International Journal of Computer Applications (0975 – 8887)
Volume 105 – No. 1, November 2014

Edge-Odd Graceful Graphs Related to Ladder and Complete Graph with Four Vertices

A. Solairaju
Associate Professor of Mathematics,
Jamal Mohamed College,
Trichy (T.N.),

D. Senthil Kumar
Lecturer in Mathematics,
Sri Sai Ram College of Education,
Koodapakkam, Puducherry, India

ABSTRACT
A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f : E(G) → {1, 3, 5,…,2q-1} so that induced map f_+:V(G) → [0, 1, 2, 3, …, (2k-1)] defined by $f_+(x) = \sum f(xy) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd. In this article, the edge-odd gracefulness of ($P_2 \times P_n$) ∇ P_n [n copies of doors]

Keywords
Graceful Graph, Edge-odd graceful labeling, Edge-odd Graceful Graph

1. INTRODUCTION
Solairaju and Chitra [2008, 2009] obtained edge-odd graceful labeling of some graphs related to paths and circuits of each length 4. Solairaju, Vimala, and Sasikala [2008a, 2008b] gracefulness of a spanning tree of the graph of Cartesian product of S_m and S_n, Solairaju et.al. [2009] that the cartesian product of path P_2 and circuit C_n for all integer n, $S_{m,n}$, C_m Θ S_n for n is even and the crown graph C_3 Θ P_n and C_3 Θ $2P_n$ are is edge-odd graceful. Here the edge-odd graceful labeling of P_m Θ S_n, m = 5, 6, 7, 8 is obtained.

2. THE CONNECTED GRAPH (P2 × PN) ∇ PN,, AND (P2 × PN) ∇ 2 PN,
In this section, the following definitions are first listed.

Definition: 2.1: **Graceful Graph:** A function *f* of a graph *G* is called a graceful labeling with m edges , if *f* is an injection from the vertex set of *G* to the set {0, 1, 2,…, m} such that when each edge *uv* is assigned the label | *f(u) - f(v)*| and the resulting edge labels are distinct. Then the graph G is graceful.

Definition: 2.2: **Edge-odd Graceful Graph:** A *(p, q)* connected graph is edge-odd graceful if there exists an injection map *f :* E(G) *{1,3,…, 2q-1}* so that induced map f_+:V(G) *{0,1,2,…,(2k-1)}* defined by $f_+(x) \equiv \sum f(x, y)$ (mod 2k), where the vertex x is incident with other vertex y and k=max{p,q} makes all the edges distinct and odd. Hence the graph G is edge-odd graceful.

Definition 2.3: ($P_2 \times P_n$) ∇ P_n is a connected graph defined by the following figure 1.

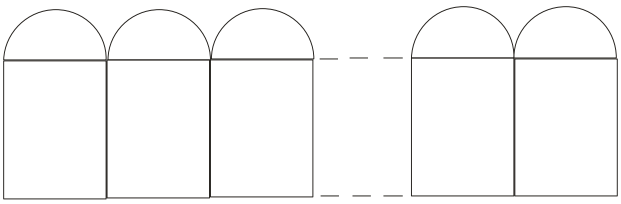

Figure 1

Theorem 2.1: The connected graph ($P_2 \times P_n$) ∇ P_n (P_n - a path with 'n' vertices) is edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph ($P_2 \times P_n$) ∇ P_n are as follows:

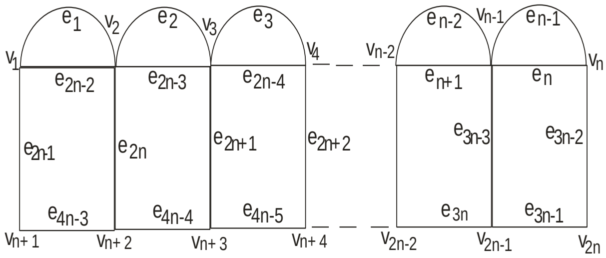

Figure 2: Edge-odd Graceful Graph ($P_2 \times P_n$)∇ P_n

To find edge-odd graceful, define f :E(G)→ {1,3,5,..,2q} by f (**e** $_i$) = (2i-1) , i = 1 to (4n-3) →(1). The induced map f_+: V(G) → {1,2,3,..,2q} by $f_+(v) \equiv \sum f(uv)$ (mod 2q) where this sum run over all edges through v Both of f and f_+ finds the distinct labels for vertices and also the edge labeling is distinct. Here the edge –odd graceful labeling of ladder ($P_2 \times P_n$) ∇ P_n is obtained

Example 1: The connected graph ($P_2 \times P_{12}$) ∇ P_{12} is edge – odd graceful.

Due to the rules (1) in theorem (2.4), edge odd graceful labeling of ($P_2 \times P_{12}$)∇P_{12} is obtained as follows:

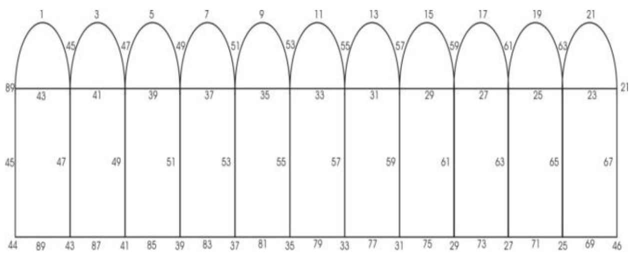

Figure 3: Edge-odd Graceful Graph (P_2xP_{12})∇P_{12}

Definition 2.4: The graph ($P_2 \times P_n$) ∇ 2 P_n is a connected graph defined by the following graph (figure 4):

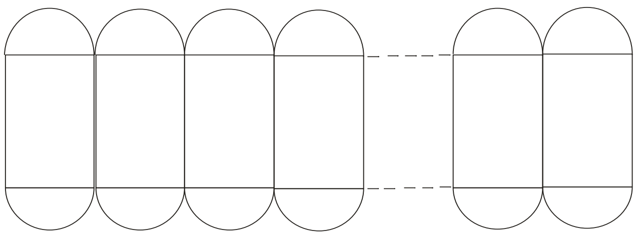

Figure 4

Theorem 2.2: The connected graph $(P_2 \times P_n)\ \nabla\ 2P_n$ (P_n -a path with 'n' vertices) is edge – odd graceful

Here the edge –odd graceful labeling of ladder $(P_2 \text{ x } P_n)\ \nabla\ 2P_n$ is obtained

Proof: The arbitrary labels of edges for the n-doors graph $(P_2 \times P_n)\ \nabla 2\ P_n$

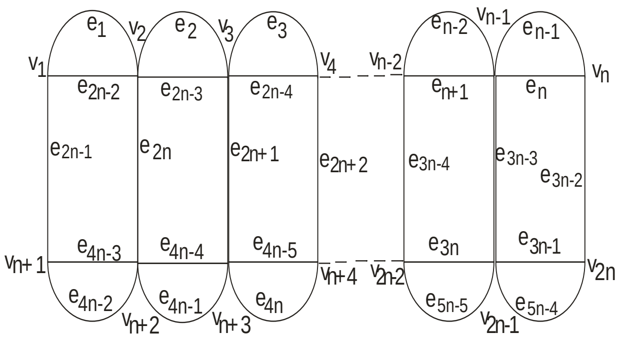

Fig. 5: Edge-odd Graceful Graph $(P_2 \times P_n)\nabla\ 2P_n$

To find edge-odd graceful , define f :E(G)→ {1,3,5,…,2q} by

$f(\mathbf{e}_i) = (2i\text{-}1)$, i= 1,2,3,…,2n-2,2n+1,…, (5n-4) →(1)

$f(\mathbf{e}_{2n-1}) = 4n\text{-}1$

$f(\mathbf{e}_{2n}) = 4n\text{-}3$ →(2)

The induced map f_+: V(G) → {1,2,3,…, 2q} by $f_+(v) \equiv \sum f(uv) \pmod{2q}$ where this sum run over all edges through v Both of f and f_+ finds the distinct labels for vertices and also the edge labeling is distinct.

Example 2: The connected graph $(P_2 \times P_{12})\ \nabla 2P_{12}$ is edge –odd graceful.

Due to the rules (1) and (2) in (2.7), edge odd graceful labeling of $(P_2 \times P_{12})\ \nabla\ 2P_{12}$ is obtained as follows:

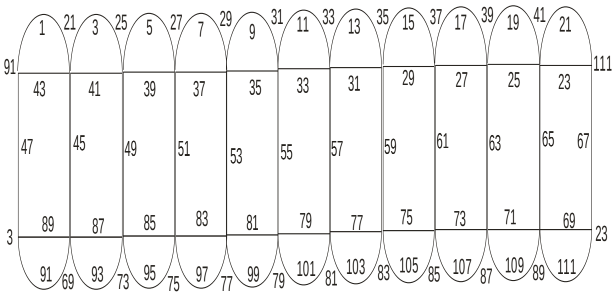

Fig.7: Edge-odd Graceful Graph $(P_2 \times P_{12})\ \nabla\ 2P_{12}$

3. THE CONNECTED GRAPHS (N-1)*k_4, (N-1)*k_4 ∇ PN, AND (N-1)*k_4 ∇ 2PN

Definition 3.1: The connected graph **(n-1)*k_4** is defined as follows in figure 8.

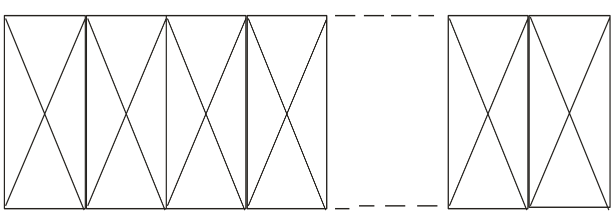

Figure 8

Theorem 3.1: The connected graph (n-1)*k_4 (P_n -a path with 'n' vertices) is edge – odd graceful.

Proof: The arbitrary labels of edges for the n-doors graph (n-1)*K_4

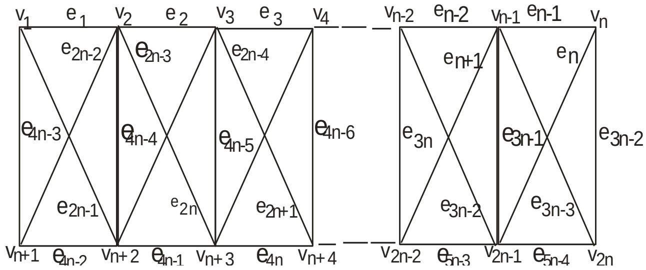

Figure 9: Edge-odd Graceful Graph n-1k_4

To find edge-odd graceful, define, f: E(G) → {1, 3, 5,., 2q} by

$f(\mathbf{e_i}) = (2i\text{-}1)$, i= 2,…,(n-2), n,…,(5n-4) →(1)

$f(e_1) = (2n\text{-}3)$

$f(e_{n-1}) = 1$ →(2)

The induced map f_+: V(G) → {1,2,3,..,2q} by $f_+(v) \equiv \sum f(uv) \pmod{2q}$ where this sum run over all edges through v. Both of f and f_+ finds the distinct labels for vertices and also the edge labeling is distinct. Here the edge –odd graceful labeling of ladder (n-1) k_4 is obtained

Example 3: The connected graph 9 k_4 is edge –odd graceful.

Due to the rules (1) and (2) in (3.2), edge odd graceful labeling of 9k_4 is obtained as follows:

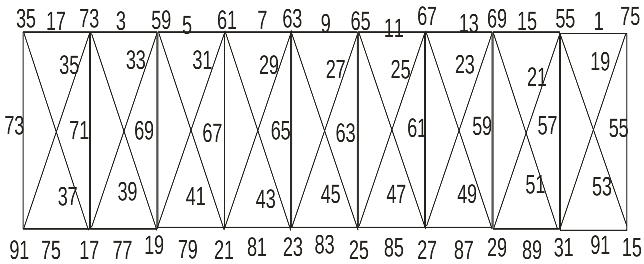

Figure 10: Edge-odd Graceful Graph 9k_4

Definition 3.2: ((n-1) k_4)∇ P_n is a connected graph defined by the following figure 11.

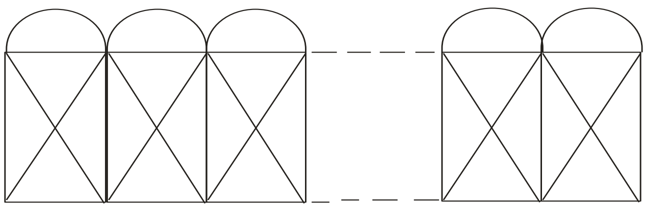

Figure 11

The following result is now stated:

Theorem 3.2: The connected graph ((n-1) k_4) ∇ P_n (P_n -a path with 'n' vertices) is edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph (n-1) k_4)∇ P_n is as follows:

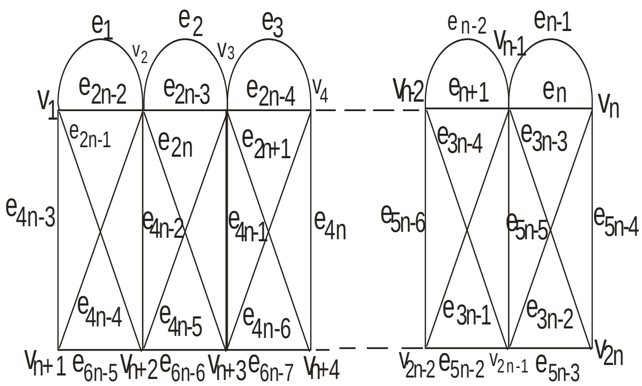

Fig.12: Edge-odd Graceful Graph (n-1xk$_4$)∇ P$_n$

To find edge-odd graceful, define by f :E(G) → {1,3,5,…, 2q} by f ($\mathbf{e_i}$) = (2i-1) , i= 1 to (6n-5) →(1)

The induced map f$_+$: V(G) → {1,2,3,…, 2q} by f$_+$ (v) ≡ ∑f (uv) (mod2q) where this sum run over all edges through v.

Both of f and f$_+$ finds the distinct labels for vertices and also the edge labeling is distinct.. Here the edge –odd graceful labeling of ladder (n-1) k$_4$) ∇ P$_n$ is obtained.

Example 4: The connected graph (11 k$_4$) ∇ P$_{12}$ is edge – odd graceful.

Due to the rules (1) in (3.4), edge odd graceful labeling of (11 × k$_4$)∇ P$_{12}$ is obtained as follows :

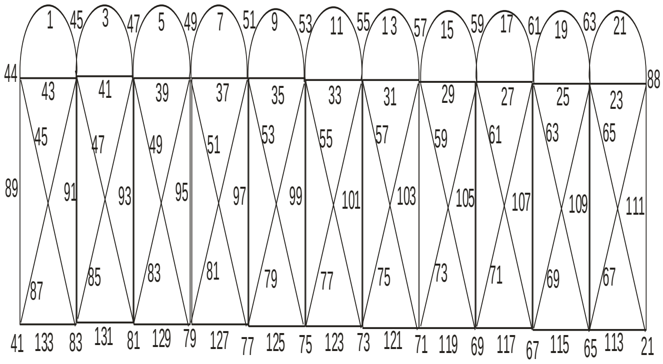

Fig. 13: Edge-odd Graceful Graph (11x k$_4$)∇ P$_{12}$

Definition 3.3: ((n-1) k$_4$) ∇ 2P$_n$ is a connected graph defined by the following figure 14.

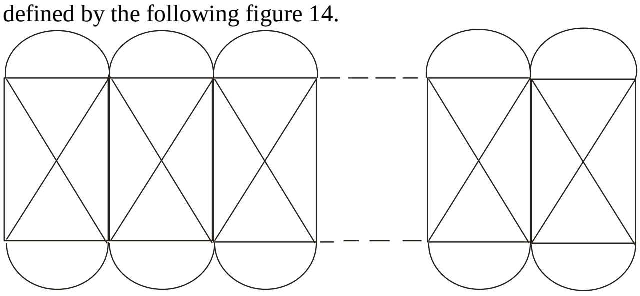

Figure 14

Theorem 3.3: The connected graph (n-1) k$_4$) ∇ 2P$_n$ (Pn -a path with 'n' vertices) is edge – odd graceful

Proof: The arbitrary labels of edges for the n-doors graph (n-1) k$_4$) ∇ 2P$_n$.

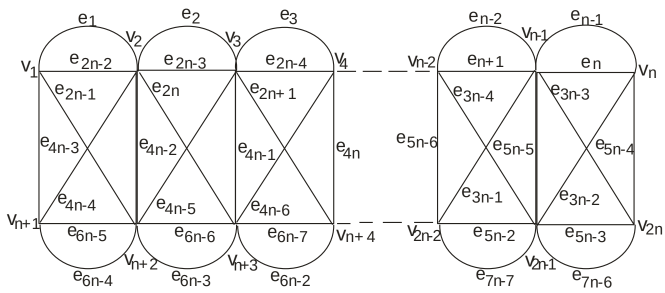

Fig. 15: Edge-odd Graceful Graph (n-1xk$_4$)∇2P$_n$

To find edge-odd graceful, define, f :E(G)→ {1,3,5,…, 2q} by

f ($\mathbf{e_i}$) = (2i-1) , i= 1,2,3,…, n,n+1,…,2n,(2n+1),…,(6n-5),(6n-3),…,(7n-7) →(1)

f($\mathbf{e}_{6n-4}$)=14n-13

f($\mathbf{e}_{7n-6}$)=12n-9 →(2)

The induced map f$_+$:V(G)→ {1,2,3,…,2q} by f$_+$ (v) ≡ ∑f (uv) (mod 2q) where this sum run over all edges through v Both of f and f$_+$ finds the distinct labels for vertices and also the edge labeling is distinct. Here the edge –odd graceful labeling of ladder (n-1) k$_4$) ∇ 2P is obtained.

Example 5: The connected graph (8 k$_4$) ∇ 2P$_9$ is edge – odd graceful.

Due to the rules (1) and (2) in (3.8), edge odd graceful labeling of (8 k$_4$) ∇ 2P$_9$ is obtained as follows :

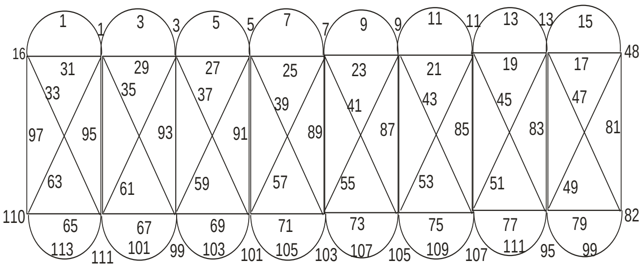

Figure 16: Edge-odd Graceful Graph (8 × k$_4$)∇2P$_9$

4. REFERENCES

[1] A.Solairaju and K. Chitra, "New classes of graceful grphs by merging a finite number of C4, Acta Ciencia Indica, Vol. XXXIV, No.2, 959, 2008.

[2] A.Solairaju and K. Chitra, "Edge-odd Graceful Lebeling of some Graphs", Electronics Notes in Discrete Mathematics, 33, April 2009, 15-20.

[3] A. Solairaju, A.Sasikumar, C. Vimala, "Gracefulness of a Spanning Tree of The Graph of Product of P$_m$ and C$_n$", The Global Journal of Pure and Applied of Mathematics of Mathematical Science, 1, No-2 (July – Dec 2008a), 133-136.

[4] A.Solairaju, C. Vimala, A.Sasikala, "Gracefulness of a Spanning Tree of the Graph of Cartesian Product of S$_m$ and S$_n$ ", The Gobal Journal of pure and Applied Mathematics of Mathematical Sciences, 1, No-2 (July-Dec 2008b), 117-120.

[5] A.Solairaju, A.Sasikala, C. Vimala, "Edge-odd Gracefulness of a Spanning Tree of Cartesian Product of P$_m$ and C$_n$ ", Pacific-Asian Journal of Mathematics, 3,No.1-2. (Jan-Dec. 2009), 39 - 42.