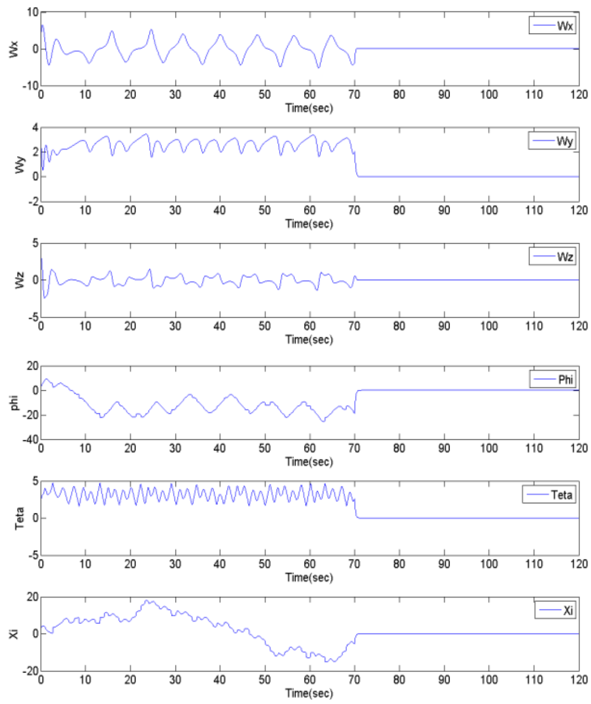

Fig.6 State variables of the hyper-chaotic satellites with controller proposed

Theorem1. The hyper-chaotic satellite with FSM controller proposed in the (12), the error state trajectory then converges to zero, if all initial condition $x_0 \in \Re^n$, and all satellite dynamics is measurable.

Proof1. Let the Lyapanov function candida is:

$$V = 0.5(S_1^2 + S_2^2 + S_3^2 + S_4^2 + S_5^2 + S_6^2) \quad (14)$$

Differentiating the Lyapunov function equation (14) with respect to time yields,

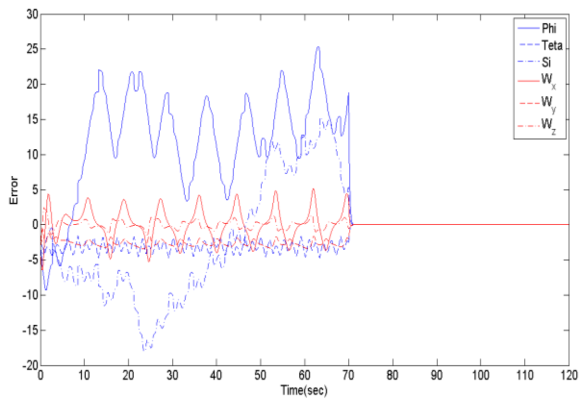

Fig.7 Error Controller

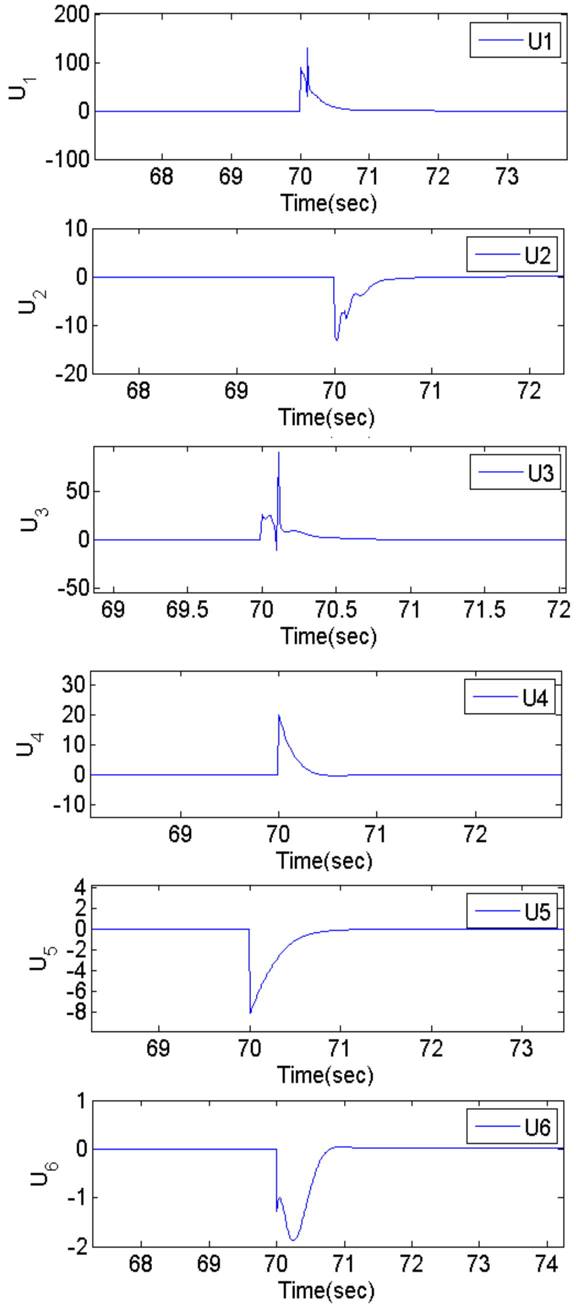

Fig.8 FSM Controller proposed

$$\begin{aligned}\dot{V} &= S_1\dot{S}_1 + S_2\dot{S}_2 + S_3\dot{S}_3 + S_4\dot{S}_4 + S_5\dot{S}_5 + S_6\dot{S}_6 \\ &= S_1(\dot{e}_1 + \lambda_1 e_1 - U_1) + S_2(\dot{e}_2 + \lambda_2 e_2 - U_2) \\ &+ S_3(\dot{e}_3 + \lambda_3 e_3 - U_3) + S_4(\dot{e}_4 + \lambda_4 e_4 - U_4) \\ &+ S_5(\dot{e}_5 + \lambda_5 e_5 - U_5) + S_6(\dot{e}_6 + \lambda_6 e_6 - U_6)\end{aligned} \tag{15}$$

Substituting U_i, i = 1, 2, 3, 4, 5, 6 into $\dot{V}$ yields,

$$\begin{aligned}\dot{V} &= S_1 k_1 FSM + S_2 k_2 FSM + S_3 k_3 FSM \\ & S_4 k_4 FSM + S_5 k_5 FSM + S_6 k_6 FSM \\ &= -k_1|S_1| - k_2|S_2| - k_3|S_3| \\ & k_4|S_4| - k_5|S_5| - k_6|S_6| < 0\end{aligned} \tag{16}$$

This is a negative definite function. Hence, by Lyapunov stability theory, it is immediate that the error dynamics (8) is globally asymptotically stable for all initial conditions $e(0) \in R^n$. Complete the proof.

4. SIMULATION RESULT

In this section, the fourth-order Runge-Kutta integration with step time $h = 10^{-3}$ method is used to solve the satellite system (7). Initial values of the hyper-chaotic system are $x_0 = [3 \quad 3 \quad 3 \quad 3 \quad 3 \quad 3]^T$. The time responses of state variable of the hyper-chaotic satellite are show in Fig.1 and Fig.2. Phase Portraits of hyper-chaotic satellite is represented in Fig3. In the simulation, the parameters of the control inputs given by Eq. (12) are set to:

$\lambda = 5$, $\quad k = [0.05 \quad 0.05 \quad 0.05 \quad 0.05 \quad 0.05 \quad 0.05]^T$.

Fig.4 and Fig.5 shows the Membership functions of the input–output variables for FSM controller proposed. The controller is active at time t = 70(sec) the state variables are controlled to the equilibrium point unstable. Fig6 and Fig.7 presents state variables of the hyper-chaotic satellites with controller proposed and error, respectively, these errors converge asymptotically to zero. The simulation results demonstrate that the proposed Fuzzy Sliding Mode controller is effective in the to the equilibrium point unstable. The proposed controller does not chattering, means that the controller can be implemented in the real world. (See Fig.8)

5. CONCLUSION

The attitude equation of a satellite, includes kinematic and dynamic equations of a satellite, is a six-dimensional nonlinear system which includes some types of complex and nonlinear dynamical behaviors. The object of the attitude control system is to regulate the angular angles and angular velocities of a rigid-body in the space. In this paper, was introduction a Fuzzy Sliding Mode control design method for stabilization of the hyper-chaotic satellite. Using Lyapunov theory, the stability control system is proved. The numerical simulation results show that the proposed FSM controller the system could hyper-chaotic motion of satellites converge to the unstable equilibrium points. According to Fig.7, the controller after the applying the chaotic dynamics of satellites quickly forced the move towards a stable equilibrium points.

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