Shortest Path Problem Under Intuitionistic Fuzzy Setting

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ABSTRACT

In this article a fuzzy network has been considered whose edge weights are characterized by trapezoidal intuitionistic fuzzy numbers (TRIFNs).The network is acyclic with topological ordering. The shortest path and the corresponding path distance have been computed with the help of Bellman dynamic programming formulation. The method is illustrated by a suitable numerical example.

Keywords:

Value, ambiguity, ranking, trapezoidal intuitionistic fuzzy number(TRIFN),shortest path

1. INTRODUCTION

The shortest path problem is one of the most fundamental combinatorial optimization problem that appears in many applications as a sub problem. The length of arcs in the network represents distance or costs or traversing time. In real life situations, these arc lengths cannot be determined precisely i.e.,the quantities represented in the edge lengths of a network are not well defined. To determine the exact values of these quantities is a very difficult job and sometimes it is too much inconvenient for a decision maker to make concrete decision. In order to deal with such situations, many authors have considered shortest path problem in fuzzy context[[2], [4], [5], [7] [9]]. In 1980 Dubious and Prade [2] first introduced fuzzy shortest path problem. Though it make a breakthrough in the field of fuzzy optimization, the method suggested in [2] have been found to be inapplicable in few practical cases. Ocada and Soper [9] introduced an algorithm to find fuzzy non dominated path from any node of a network to a specified node of a network. The method suggested by [9] gives an order relation between fuzzy numbers based on fuzzy min concept. Klein [7] introduced one shortest path algorithm for acyclic network. De and Bhincher [4] computed shortest path in a fuzzy network with case study on Rajasthan Roadway network. De and Bhincher [5] described two different methods to obtain shortest path in a fuzzy network- Bellman dynamic programming approach and multi objective linear programming method. In recent years, number of works can be found in literature where intuitionistic fuzzy numbers are used to characterize the edge weights [[1], [6], [8], [11]]. Mukherjee [8] developed a heuristic methodology by modifying intuitionistic fuzzy Dijkstras algorithm (MIFDA) and used Intuitionistic Fuzzy Hybrid Geometric (IFHG) operator for solving shortest path problem in intuitionistic fuzzy setting. Biswas [1] generalized the concept of classical Dijkstras algorithm in context of intuitionistic fuzzy shortest path problem. Jayagowri and Geetharamani [6] proposed an optimized path for trapezoidal intuitionistic fuzzy network. In this paper, a shortest path problem have been solved in intuitionistic fuzzy environment considering value and ambiguity indices for trapezoidal fuzzy numbers and using Bellman dynamic programming.

The rest of the paper is constructed as follows: Section 2 contains few basic definitions and prerequisites. In section 3 we have discussed shortest route problem for an acyclic network whose edge weights are characterized by trapezoidal intuitionistic fuzzy numbers(TRIFNs). Lastly, we give our conclusive remarks.

2. PRELIMINARIES

We collect some basic definitions and notations related to trapezoidal intitionistic fuzzy number (TRIFN).

Definition 2.1. *[10] A TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a special Intuitionistic Fuzzy set on a set of real number $\mathbb{R}$, whose membership function and non membership function are defined as follows:*

$$\mu_{\tilde{a}}(x) = \begin{cases} \dfrac{(x-a_1)}{(a_2-a_1)} w_{\tilde{a}} & a_1 \le x \le a_2 \\ w_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{(a_4-x)}{(a_4-a_3)} w_{\tilde{a}} & a_3 \le x \le a_4 \\ 0 & a_4 < x or a_1 > x \end{cases} \quad (2.0.1)$$

and

$$\nu_{\tilde{a}}(x) = \begin{cases} \dfrac{(a_2-x) + u_{\tilde{a}}(x-a_1)}{(a_2-a_1)} & a_1 \le x \le a_2 \\ u_{\tilde{a}} & a_2 \le x \le a_3 \\ \dfrac{(x-a_3) + u_{\tilde{a}}(a_4-x)}{(a_4-a_3)} & a_3 \le x \le a_4 \\ 1 & a_4 < x or a_1 > x \end{cases}$$

respectively.
The values $w_{\tilde{a}}$ and $u_{\tilde{a}}$ represents the maximum degree of membership and minimum degree of non membership, respectively, such that the conditions $0 \le w_{\tilde{a}} \le 1$, $0 \le u_{\tilde{a}} \le 1$ and $0 \le w_{\tilde{a}} + u_{\tilde{a}} \le 1$ are satisfied. The parameters $w_{\tilde{a}}$ and $u_{\tilde{a}}$ reflects the confidence level and non confidence level of the TRIFN

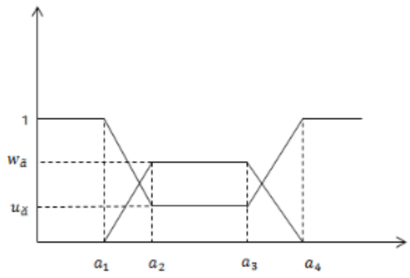

Fig. 1. Trapezoidal Intuitionistic Fuzzy Number

$\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$,respectively.
The function $\pi_{\tilde{a}}(x) = 1 - \mu_{\tilde{a}}(x) - \nu_{\tilde{a}}(x)$ is called an IF index of an element x in $\tilde{a}$.It is the degree of the indeterminacy membership of the element x in $\tilde{a}$.

Definition 2.2. *[3] A α-cut set of a TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a crisp sub set of $\mathbb{R}$, denoted and defined as $\tilde{a}_\alpha = \{x | \mu_{\tilde{a}}(x) \geq \alpha\}$ where $0 \leq \alpha \leq w_{\tilde{a}}$. The α-cut set of a TRIFN $\tilde{a}$ can be represented as the closed interval $[L_{\tilde{a}}(\alpha), R_{\tilde{a}}(\alpha)] = [a_1 + \frac{\alpha(a_2 - a_1)}{w_{\tilde{a}}}, a_4 - \frac{\alpha(a_4 - a_3)}{w_{\tilde{a}}}]$.*

Definition 2.3. *[3] A β-cut set of a TRIFN $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$ is a crisp sub set of $\mathbb{R}$, denoted and defined as $\tilde{a}_\beta = \{x | \nu_{\tilde{a}}(x) \leq \beta\}$ where $u_{\tilde{a}} \leq \beta \leq 1$. The β-cut set of a TRIFN $\tilde{a}$ can be represented as the closed interval $[L_{\tilde{a}}(\beta), R_{\tilde{a}}(\beta)] = [\frac{(1-\beta)a_2 + (\beta - u_{\tilde{a}})a_1}{1 - u_{\tilde{a}}}, \frac{(1-\beta)a_3 + (\beta - u_{\tilde{a}})a_4}{1 - u_{\tilde{a}}}]$.*

The value and ambiguity of a trapezoidal intuitionistic fuzzy number are defined as follows: [3]

Definition 2.4. *[3] Let $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ be an α-cut set and a β-cut set of a trapezoidal intuitionistic fuzzy number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$, respectively. Then the values of the membership function $\mu_{\tilde{a}}(x)$ and the non-membership function $\nu_{\tilde{a}}(x)$ for the trapezoidal intuitionistic fuzzy number $\tilde{a}$ are defined as follows:*

$$V_\mu(\tilde{a}) = \frac{a_1 + a_4 + 2(a_2 + a_3)}{6} w_{\tilde{a}} \tag{2.0.2}$$

$$V_\nu(\tilde{a}) = \frac{a_1 + a_4 + 2(a_2 + a_3)}{6} (1 - u_{\tilde{a}}) \tag{2.0.3}$$

With the condition that $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$,it follows that $V_\mu(\tilde{a}) \leq V_\nu(\tilde{a})$. Thus, the values of membership and non-membership functions of a TRIFN $\tilde{a}$ may be concisely expressed as an interval $[V_\mu(\tilde{a}), V_\nu(\tilde{a})]$.

Definition 2.5. *[3] Let $\tilde{a}_\alpha$ and $\tilde{a}_\beta$ be an α-cut set and a β-cut set of a trapezoidal intuitionistic fuzzy number $\tilde{a} = \langle (a_1, a_2, a_3, a_4); w_{\tilde{a}}, u_{\tilde{a}} \rangle$, respectively. Then the ambiguities of the membership function $\mu_{\tilde{a}}(x)$ and the non-membership function $\nu_{\tilde{a}}(x)$ for the trapezoidal intuitionistic fuzzy number $\tilde{a}$ are defined as follows:*

$$A_\mu(\tilde{a}) = \frac{(a_4 - a_1) - 2(a_2 - a_3)}{3} w_{\tilde{a}} \tag{2.0.4}$$

$$A_\nu(\tilde{a}) = \frac{(a_4 - a_1) - 2(a_2 - a_3)}{3} (1 - u_{\tilde{a}}) \tag{2.0.5}$$

With the condition that $0 \leq w_{\tilde{a}} + u_{\tilde{a}} \leq 1$,it follows that $A_\mu(\tilde{a}) \leq A_\nu(\tilde{a})$. Thus, the ambiguities of membership and non-membership functions of a TRIFN $\tilde{a}$ may be concisely expressed as an interval $[A_\mu(\tilde{a}), A_\nu(\tilde{a})]$.

Definition 2.6. *[3] The value and ambiguity indices of a TRIFN are given by the formulae:*

$$V(\tilde{a}) = \frac{V_\mu(\tilde{a}) + V_\nu(\tilde{a})}{2} \tag{2.0.6}$$

$$A(\tilde{a}) = \frac{A_\mu(\tilde{a}) + A_\nu(\tilde{a})}{2} \tag{2.0.7}$$

Definition 2.7. *The ranking function for TRIFN $\tilde{a}$ is defined as*

$$R(\tilde{a}) = V(\tilde{a}) + A(\tilde{a}) \tag{2.0.8}$$

3. BELLMAN DYNAMIC PROGRAMMING FORMULATION

Let $G = (V, e)$ be an acyclic directed connected graph of n vertices numbered from 1 to n such that '1' is the souce node and 'n' is the destination node. Here the nodes of the network are arranged with topological ordering $(e_{ij}, i < j)$. Now the shortest path can be determined by Bellman dynamic programming formulation by forward pass computation method. The Bellman dynamic programming formulation is described as follows:

$$f(1) = 0 \tag{3.0.9}$$

$$f(i) = \min_{i<j}\{f(i) + d_{ij}\}$$

where d_{ij}= weight of the directed edge e_{ij}
$f(i)$= length of the shortest path of i^{th} node from the source node 1.
We consider a directed connected graph whose edge weights are characterized by TRIFNs and calculate the corresponding shortest path as follows:

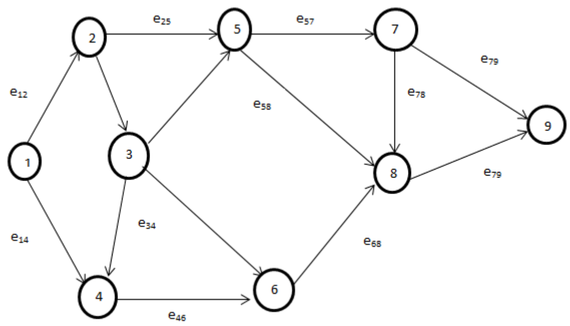

Fig. 2. Intuitionistic fuzzy weighted ghraph

$e_{12} = \langle (50, 60, 65, 70); 0.8, 0.2 \rangle$
$e_{14} = \langle (30, 40, 48, 60); 0.6, 0.3 \rangle$
$e_{23} = \langle (20, 35, 40, 45); 0.6, 0.2 \rangle$
$e_{25} = \langle (10, 16, 19, 26); 0.7, 0.3 \rangle$

$e_{34} = \langle(12, 18, 20, 24); 0.5, 0.3\rangle$
$e_{35} = \langle(8, 9, 9, 10); 0.9, 0.1\rangle$
$e_{36} = \langle(12, 17, 22, 27); 0.6, 0.4\rangle$
$e_{46} = \langle(42, 45, 50, 55); 0.8, 0.1\rangle$
$e_{57} = \langle(53, 65, 75, 83); 0.7, 0.3\rangle$
$e_{58} = \langle(50, 60, 72, 80); 0.7, 0.3\rangle$
$e_{68} = \langle(50, 68, 78, 90); 0.4, 0.5\rangle$
$e_{78} = \langle(20, 29, 36, 40); 0.8, 0.2\rangle$
$e_{79} = \langle(45, 50, 55, 60); 0.6, 0.4\rangle$
$e_{89} = \langle(30, 41, 51, 62); 0.7, 0.2\rangle$

For each directed edge e_{ij}, the weight has been represented by corresponding rank of the alternatives i.e.,$d_{ij} = V(e_{ij}) + A(e_{ij})$. The weights of the respective edges are as follows:

edges(e_{ij})	V_{ij}	A_{ij}	d_{ij}
e_{12}	49.33	8.00	57.33
e_{14}	28.81	9.96	38.77
e_{23}	25.08	8.16	33.24
e_{25}	12.36	5.13	17.49
e_{34}	15.86	3.19	19.05
e_{35}	8.10	0.60	8.70
e_{36}	11.70	5.00	16.70
e_{46}	40.65	6.90	47.55
e_{57}	48.53	11.66	60.19
e_{58}	45.96	12.60	58.56
e_{68}	32.40	9.00	41.40
e_{78}	25.33	9.06	34.39
e_{79}	31.50	5.00	36.50
e_{89}	34.50	6.49	40.99

A table has been prepared to display the possible shortest distance from each node to the other within the given network with topological ordering and the distance between two same nodes are defined to be zero: (Please refer 4)

$$\begin{aligned}
f(1) &= 0 \\
f(2) &= \min_{i<2}\{f(i) + d_{i2}\} = f(1) + d_{12} = d_{12} = 57.33 \\
f(3) &= \min_{i<3}\{f(i) + d_{i3}\} = f(3) + d_{23} = d_{12} + d_{23} \\
&= 57.33 + 33.24 = 90.57 \\
f(4) &= \min_{i<4}\{f(i) + d_{i4}\} = \min\{f(1) + d_{14}, f(3) + d_{34}\} \\
&= \min\{d_{14}, d_{12} + d_{23} + d_{34}\} = d_{14} = 38.77 \\
f(5) &= \min_{i<5}\{f(i) + d_{i5}\} = \min\{f(2) + d_{25}, f(3) + d_{35}\} \\
&= \min\{d_{12} + d_{25}, d_{12} + d_{23} + d_{35}\} = d_{12} + d_{25} = 75.37 \\
f(6) &= \min_{i<6}\{f(i) + d_{i6}\} = \min\{f(3) + d_{36}, f(4) + d_{46}\} \\
&= \min\{d_{12} + d_{23} + d_{36}, d_{14} + d_{46}\} = d_{14} + d_{46} = 86.32 \\
f(7) &= \min_{i<7}\{f(i) + d_{i7}\} = f(5) + d_{57} = d_{12} + d_{25} + d_{57} = 135.01 \\
f(8) &= \min_{i<8}\{f(i) + d_{i8}\} = \min\{f(5) + d_{58}, f(6) + d_{68}\} \\
&= \min\{d_{12} + d_{25} + d_{58}, d_{14} + d_{46} + d_{68}\} \\
&= d_{14} + d_{46} + d_{68} = 127.72 \\
f(9) &= \min_{i<9}\{f(i) + d_{i9}\} = \min\{f(7) + d_{79}, f(8) + d_{89}\} \\
&= \min\{d_{12} + d_{25} + d_{57} + d_{79}, d_{14} + d_{46} + d_{68} + d_{89}\} \\
&= d_{14} + d_{46} + d_{68} + d_{89} = 168.71
\end{aligned}$$

Hence,the shortest path is $d_{14} + d_{46} + d_{68} + d_{89}$ and the corresponding shortest distance is 168.71 units.

4. CONCLUSION

This paper presented a method for solving shortest path problem based on value and ambiguity of TRIFNs and for this purpose we have employed Bellman dynamic programming problem. An acyclic directed graph with edge weights characterized by TRIFNs has been considered and the shortest path and corresponding shortest distance have been evaluated.

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Edges (e_{ij})	1	2	3	4	5	6	7	8	9
1	0	57.33	90.57	38.77	75.37	86.32	135.01	127.72	168.71
2	-	0	33.24	52.74	17.49	49.94	77.68	76.05	113.91
3	-	-	0	19.05	8.70	16.70	68.89	58.10	99.09
4	-	-	-	0	-	47.55	-	88.95	129.94
5	-	-	-	-	0	-	60.19	58.56	99.55
6	-	-	-	-	-	0	-	41.40	82.39
7	-	-	-	-	-	-	0	34.39	36.50
8	-	-	-	-	-	-	-	0	40.99
9	-	-	-	-	-	-	-	-	0

Fig. 3. Distance Table