

# Spectrum Sensing based on Direction of Arrival Estimation

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## ABSTRACT

Spectrum sensing is the main aspect of Cognitive Radio. There are several spectrum sensing techniques but the idea of the Direction of Arrival of the signal when the signal is sensed is equally important. Many methods have been implemented and new methods are coming up to estimate the Direction of Arrival of a signal. Implementation of Direction of Arrival using MUSIC, Root-MUSIC, and ESPRIT Algorithm is done in TV Band frequency spectrum and results have been verified and also plotted. Goal is to implement as many Direction of Arrival techniques and compare them to find the best technique with precise result and less complexity.

## GENERAL TERMS

Cognitive Radio, Direction of Arrival, Beam forming, Music, Root-Music, ESPRIT, Primary user, Secondary user

## Keywords

Cognitive Radio, Direction of Arrival, Beam forming, Music, Primary user, Secondary user, Minimum Square Error, Mean Square Error

## 1. RELEVANCE

This project is relevant in the field of the Mobile communication. Various Direction of Arrival techniques which surrounds Cognitive Radio will be discussed. For programming of these algorithms MATLAB is used.

## 2. LITERATURE SURVEY

The work in the field of Cognitive Radio is gaining significance; several algorithms for spectrum sensing as well as estimation of Direction of Arrival have come up in recent years.

Several IEEE papers have been referred to understand the Spectrum sensing and Direction of Arrival techniques such as MUSIC Algorithm etc. along with comparison of the DOA techniques. Some of these papers are mentioned below

1. "Spectrum Sensing Based On Estimation of Direction of Arrival" by Jingjing Xie, Zhizhong Fu, Haiying Xian UESTC, Chengdu, Sichuan, China. [3] In this paper details of spectrum sensing along with Mathematical models of MUSIC Algorithm are mentioned. It's been realized that an angle dimension is created as a new spectrum opportunity and not only that signals need to be detected but also direction of arrival of the primary user is estimated. This method increases the spectrum opportunities at the cost of complexity of implementation.
2. "Evaluation of MUSIC Algorithm for a Smart Antenna System for Mobile Communications" by T. Nageswara Rao, Department of Electronics & Communication Engineering, Kokad, V. Srinivasa

Rao, Department of Electronics & Communication Engineering, kokad. [6]

3. Paper is focused on MUSIC Algorithm, Its theoretical model and Mathematical model is discussed here. Parametric study of static case is discussed and dynamic case is also touched up on.
4. "ROOT-MUSIC based Direction-Of-Arrival estimation method for arbitrary non-uniform arrays" by Bhasker D Rao and K.V.S Hari. [7] In this paper brief introduction of ROOT-MUSIC algorithm, mathematical model, performance analysis and simulation results are discussed. Fourier Domain root-Music approach is used here and several simulations have been formulated to show results of the Algorithm.
5. "Direction of Arrival Estimation using ESPRIT with Sparse Arrays" by Volodymyr Vasylyshyn Papers is focused on ESPRIT algorithm. [8] In this combination of ESPRIT with MUSIC is used which allows to identify true direction cosine estimate from a set of ambiguous candidates.
6. "An Approach to MUSIC Algorithm in smart antenna systems" by R. S Kawitkar and D. G. Wakde. [9] This paper is focused MUSIC Algorithm for adaptive antenna signal processing also the concepts of spectral MUSIC and constrained MUSIC is detailed here.
7. "Statistical analysis of MUSIC and ESPRIT estimates of sinusoidal frequencies" by Petre Stoica and Torsten. In this paper a detail comparison of both the algorithm is done and it is proved that if we increase M then accuracy is significantly increased and ESPRIT is slightly more accurate than MUSIC.
8. "Performance Analysis of MUSIC and ESPRIT DOA Estimation algorithms for adaptive array smart antenna in mobile communication" by T.V Lavate and Prof. V. K. kokate. In this paper performance analysis is done for both the algorithms. It is verified that ESPRIT is better than MUSIC in terms of accuracy.

## 3. MOTIVATION

The efficient use of the spectrum is very important for both Primary and Secondary Users in the field of Mobile Communication. Also spectrum sensing is used by secondary user for seeking opportunity of spectrum allocation. The use of Smart Antennas can not only save a lot of money but also will utilize the spectrum which is not in use.

So the knowledge and comparison of several techniques for estimation of Direction of Arrival is much needed.

#### 4. PROBLEM DEFINATION

In this paper different Direction of Arrival techniques are studied such as MUSIC Algorithm, root-Music Algorithm and ESPRIT Algorithm and implementation is done for a frequency 478 MHz .Study and comparison based on their efficiency, Angle of Arrival after changing SNR Ratio, Pseudo-spectrum based on the Angle of Arrival.

#### 5. SCOPE AND OBJECTIVES

Comparison will be done of all three Algorithms based on several SNR, Number of array elements in TV Band. Advantages and Disadvantage of each algorithm will be done. In this paper analysis of MUSIC Algorithm is carried forward for a frequency 478 MHz Its steering vector matrix is formed and Eigen value decomposition is done and then using the angle of arrival, pseudo spectrum is plotted and verified for given angles and Root-MUSIC algorithm is touched up on too.

#### 6. TECHNICAL APPROACH

Formation of Correlation Matrix: Calculating Noise and Signal correlation Matrices and formulating it to form the correlation Matrix. Eigen Value Decomposition: After forming the Correlation Matrix we do Eigen value Decomposition of the Matrix. We find out Eigen Value matrix and Eigen Vector Matrix. Formation of Subspace Matrix: We formed subspace matrix using Eigen Vectors associated with the Noise and Signal. Plotting the pseudo spectrum: Pseudo Spectrum of MUSIC Algorithm was plotted.

#### 7. DIRECTION OF ARRIVAL TECHNIQUES

##### 7.1 Introduction

There can be many possible propagation paths and angles of arrival for a single source. If there are several transmitters, each source will create multipath components at the receiver end. Therefore it is very important for receiver to estimate angle of arrival to decipher its presence and angular position. This information can be used in further beam forming process as well. DOA estimation involves time series analysis, spectral analysis, Eigen structures methods, linear prediction methods, adaptive array methods etc. Some of the well known DOA estimation techniques are

- MUSIC Algorithm
- Root-MUSIC Algorithm
- ESPRIT Algorithm
- Min-Norm Algorithm
- Maximum Entropy Algorithm

#### 7.2 MUSIC ALGORITHM

MUSIC is an acronym which stands for Multiple Signal Classification. This approach was first shown by Schmidt also it is a very popular Eigen structure method. MUSIC provides unbiased estimates of the number of signals, angles of arrival, accuracy of the angles and also the strength of the waveforms. MUSIC makes the assumption that the noise in each channel is uncorrelated making the noise correlation matrix diagonal.[1] The incident signals may be somewhat correlated creating a non-diagonal signal correlation matrix. However, under high signal correlation the traditional MUSIC algorithm breaks down and other methods must be implemented to correct this weakness.

One must know in advance the number of incoming signals or one must search the Eigen values to determine the number of incoming signals. If the number of signals is D, the number of signal Eigen values and Eigenvectors is D, and the number of noise Eigen values and Eigenvectors is M- D (M is the number of array elements). Because MUSIC exploits the noise Eigenvector subspace, it is sometimes referred to as a subspace method.

We have done music algorithm for DOA estimation in which following parameters are involved

Random signal:

Random signal used in MUSIC is given by,

$$S = \text{sign}(\text{randn}(\text{no}, M)) \tag{1}$$

Random noise :

Random noise is given by,

$$n = \text{sqrt}(\text{sig}^2)(\text{randn}(M, k)) \tag{2}$$

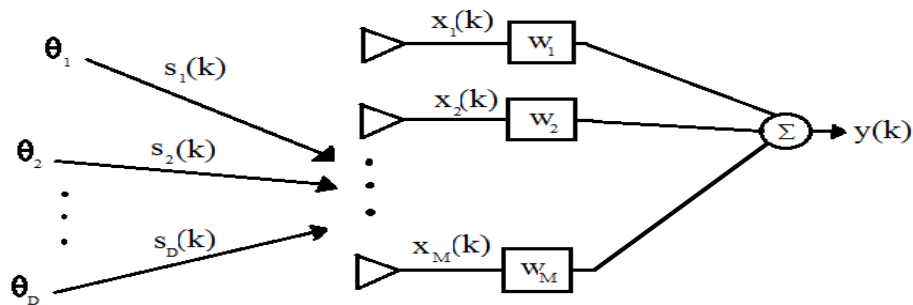


Fig. 1: M-element array with arriving signals.

Figure 1 shows D signals arriving from D directions. They are received by an array of M elements with M potential weights. Each received signal X(k) includes additive, zero mean, Gaussian noise. Time is represented by the k<sup>th</sup> time sample. Thus, the array output Y can be given in the following form:

$$y(k) = W^T x(k) \tag{3}$$

$$\text{Where, } \mathbf{x}(k) = \begin{bmatrix} a_1(\theta) a_2(\theta) \dots a_n(\theta) \\ s_1(k) \\ s_2(k) \\ \vdots \\ s_D(k) \end{bmatrix} + \mathbf{n}(k) \quad (4)$$

Generally, correlation can be given in terms of expected value as

$$R_{xx} = E \begin{bmatrix} \overrightarrow{\overrightarrow{H}} \\ \overrightarrow{x.x} \end{bmatrix} \quad (5)$$

$$= E \begin{bmatrix} \overrightarrow{\overrightarrow{H}} & \overrightarrow{\overrightarrow{H}} & \overrightarrow{\overrightarrow{H}} \\ (A s + n)(s^H & A^H & + n^H) \end{bmatrix} \quad (6)$$

$$= A E \begin{bmatrix} \overrightarrow{\overrightarrow{H}} \\ \overrightarrow{s.s} \end{bmatrix} A^H + E \begin{bmatrix} \overrightarrow{\overrightarrow{H}} \\ \overrightarrow{n.n} \end{bmatrix} \quad (7)$$

$$R_{xx} = A R_{ss} A^H + R_{nn} \quad (8)$$

Correlation:

Practically, Correlation is given by expected value and its delayed value but we have assumed ergodicity of the mean and ergodicity of the correlation.

Signal auto-correlation is given by,

$$R_{ss} = \sum_{k=1}^K s(k) s^H(k) \quad (9)$$

Signal-noise correlation is given by,

$$R_{sn} = \sum_{k=1}^K s(k) n^H(k) \quad (10)$$

Signal-noise correlation is given by,

$$R_{ns} = \sum_{k=1}^K n(k) s^H(k) \quad (11)$$

Noise auto-correlation is given by,

$$R_{nn} = \sum_{k=1}^K n(k) n^H(k) \quad (12)$$

Combined array correlation matrix is given by,

$$R_{rr} = A R_{ss} A^H + A R_{sn} A^H + A R_{ns} A^H + R_{nn} \quad (13)$$

Array steering vector

In the literature the array vector has been alternatively called: the array steering vector, the array propagation vector, the array response vector, and the array manifold vector. For simplicity's sake, we will call  $\mathbf{a}(\theta)$  the array vector.

Array steering vector is given by,

$$A = [a_1(\theta) a_2(\theta) \dots a_{n_0}(\theta)] \quad (14)$$

Pseudo spectrum

Pseudo spectrum is given by [2],

$$P(k) = 1 / | \mathbf{a}^H \mathbf{E} \mathbf{R}_{rr} \mathbf{E}^H \mathbf{a} | \quad (15)$$

Relation between SNR and noise variance

$$\text{SNR} = 10 \log(s^2 / \text{sig}^2) \quad (16)$$

### 7.3 ROOT-MUSIC ALGORITHM

The MUSIC algorithm in general can apply to any arbitrary array regardless of the position of the array elements. Root-MUSIC implies that the MUSIC algorithm is reduced to finding roots of a polynomial as opposed to plotting the pseudo spectrum or searching for peaks in the pseudo spectrum. Barabell simplified the MUSIC algorithm for the case where the antenna is a ULA.

In Root MUSIC the equation of MUSIC is simplified and roots of the equation are used to determine the plot of the Root-MUSIC Algorithm.

Pseudo-spectrum for Root MUSIC can be given by [7]

$$P_{\text{RMU}} = 1 / | \mathbf{a}^H(\theta) \mathbf{C} \mathbf{a}(\theta) | \quad (17)$$

Where, C is given as

$$C = \mathbf{E} \mathbf{R}_{rr} \mathbf{E}^H \quad (18)$$

### 7.4 ESPRIT ALGORITHM

ESPRIT stands for Estimation of Signal Parameters via Rotational Invariance Techniques and was proposed by Roy and Kailath in 1989. The goal of the ESPRIT technique is to exploit the rotational invariance in the signal subspace which is created by two arrays with a translational invariance structure. ESPRIT inherently assumes narrowband signals so that one knows the translational phase relationship between the multiple arrays to be used. Both MUSIC and ESPRIT assumes there are  $D < M$  narrow-band sources centered at the center frequency  $f_0$ . [2] These signal sources are assumed to be of a sufficient range so that the incident propagating field is approximately planar. The sources can be either random or deterministic and the noise is assumed to be random with zero-mean. The concept of Doublets is introduced in ESPRIT. These can be separate arrays or can be composed of sub arrays of one larger array. It is important that these arrays are displaced translationally but not rotationally.

### 7.5 ALGORITHM FOR MUSIC

Step 1: Accept M Antenna array elements.

Step 2: Accept D number of signal.

Step 3: Accept 'no' number of angles.

Step 4: Accept sig<sup>2</sup> noise variance.

Step 5: Convert accepted angles into radian.

Step 6: Calculate 'a' steering vector matrix.

Step 7: Do  $A = \mathbf{a}'$  for matching dimension.

Step 8: Accept number of time samples.

Step 9: Generate signal S of frequency 478MHz.

Step 10: Calculate source correlation matrix,  $R_{ss}$ .

Step 11: Calculate the 'K' time samples of the noise for the 'M' array elements.

Step 12: Calculate 'R<sub>nn</sub>' noise correlation matrix.

Step 13: Calculate  $R_{nn}$  the noise/signal correlation matrix.

Step 14: Calculate  $R_{sn}$  the signal/noise correlation matrix.

Step 15: Do Eigen vector decomposition.

Step 16: Combine all correlation matrix to get total correlation matrix,  $R_{rr}$ .

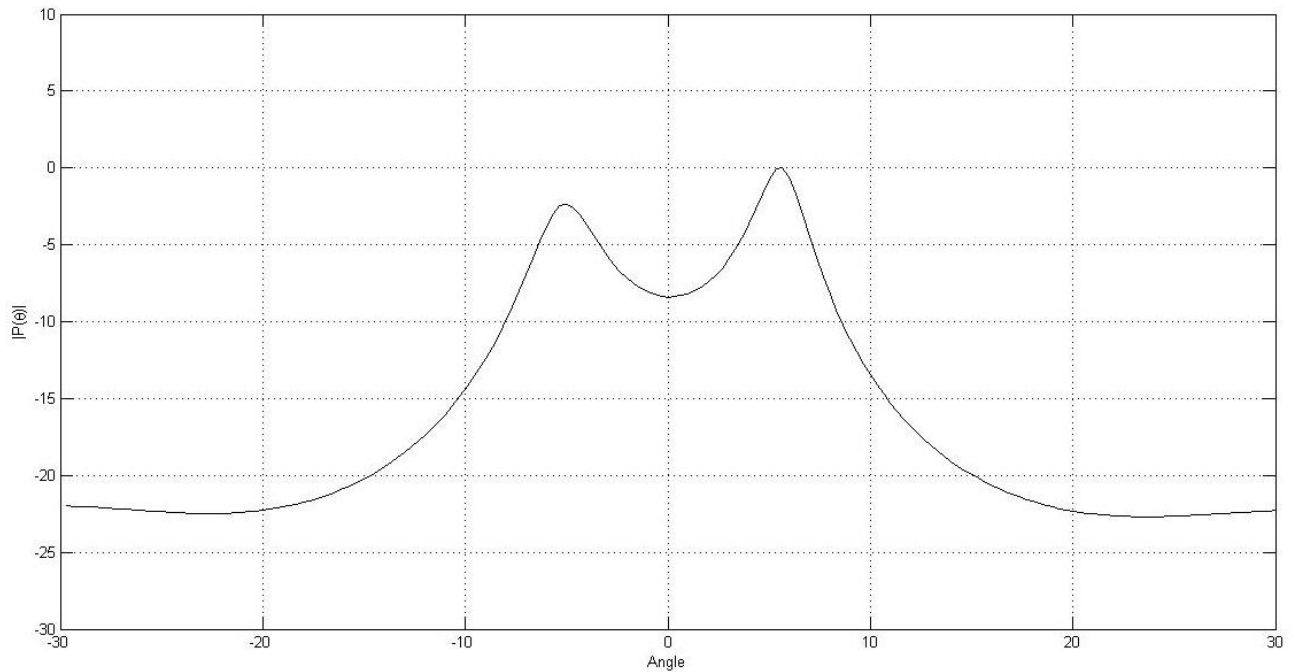
Step 17: Sort column matrix Y according to Eigen values.

Step 18: Generate noise subspace matrix of Eigen vectors.

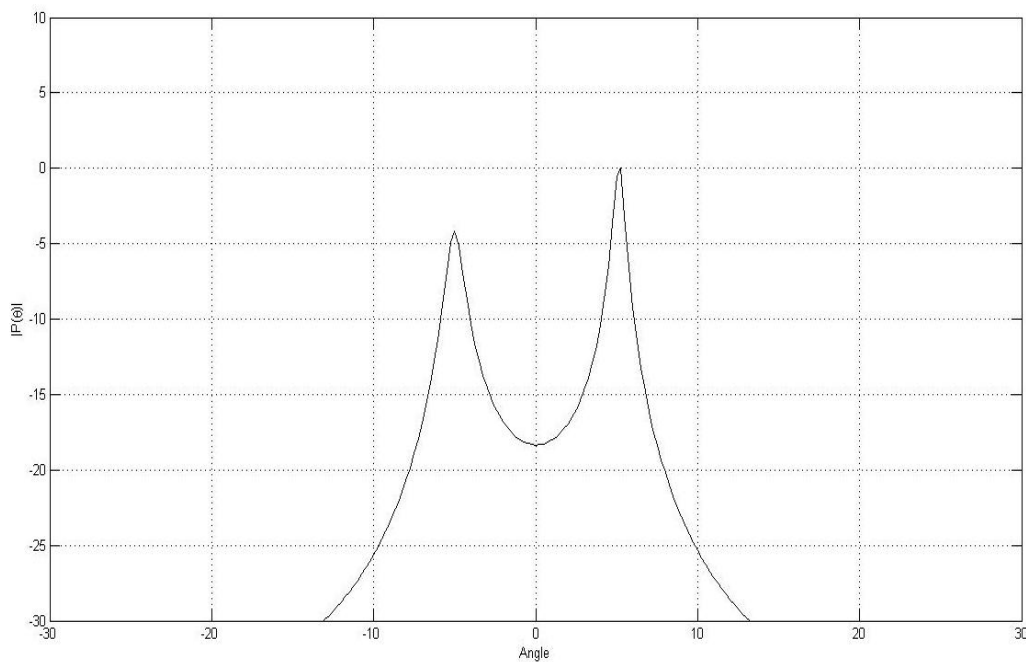
Step 19: Generate steering vector for desired angle.

Step 20: Generate music pseudo-spectrum of normalized P in db.

## 8. RESULTS AND COMPARISONS



**Fig. 2: Spectrum for angles (-5 and 5), frequency 478 MHz and noise variance (1) with number of elements  $M=6$  of MUSIC algorithm**



**Fig. 3: Spectrum for angles (-5 and 5), frequency 478 MHz and noise variance (0.1) WITH number of elements  $M= 6$  for MUSIC Algorithm**

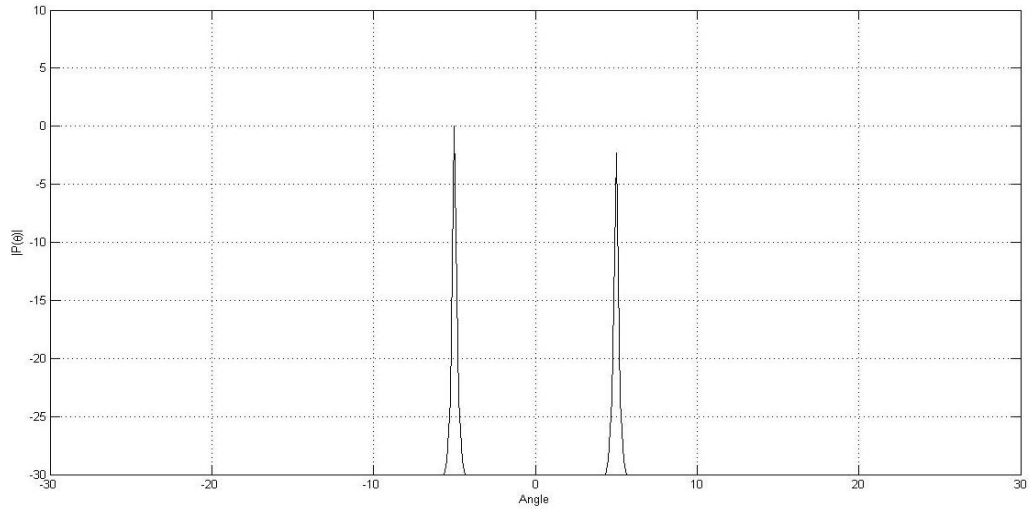


Fig. 4: Spectrum for angles (-5 and 5), frequency 478 MHz and noise variance (.1) with number of elements  $M=100$  for MUSIC Algorithm

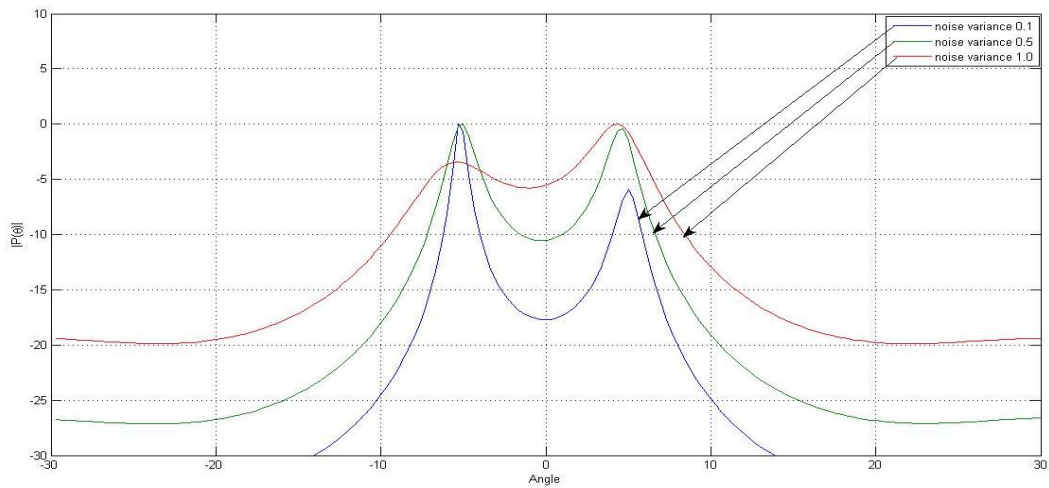


Fig. 5: Spectrum for angles (-5 and 5), frequency 478 MHz and noise variance (0.1, .5, 1) with number of elements  $M=6$  for MUSIC Algorithm

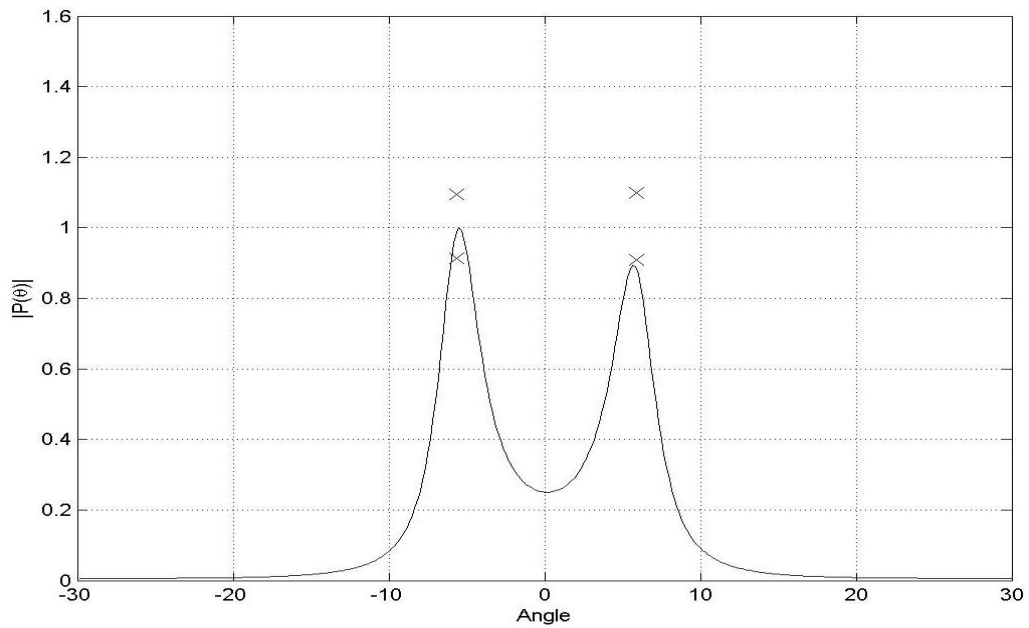


Fig. 6: Spectrum for angles (-5 and 5), frequency 478 MHz and noise variance (0.1) with number of elements  $M=6$  for Root-MUSIC Algorithm.

Figures	Results
Fig. 2	Displays the output for MUSIC Algorithm for Direction of Arrival angles 5 and -5 for M = 6. The graph is not sharp and it curves over 5 and -5 due to high value of Noise Variance.
Fig. 3	Displays the output for MUSIC Algorithm for Direction of Arrival angles 5 and -5 and for M = 6. Unlike figure 2 this graph is sharpened over 5 and -5 because of the lower value of Noise Variance.
Fig. 4	Displays the output of MUSIC Algorithm for Direction of Arrival angles 5 and -5 and for M = 100. In this graph the peaks are sharpened to great extent almost like a straight line and no curve. It happens because of high number of arrays in the antenna.
Fig. 5	This figure compares the graph for different Noise Variances, we can see lower the Noise Variance sharper the peaks.
Fig. 6	Displays the output of Root-MUSIC Algorithm for similar parameters as MUSIC we can conclude it is very similar to MUSIC but not as accurate as it.

## 9. CONCLUSIONS

Spectrum sensing was studied along with Direction of Arrival techniques like MUSIC Algorithm, Root-MUSIC Algorithm and ESPRIT Algorithm. Implementation of MUSIC, Root-MUSIC, and ESPRIT algorithm in TV Band was done by sinusoidal wave of frequency 478 MHz was taken and tested. Precise results were achieved. Graphs were plotted for different noise variance and for different angle of arrival. As the noise variance is increased the pseudo spectrum sharpens and with increase in noise variance it flattens.

## 10. FUTURE PLAN

With the increase in noise we do not achieve precise values of the angle of arrival. So use of attenuation of noise signal in the practical implementation is to be carried out. Also effect of noise along with multipath fading also can be seen.

Smart antennas concept is being used in this project. But smart antenna does not have intelligence. So some learning Algorithms having some intelligence can be seen.

## 11. REFERENCES

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