

# Primary User Localization Schemes in Cooperative Sensing

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## ABSTRACT

In cognitive radio networks secondary users access licensed frequency bands if they are free to use. Since it is inevitable that transmissions of the licensed users must not be interfered with different sensing and transmission techniques have to be utilized by the secondary users. For example the knowledge of a primary user location can be exploited to reduce the interference with that particular user. Thereby applicable techniques are directed transmissions and transmission power control of the secondary users. However the localization is not trivial and is tainted with uncertainties due to estimation errors.

In this paper several different primary user localization schemes based on Received Signal Strength (RSS) measurements which try to reduce the localization error to a minimum with reasonable effort are presented. Some of them are adopted from wireless sensor networks.

## General Terms:

Cognitive Radio, Cooperative Sensing, Primary User Localization

## Keywords:

cognitive radio networks, cooperative sensing, user localization, received signal strength, least squares, weighted centroid

## 1. INTRODUCTION

Spectrum is one of the finite but vital resources for wireless radio communications. This scarcity is a challenge to the growing demand for wireless connectivity. Although most parts of the usable spectrum have already been allocated by the regulatory authorities for currently existing services there is still room for improvement because the spectrum bands are underutilized. Cognitive radio networking is an approach to make use of the spectrum in a more efficient way.

In cognitive radio networks so-called unlicensed secondary users are allowed to access the licensed spectrum bands if they are not occupied by the primary users. The periods in which the licensed frequency bands are not in use are called white spaces. Prior to the access the secondary user has to sense which bands are unused. The spectrum sensing process has to ensure that no harmful inter-

ference with primary users occurs and detect all white spaces for good network performance.

Cooperative sensing is a technique to overcome the limitations of single user sensing like noise uncertainty, shadowing and multipath fading. Since cooperative sensing exploits the spatial diversity of the secondary users it is helpful to know the location of the primary users as well. Additionally if the location of a primary user is known directional transmission and power controlled transmission techniques to reduce the interference can be applied.

Therefore secondary users should not only be aware of the presence of a primary user but also of its location. With the knowledge of the exact user locations a Radio Environment Map (REM) can be compiled and used as a tool for spectrum management [1]. Figure 1 shows a typical REM of a cognitive radio network. According to the map secondary users can adjust their transmission parameters in terms of power frequency and direction to lower the interference with the primary users.

Usually the secondary users perform measurements related to the localization of the primary user and send the measurement data to a base station for further analysis. For the localization the base station combines all the measurements, extracts the primary user location and keeps track of it if the primary user moves.

Nevertheless legacy primary users cannot be assumed to cooperate with the secondary user sensing. That is why their transmit powers and corresponding locations have to be estimated. In this paper different localization methods based on measurements of the Received Signal Strength (RSS) are investigated.

## 2. SYSTEM MODEL

For modeling the network conditions several assumptions have to be made. In the simulation model all secondary users are aware of their positions and the network consists of one primary user and at least four secondary users which receive signals from the primary user. Additionally each secondary user shares its position and the measured RSS values with the base station. Moreover the shadowing effect to each secondary user is independent. Under these assumptions the primary user's position can be estimated at the cognitive radio base station.

### 2.1 Log-normal Path Loss Model

Since the Received Signal Strength (RSS) is used for for the estimation of the distances between the primary user and the secondary

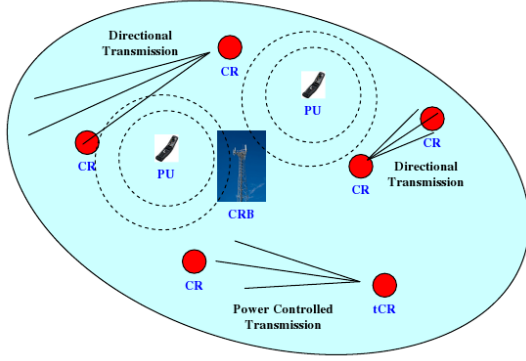


Fig. 1. Radio environment map and radio scene analysis in a cognitive radio network environment [2]

users a propagation model has to be derived. Since shadowing is severe in urban networks the log-normal path loss model is used. The distance between the position of the primary user  $[x, y]$  and the position  $[x_i, y_i]$  of the  $i$ -th secondary user is represented by the following equation where  $N$  is the total number of secondary users.

$$d_i(x, y) = \sqrt{(x - x_i)^2 + (y - y_i)^2}, \quad i = 1, 2, \dots, N \quad (1)$$

The RSS at the  $i$ -th secondary user is denoted by

$$RSS_i = K_i \frac{P_{TX}}{d_i^\alpha S_i}. \quad (2)$$

In this model  $P_{TX}$  is the transmission power of the primary user,  $\alpha$  the path-loss exponent,  $K_i$  factors which influence the RSS, e.g. antenna gain and antenna height, and  $S_i$  a lognormal random variable. Through the variable  $S_i = 10^{0.1X_i}$  shadowing effects which cause wide variations in the measured RSS values are taken into account where  $X_i$  is a Gaussian Random Variable with  $\mu = 0$  and variance  $\sigma^2$ . For obstructed areas a path-loss exponent of  $\alpha = 5$  is applicable [3]. All antenna gains are set to 1 which leads to a factor of  $K = 1$  and the network size specified as  $1 \text{ km}^2$ .

The noise-affected distance  $\tilde{d}_i$  for each secondary user to the primary user is then computed at the base station by

$$\tilde{d}_i = \sqrt[\alpha]{K_i \frac{P_{TX}}{RSS_i}} \quad (3)$$

## 2.2 Sample Mean

Due to the shadowing effect each transmission is disturbed and varying measurements may cause wrong distance estimations. For reducing the disturbances the mean value of several measurements computed by

$$m_i = \frac{1}{M} \sum_{j=1}^M RSS_{i,j} \quad (4)$$

where  $M$  is the total number of samples and  $RSS_{i,j}$  is the  $j$ -th sample taken at the  $i$ -th user. The effect of the sampling will be discussed in the simulation results.

## 3. LOCALIZATION METHODS

For the localization of the PU the distances have to be estimated at first like it has been described in the previous section. Since the distances are affected by noise it is impossible to derive an exact primary user location.

The estimation error is minimized by Minimum Mean Square Error Method (MMSE). The error equation is formulated by

$$e_i = \tilde{d}_i - d_i \quad (5)$$

Inserting 1 in 5 leads to

$$e_i = \tilde{d}_i - \sqrt{(x - x_i)^2 + (y - y_i)^2} \quad (6)$$

To minimize the error let  $e_i = 0$ . Applying the cosine rule to 6 leads to

$$\tilde{d}_i^2 - x_i^2 - y_i^2 - (\tilde{d}_N^2 - x_N^2 - y_N^2) = 2(x_N - x_i)x + 2(y_N - y_i)y. \quad (7)$$

The formula 7 can be expressed in matrix form

$$A\theta = b \quad (8)$$

where

$$A = \begin{bmatrix} 2(x_N - x_i) & 2(y_N - y_i) \\ \vdots & \vdots \\ 2(x_N - x_{N-1}) & 2(y_N - y_{N-1}) \end{bmatrix} \quad (9)$$

$$b = \begin{bmatrix} \tilde{d}_1^2 - x_1^2 - y_1^2 - (\tilde{d}_N^2 - x_N^2 - y_N^2) \\ \vdots \\ \tilde{d}_{N-1}^2 - x_{N-1}^2 - y_{N-1}^2 - (\tilde{d}_N^2 - x_N^2 - y_N^2) \end{bmatrix} \quad (10)$$

$$\theta = \begin{bmatrix} x \\ y \end{bmatrix} \quad (11)$$

For solving the linear problem different methods can be applied, e.g. Least Squares (LS) method. Let the estimated position be

$$\tilde{\theta} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \end{bmatrix} \quad (12)$$

then the performance of the estimation methods can be investigated by the root of the Mean Square Error (MSE)

$$RMSE = \sqrt{(\tilde{x} - x)^2 + (\tilde{y} - y)^2} \quad (13)$$

### 3.1 Least Squares Method

For the solution of the linear problem 8 the Least Squares method can be used.

Minimizing the sum of the squares of the residuals

$$S = (b - A\tilde{\theta})^T (b - A\tilde{\theta}) \quad (14)$$

requires solving the normal equation

$$A^T A \tilde{\theta} = A^T b \quad (15)$$

$$\tilde{\theta} = (A^T A)^{-1} A^T b \quad (16)$$

### 3.2 Weighted Least Squares Method

For reducing the error in the estimation weights can be applied to the previously introduced LS method. Since the error tends to be larger for far away secondary users the measurements of these users should be taken less into account.

Let the weighting matrix be

$$w = \text{diag}(\tilde{d}_i^{-1}), \quad i = 1, \dots, N \quad (17)$$

and the normal equation transforms to

$$\tilde{\theta} = (A^T w A)^{-1} A^T w b. \quad (18)$$

### 3.3 Nonlinear Least Squares Model

In the Nonlinear Least Squares Model (NLSQ) the goal is to minimize the sum of the squares of the errors on the distances. The problem can be denoted as

$$F(\tilde{\theta}) = \sum_{i=1}^N (\tilde{d}_i - d_i)^2 \quad (19)$$

where  $\tilde{\theta}$  is the estimated PU location. For solving the problem  $\nabla F(\tilde{\theta}) = 0$  the Jacobian matrix  $J(\tilde{\theta})$  is defined.

$$J(\tilde{\theta}) = \begin{bmatrix} \frac{\partial \tilde{d}_1(\tilde{\theta})}{\partial x} & \frac{\partial \tilde{d}_1(\tilde{\theta})}{\partial y} \\ \vdots & \vdots \\ \frac{\partial \tilde{d}_N(\tilde{\theta})}{\partial x} & \frac{\partial \tilde{d}_N(\tilde{\theta})}{\partial y} \end{bmatrix} \quad (20)$$

With the Jacobian matrix the problem translates to  $2J(\tilde{\theta})^T f(\tilde{\theta}) = 0$  where

$$f(\tilde{\theta}) = \begin{bmatrix} \tilde{d}_1(\tilde{\theta}) - d_1 \\ \vdots \\ \tilde{d}_N(\tilde{\theta}) - d_N \end{bmatrix} \quad (21)$$

and

$$J(\tilde{\theta})^T f(\tilde{\theta}) = \begin{bmatrix} \sum_{i=1}^N \frac{(x-x_i)(\tilde{d}_i(\tilde{\theta})-d_i)}{d_i(\tilde{\theta})} \\ \sum_{i=1}^N \frac{(y-y_i)(\tilde{d}_i(\tilde{\theta})-d_i)}{d_i(\tilde{\theta})} \end{bmatrix} \quad (22)$$

For solving this problem the Gauss-Newton algorithm is used since it does not require second derivatives. The algorithm iteratively finds the minimum of the function starting with an initial guess of the PU location. The initial guess  $\tilde{\theta}_0$  is any randomly determined position with  $\theta_k$  being the k-th estimate of the position.

$$\tilde{\theta}_{k+1} = (\tilde{\theta}_k - J(\tilde{\theta}_k)^T J(\tilde{\theta}_k))^{-1} J(\tilde{\theta}_k)^T f(\tilde{\theta}_k) \quad (23)$$

The iteration stops if the alteration of the estimated position  $\|\tilde{\theta}_{k+1} - \tilde{\theta}_k\|$  is sufficiently small.

### 3.4 Advanced estimation problem

By way of comparison the estimation problem taken from [4] is investigated as well. In this problem the formulation of 8 differs in the matrices  $A$ ,  $\theta$  and  $b$  where

$$A = \begin{bmatrix} 2x_1 & 2y_1 & \frac{K_1}{RSS_1^{\frac{2}{\alpha}}} & -1 \\ \vdots & \vdots & \vdots & \vdots \\ 2x_N & 2y_N & \frac{K_N}{RSS_N^{\frac{2}{\alpha}}} & -1 \end{bmatrix} \quad (24)$$

$$\theta = \begin{bmatrix} x \\ y \\ p \\ R^2 \end{bmatrix} \quad (25)$$

$$b = \begin{bmatrix} x_1^2 + y_1^2 \\ \vdots \\ x_N^2 + y_N^2 \end{bmatrix} \quad (26)$$

The problem is solved by the Least Squares method as shown in 3.1. In contrast to the other formulation this one takes the knowledge about the factors  $K_i$  and  $\alpha$  into account. However it can not always be assumed that these parameters are known.

### 3.5 Weighted Centroid Localization

So called Weighted Centroid Localization (WCL) algorithms are well known from Wireless Sensor Networks (WSN) [5] and can be adopted for PU localization in CRNs as well. The main advantage of this particular technique is its simplicity since the characteristics of the PU signal or the radio channel can remain unknown — only the RSS must be measured.

The PU position is estimated by the algorithm as

$$(x_0, y_0) = \left( \frac{\sum_{i=1}^N w_i x_i}{\sum_{i=1}^N w_i}, \frac{\sum_{i=1}^N w_i y_i}{\sum_{i=1}^N w_i} \right) \quad (27)$$

where  $w_i$  is the weighting parameter for the i-th secondary user. Choosing the correct weighting parameters is crucial for the PU localization and many different techniques for calculation have been proposed, e.g. distance-based or based on the received energy. In this approach the RSS of each secondary user is used for getting the according weighting factor. Due to the path loss and the shadowing effects it makes sense to rely more on the nodes with a stronger RSS value. For each SU the corresponding weighting factor is therefore given by

$$w(i) = \frac{RSS(i) - RSS_{min}}{RSS_{max} - RSS_{min}} \quad (28)$$

where  $RSS(i)$ ,  $RSS_{min}$  and  $RSS_{max}$  are the sampled RSS values for the secondary users.

## 4. SIMULATION RESULTS

The performance of the different localization techniques is evaluated with respect to the number of secondary users in the network and the number of samples taken. Every simulation is repeated 100 times and the mean RMSEs are compared. At first it is assumed that all usually unknown parameters like the path-loss exponent are known. The only effect influencing the measurements is shadowing.

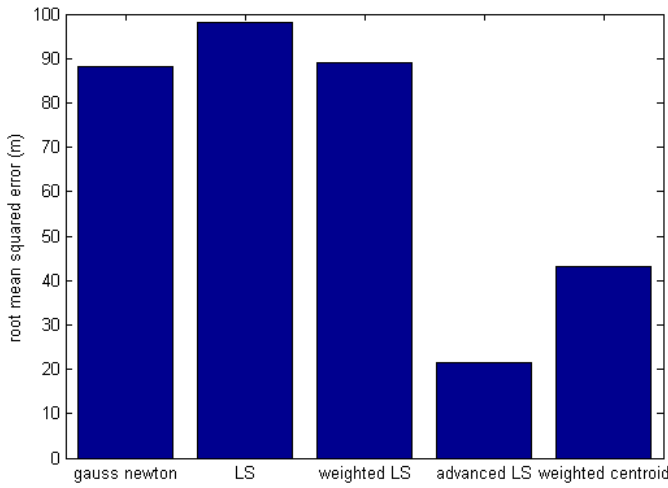


Fig. 2. Mean RMSE

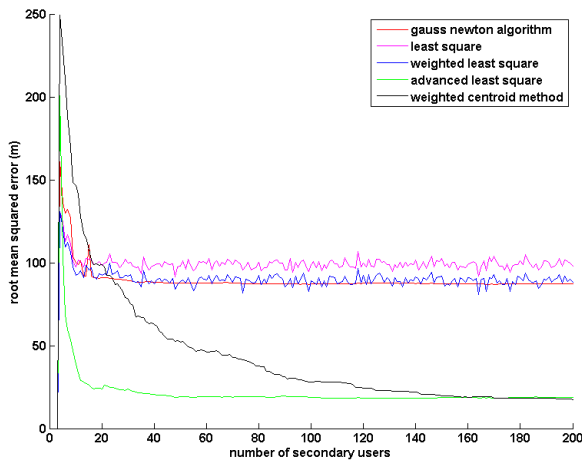


Fig. 3. RMSE with respect to the number of secondary users in the network

#### 4.1 Number of Secondary Users

For the investigation of the effect of the number of secondary users in the network the number of samples is fixed to 250. More secondary users in the network lead to an increased performance of all localization schemes since more information for the localization is available.

The overall performance of the advanced LS model and the weighted centroid method are on par with 200 users in the network and better than the linear and non-linear LS methods. However if less than 150 users are present in the network the advanced LS method outperforms the weighted centroid method. If only a few secondary users are present in the network the LS methods perform better than the weighted centroid method but they do not improve further after the number of secondary users exceeds 20.

#### 4.2 Number of Samples

All simulations related to the effect of an increasing number of samples are done with a fixed number of 50 secondary users.

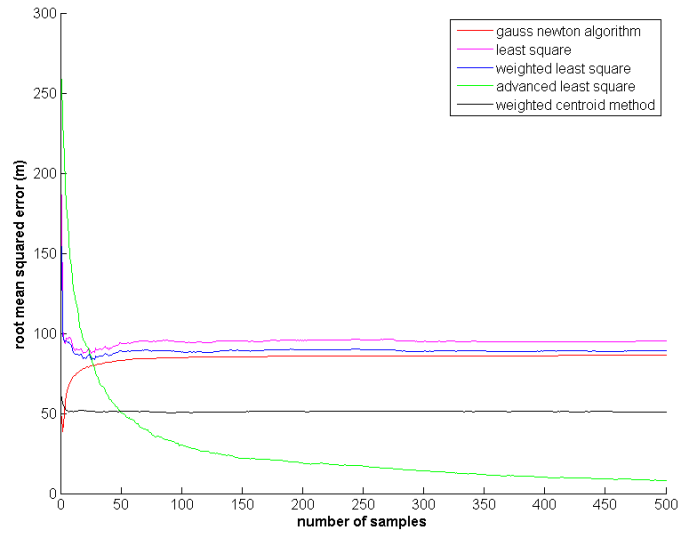


Fig. 4. RMSE with respect to the number of samples

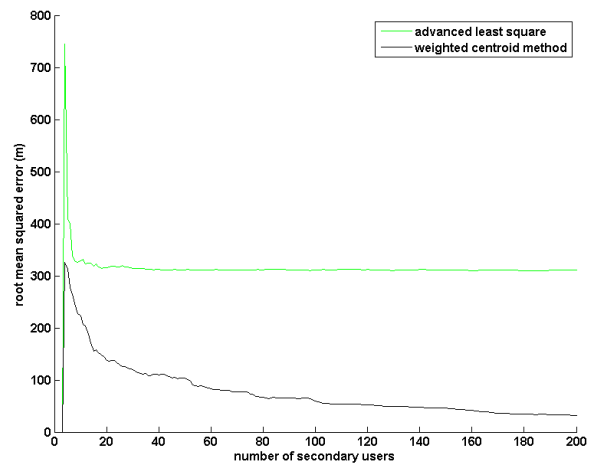


Fig. 5. RMSE with respect to the number of secondary users under varying path-loss exponent

An increase of the number of samples leads to an improved performance of the localization since the shadowing effect is moderated. The advanced LS method is influenced the most whereas the others do not improve any more with more than 50 samples taken. With less than 50 samples the weighted centroid performance performs the best. However if the number of samples is too big the network performance decreases because an increased sensing time leads to a decrease of the effective transmission time.

#### 4.3 Localization with varying path-loss exponent

In the previous simulations the path-loss was fixed and therefore assumed to be known at the secondary users. In this section the performance under a varying path-loss exponent is investigated. In-

stead of using a fixed exponent it is assumed to be randomly distributed since every user experience a different path-loss.

The weighted centroid does not use an estimated path-loss exponent for the computation. Therefore this method is not affected by the changing exponent and performs as before. in contrast to that the advanced LS method performs worse at any point.

## 5. SUMMARY AND CONCLUSION

The simulations have shown that the advanced LS method and the weighted centroid method outperform the other LS methods. The advanced LS method performs best if the path-loss exponent is known. However it cannot be guaranteed that the path-loss exponent will be available or estimated correctly for the localization. Moreover a large number of samples is needed for a low estimation error leading to a decrease of the effective transmission time.

The weighted centroid method on the contrary does not rely on any knowledge of the channel, the primary user transmission power or on a large number of sampled RSS values. Nevertheless it needs a high density of secondary users to estimate the primary user location correctly. Since this might not be true for an everyday use case a combination of different schemes depending on the density of secondary users and available channel parameters should be used instead.

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