International Journal of Computer Applications (0975 – 8887)
Volume 105 – No. 1, November 2014

Graceful Labelings of Graphs Related to Circuits of Length 4

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ABSTRACT
The aim of the paper is to find graceful labeling for the graphs nC_4, nC_4 o $2P_n$, nP_2 ◊ $P_{2n,}$ its mirror image, nC_4 ◊ $2P_{2n} \cup (n-1)P_2$, and nC_4 ◊ $2P_{2n} \cup (n-1)P_2 \cup (n-1)P_3$.

Keywords
Graceful graphs, edge-odd graceful labeling, and edge-odd graceful graph

1. INTRODUCTION AND PRELIMINARIES
A (p, q)-graph is a graceful graph if there exists an injective map f: V(G)→{0,1,2, …,k} so that induced map f_+: E(G) → {1, 2, 3, …, q} defined by $f_+(xy) \equiv |f(x) – f(y)|$, where the vertex x is incident with other vertex y that f and f_+ make all are distinct

A (p, q) connected graph is edge-odd graceful graph if there exists an injective map f: E(G) → {1, 3, …, 2q-1} so that induced map f_+: V(G) → {0, 1, 2,…,(2k-1)} defined by $f_+(x) \equiv \Sigma f(xy) \pmod{2k}$, where the vertex x is incident with other vertex y and k = max {p, q} makes all the edges distinct and odd.

A. Solairaju and K.Chitra [2008a, 2008b, 2009] obtained edge-odd graceful labeling of some graphs related to paths. A. Solairaju, Vimala, and Sasikala [2008a, 2008b] proved that edge-odd gracefulness of strong product of P_2 and C_n, and Cartesian product of P_2 and W_n are edge -odd graceful.

1.1 Graphs Related to Circuits of Length 4
Definition 1.1: The graph nC_4 is a disconnect graph involving n copies of C_4 with some arbitrary labeling of vertices as follows:

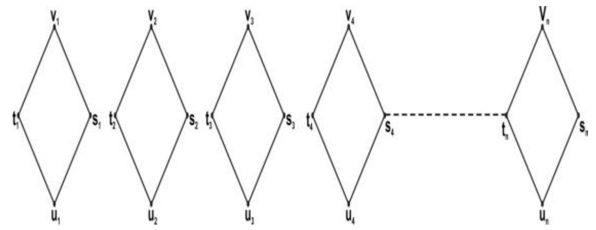


Figure-1

2. GRAPH OBTAINED FROM CIRCUITS MERGING WITH PATHS, THE FOLLOWING RESULT IS THEN OBTAINED
Definition 2.1: The graph nC_4 o $2P_n$ is a connected graph obtained from the above disconnected graph together with adjacent edges $v_i\, v_{i+1}$ [i =1 to (n-1)] and adjacent edges $u_i\, u_{i+1}$ [i =1 to (n-1)] Some labeling of vertices and edges of the graph n_{c4} o $2P_n$ is given below:

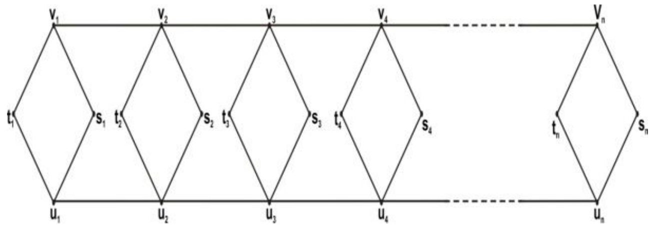


Figure 2

Theorem 2.1: The connected graph nC_4 o $2P_n$ is graceful.

Proof: The labeling of graph n_{c4} o $2P_n$ is followed in the figure (2)

Define a map f: V(G)□ {0,1,2,…,q} by $f(v_1) = 0; f(u_1) = 2; f(v_2) = (q-4); f(u_2) = (q–3)$

$f(v_i) = 3(i - 1)$, i is odd; $f(v_{i+2}) = f(v_2)-3i$, i is even where i = 3 to n

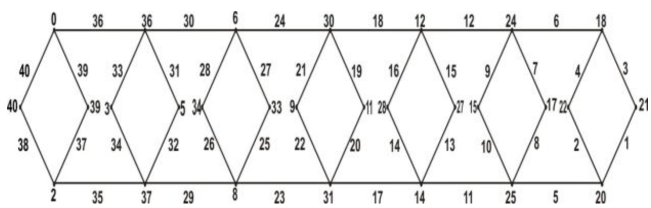
$f(u_i) = f(u_1) +3(i-1)$, i is odd; $f(u_{i+2}) = f(u_2) - 3i$, i is even; where i = 3 to n.

$f(t_1) = q; f(s_1) = (q-1); f(t_2) = 3; f(s_2) = 5;$

$f(ti) = f(t_1) – 3(i-1); f(s_i) = f(s_1) – 3(i-1)$, i is odd; where i varies 3 to n

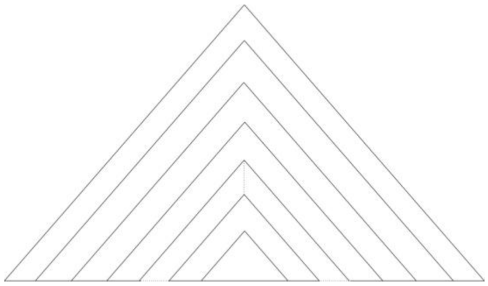
$f(t_{i+2}) = f(t_2) + 3i$; $f(s_{i+2}) = f(s_2) + 3i$, i is even; where I varies 3 to n.

Define f+ : E(G) → { 1, 2, …, q} by $f(uv) = |f(u) – f(v)|$ for all u , v ∈ V(G). The maps f and $f+$ satisfy the conditions of graceful labeling for nC_4



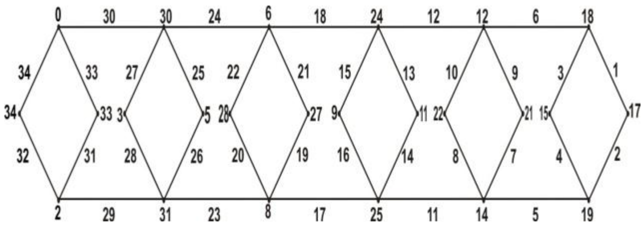
3. THE GRACEFULNESS OF THE GRAPH NP2 ◊ P2N

Definition 3.1: The graph nP_2 ◊ P_{2n} is defined as a connect graph involving n copies of P_2 and a copy of P_{2n} as follows:



Theorem 3.1: The graph nP_2 ◊ P_{2n} is defined as a connect graph involving n copies of P_2 and a copy of P_{2n} with some arbitrary labeling of vertices as follows in figure 3(nC_4 o $2P_n$). Thus the graph nC_4 o $2P_n$ is graceful

Example 1: The graphs $6C_4$ o $2P_6$ and $7C_4$ o $2P_7$ are graceful.



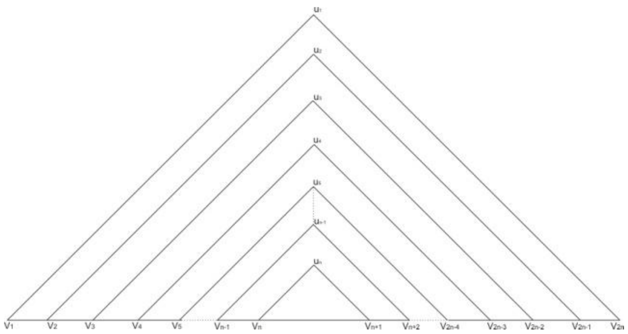


Figure 3

Define f: V (nP_2 ◊ P_{2n}) ⟶ {0,1,2,…, q} by

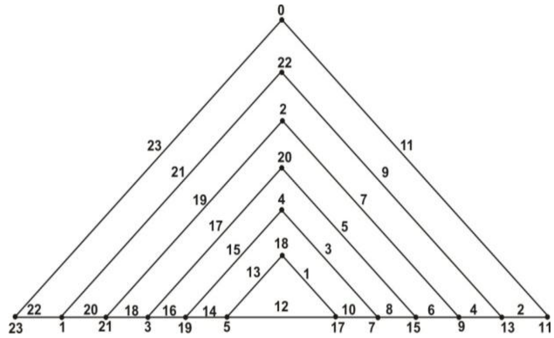
$f(V_i)$ = q – (i-1) ; if i is odd; $f(V_i)$ = (i-1) ; if i is even; i varies from 1 to 2n

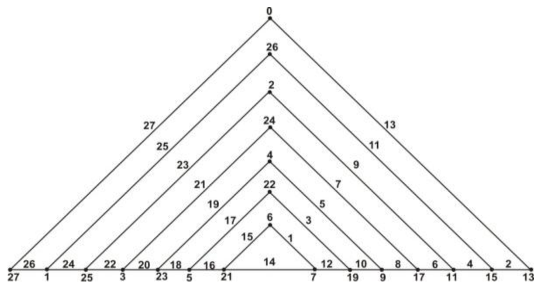
$f(u_i)$ = (i-1) ; if i is odd; $f(u_i)$ = (q-1)-(i-2) ; if i is even; i varies from 1 to n

Define f_+: E(nP_2 ◊ P_{2n}) □ {1, 2, …, q} by f(uv) = □ f(u)– f(**v**)□ for all u , v □ V(G)

The maps f and f_+ satisfy the conditions of graceful labeling for the graph nC_4 o $2P_n$. Thus the graph (nP_2 ◊ P_{2n}) is graceful.

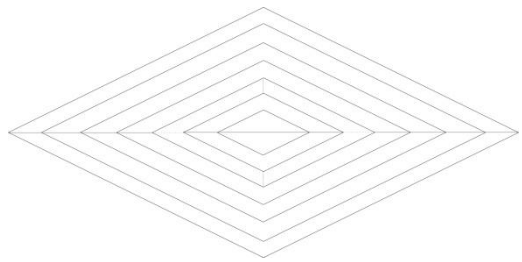
Example 2: The graphs $6P_2$ ◊ P_{12} and $6P_2$ ◊ P_{12} are graceful.





4. THE GRACEFULNESS OF THE MIRROR IMAGE OF THE GRAPH NP2 ◊ P2N

Definition: 4.1: The mirror image of the graph nP_2 ◊ P_{2n} is defined as a connect graph involving twice times of n copies of P_2 and a copy of P_{2n} as follows:



Theorem 4.1: The mirror image of the graph nP_2 ◊ P_{2n} is graceful graph.

Proof: The mirror image of the graph nP_2 ◊ P_{2n} is defined as a connect graph involving twice times of n copies of P_2 and a copy of P_{2n} with some arbitrary labeling of vertices as follows in figure 4:

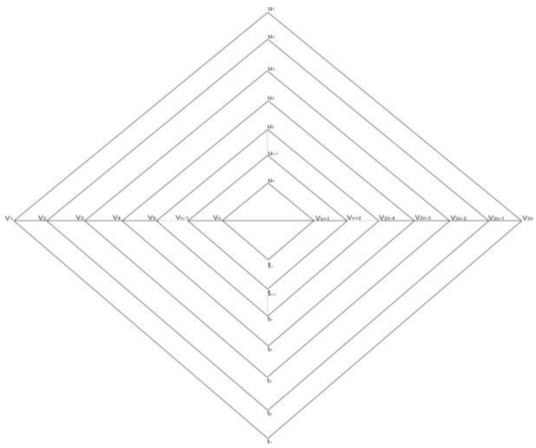


Figure 4

Define f: V(nP_2 ◊ P_{2n}) ⟶ {0, 1, 2 ,…, q} by

$f(V_i)$ = q–3[(i-1)/2)], i is odd; $f(V_i)$ = 2 + 3 [(i-1) / 2]; i is even;

where i varies from 1 to 2n;

f (u_i) = 3[(i-1)/2)], i is odd; f (u_i) = (q-1) - 3[(i-1)/2)], i is even

where i varies from 1 to n;

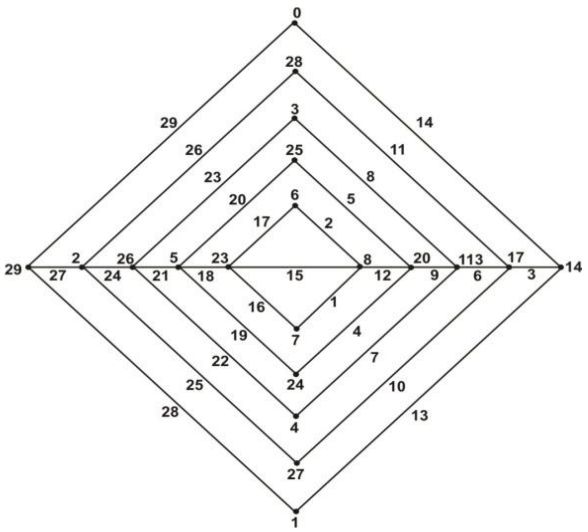
f (t_i) = 1+3[(i-1)/2], i is odd; $f(t_i)$ = (q-2) - 3[(i-1)/2)], i is even;

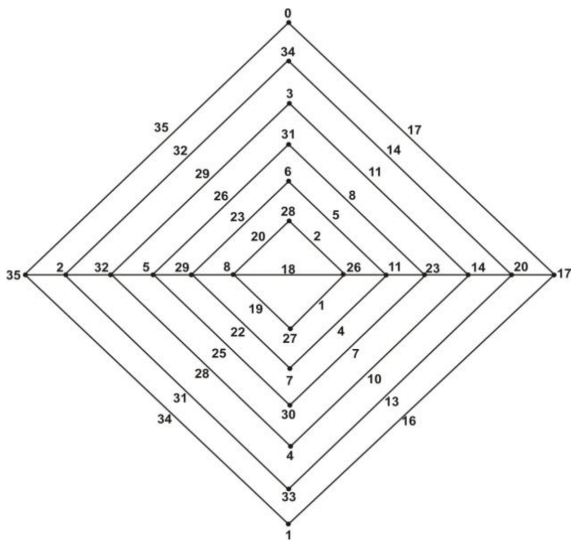
where i varies from 1 to n;

Define f_+: E(nP_2 ◊ P_{2n}) □ {1, 2, …, q} by f(uv) = □ f(u)– f(**v**)□ for all u , v □ V(G)

The maps f and f_+ satisfy the conditions of graceful labeling for the graph nP_2 ◊ P_{2n}. Thus the graph (nP_2 ◊ P_{2n}) is graceful

Example 3: The mirror images of the graphs $5P_2$ ◊ P_{10} and $6P_2$ ◊ P_{12} are graceful.





5. GRACEFULNESS OF NC4 ◊ P2N ∪ (N-1)P2

Definition 5.1: The graph nC_4 ◊ P_{2n} ∪ (n-1)P_2 is defined as a connected graph mentioned below in figure 5.

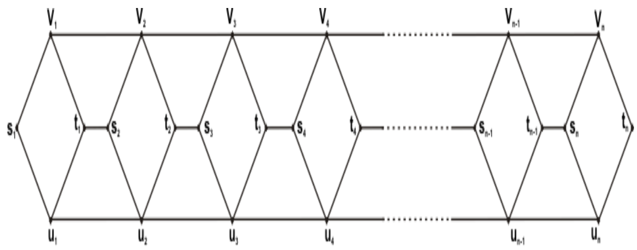


Figure 5

Theorem 5.1: The graph nC_4 ◊ P_{2n} ∪ (n-1)P_2 is graceful.

Proof: The arbitrary labeling of the given graph nC_4 ◊ P_{2n} ∪ (n-1)P_2 is mentio9ned the above figure 5

Define f: V(G)→{0,1,2,……..q} by

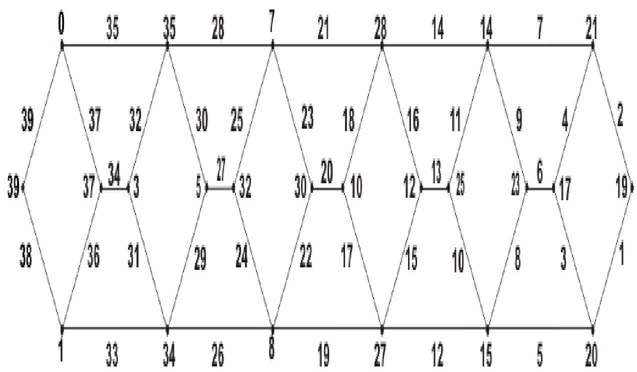
$f(v_i)= \frac{7(i-1)}{2}$, i is odd; $f(v_i)=(q+3)- \frac{7i}{2}$; i is even;

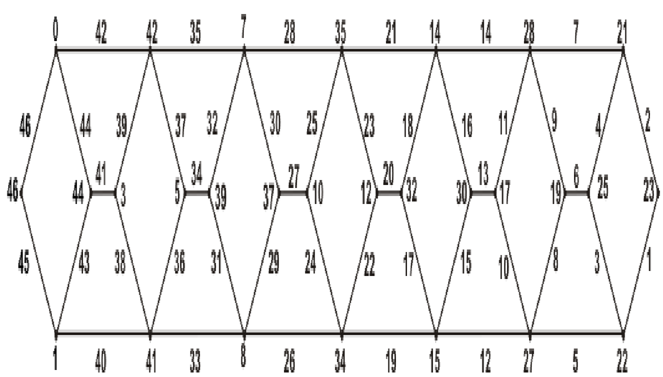
$f(u_i)= f(v_i)+1$, i is odd; $f(u_i)= f(v_i)-1$, i is even;

$f(s_i)=q- \frac{7(i-1)}{2}$, i is odd; $f(s_i)=\frac{7i}{2} – 4$, i is even;

$f(t_i)$=)= $f(s_i)$-2, i is odd; $f(t_i)$) = $f(s_i)$ + 2, i is even; where i varies from 1 to n

Example 4: The connected graphs $6C_4$ ◊ P_{12} ∪ $5P_2$ and $7C_4$ ◊ P_{14} ∪ $6P_2$ are graceful.





6. GRACEFULNESS OF GRAPH NC4 ◊ 2P2N ∪ (N-1)P2 ∪ (N-1)P3

Definition 6.1: The graph $nC_4 \lozenge 2P_{2n} \cup (n-1)P_2 \cup (n-1)P_3$ is defined as in following figure 6:

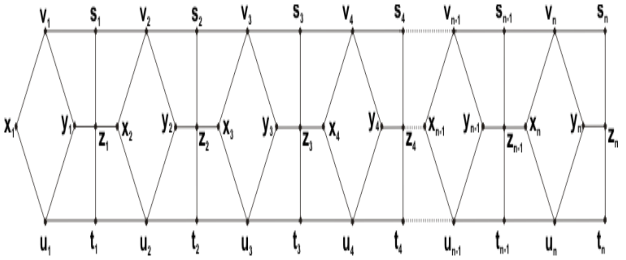


Figure 6

Theorem 6.1: The connected graph $nC_4 \lozenge 2P_{2n} \cup (n-1)P_2 \cup (n-1)P_3$ is graceful.

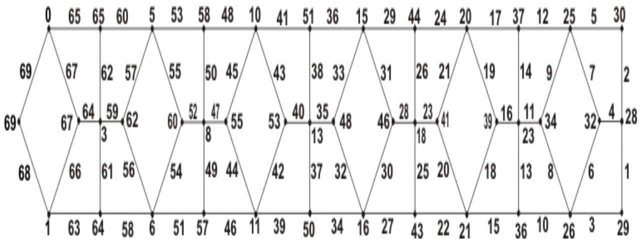
Proof: The arbitrary labelings of vertices of the graph $nC_4 \lozenge 2P_{2n} \cup (n-1)P_2 \cup (n-1)P_3$ are mentioned above in the figure 6:

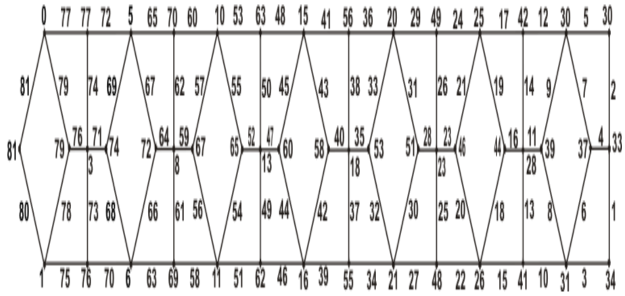
Define $f:V(G)\rightarrow\{0,1,2,\ldots, q\}$ by

$f(v_i) = 5(i-1)$; $f(u_i) = f(v_i)+1$; $f(s_i) = (q+3)-7i$;

$f(t_i) = f(s_i)-1$; $f(x_i) = (q+7)-7i$; $f(y_i) = f(x_i)-2$; $f(z_i) = 5i-2$; i varies from 1 to n

Example 5: The connected graphs $6C_4 \lozenge 2P_{12} \cup 5P_2 \cup 5P_3$ and $7C_4 \lozenge 2P_{14} \cup 6P_2 \cup 6P_3$ are graceful.





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