# Semi-Star-Alpha-Open Sets and Associated Functions

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**ABSTRACT:** The aim of this paper is to introduce various functions associated with semi*α-open sets. Here semi*α-continuous, semi*α-irresolute, contra-semi*α-continuous and contra-semi*α-irresolute functions are defined. Characterizations for these functions are given. Further their fundamental properties are investigated. Many other functions associated with semi*α-open sets and their contra versions are introduced and their properties are studied. In addition strongly semi*α-irresolute functions, contra-strongly semi*α-irresolute functions, semi*α-totally continuous, totally semi*α-continuous functions and semi*α-homeomorphisms are introduced and their properties are investigated.

**General Terms:** General topology

**Keywords:** semi*α-continuous, semi*α-irresolute, semi*α-open, semi*α-closed, pre-semi*α-open function, pre-semi*α-closed function

## 1. INTRODUCTION

In 1963, Levine [1] introduced the concept of semi-continuity in topological spaces. Dontchev [2] introduced contra-continuous functions. Crossely and Hildebrand [3] defined pre-semi-open functions. Noiri defined and studied semi-closed functions. In 1997, Contra-open and Contra-closed functions were introduced by Baker. Dontchev and Noiri [4] introduced and studied contra-semi-continuous functions in topological spaces. Caldas [5] defined Contra-pre-semi-closed functions and investigated their properties. S.Pasunkili Pandian [6] defined semi*-pre-continuous and semi*-pre-irresolute functions and their contra versions and investigated their properties. Quite recently, the authors [7, 8, 9] introduced some new concepts, namely semi*α-open sets, semi*α-closed sets, the semi*α-closure, semi*α-derived set and semi*α-frontier of a subset.

In this paper various functions associated with semi*α-open sets are introduced and their properties are investigated.

## 2. PRELIMINARIES

Throughout this paper $X$, $Y$ and $Z$ will always denote topological spaces on which no separation axioms are assumed.

**Definition 2.1**[10]**:** A subset $A$ of a topological space $(X, \tau)$ is called (i) generalized closed (briefly g-closed) if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and $U$ is open .
(ii) generalized open (briefly g-open) if $X\backslash A$ is g-closed in $X$.

**Definition 2.2:** Let $A$ be a subset of $X$. Then (i) generalized closure[11] of $A$ is defined as the intersection of all g-closed sets containing $A$ and is denoted by $Cl^*(A)$.
(ii) generalized interior of $A$ is defined as the union of all g-open subsets of $A$ and is denoted by $Int^*(A)$.

**Definition 2.3:** A subset $A$ of a topological space $(X, \tau)$ is
(i)semi-open [1] (resp. α-open[12], semi α-open[13], semi-preopen[14], semi*open, semi*α-open[7], semi*-preopen[6]) if $A\subseteq Cl(Int(A))$ (resp. $A\subseteq Int(Cl(Int(A))$, $A\subseteq Cl(Int(Cl(Int(A)))$, $A\subseteq Cl(Int(Cl(A)))$, $A\subseteq Cl^*(Int(A))$, $A\subseteq Cl^*(\alpha Int(A)$, $A\subseteq Cl^*(pInt(A))$, )
(ii)semi-closed (resp. α-closed[12], semi α-closed[15], semi-preclosed[14], semi*-closed, semi*α-closed[8], semi*-preclosed[6]) if $Int(Cl(A))\subseteq A$
(resp. $Cl(Int(Cl(A)))\subseteq A$, $Int(Cl(Int(Cl(A))\subseteq A$, $Int^*(Cl(A))\subseteq A$, $Int^*(\alpha Cl(A))\subseteq A$, $Int^*(pCl(A))\subseteq A$).
(iii) semi*α-regular [8] if it is both semi*α-open and semi*α-closed.

**Definition 2.4:** Let $A$ be a subset of $X$. Then
(i)The semi*α-interior [7] of $A$ is defined as the union of all semi*α-open subsets of $A$ and is denoted by $s^*\alpha Int(A)$.
(ii) The semi*α-closure [8] of $A$ is defined as the intersection of all semi*α-closed sets containing $A$ and is denoted by $s^*\alpha Cl(A)$.

**Definition 2.5:** A function $f:X\longrightarrow Y$ is said to be semi-continuous [1] (resp. contra-semi-continuous [4], semi*-continuous, contra-semi*-continuous, semi α-continuous [15] ) if $f^{-1}(V)$ is semi-open(resp. semi-closed, semi*-open, semi*-closed, semi α-open) in $X$ for every open set $V$ in $Y$.

**Definition 2.6:** A function $f:X\longrightarrow Y$ is said to be α-continuous (resp. semi-pre-continuous [14], semi*-pre-continuous [6]) if $f^{-1}(V)$ is α-open(resp. semi-preopen, semi*-preopen) in $X$ for every open set $V$ in $Y$.

**Definition 2.7:** A topological space $X$ is said to be
(i) $T_{1/2}$ if every g-closed set in $X$ is closed.[10]
(ii) locally indiscrete if every open set is closed.

**Theorem 2.8:** [7] (i) Every α-open set is semi*α-open.
(ii)Every open set is semi*α-open.
(iii)Every semi*-open set is semi*α-open.
(iv)Every semi*α-open set is semi α-open.
(v)Every semi*α-open set is semi*-preopen.
(vi)Every semi*α-open set is semi-preopen.
(vii)Every semi*α-open set is semi-open.

**Remark 2.9**:[8] Similar results for semi*α-closed sets are also true.

**Theorem 2.10:** [7] (i) Arbitrary union of semi*α-open sets is also semi*α-open.

(ii) If $A$ is semi*α-open in $X$ and $B$ is open in $X$, then $A\cap B$ is semi*α-open in $X$.
(iii)A subset $A$ of a space X is semi*α-open if and only if $s^*\alpha Int(A)=A$.

**Theorem 2.11:** [7] For a subset $A$ of a space X the following are equivalent:
(i) $A$ is semi*α-open in X
(ii)$A\subseteq Cl^*(\alpha Int(A))$.
(iii)$Cl^*(\alpha Int(A))=Cl^*(A)$.

**Theorem 2.12:** [8]For a subset $A$ of a space X the following are equivalent:
(i)$A$ is semi*α-closed in X.
(ii)$Int^*(\alpha Cl(A))\subseteq A$.
(iii)$Int^*(\alpha Cl(A))=Int^*(A)$.

**Theorem 2.13:** [8] (i) A subset $A$ of a space X is semi*α-closed if and only if $s^*\alpha Cl(A)=A$.
(ii)Let $A\subseteq X$ and let $x\in X$. Then $x\in s^*\alpha Cl(A)$ if and only if every semi*α-open set in $X$ containing $x$ intersects $A$.

**Definition 2.14:** [9] If $A$ is a subset of $X$, the semi*α-Frontier of $A$ is defined by
$s^*\alpha Fr(A)=s^*\alpha Cl(A)\setminus s^*\alpha Int(A)$.

**Theorem 2.15:**[9] If $A$ is a subset of $X$, then $s^*\alpha Fr(A)=s^*\alpha Cl(A)\cap s^*\alpha Cl(X\setminus A)$.

**Definition 2.16**:[15] A function $f:X\longrightarrow Y$ is said to be (i) semi α*-continuous (resp. semi α**-continuous ) if $f^{-1}(V)$ is semi α-open(resp. open) set in $X$ for every semi α-open set $V$ in $Y$.
(ii) totally semi-continuous [16] if $f^{-1}(V)$ is semi regular in $X$ for every open set $V$ in $Y$.
(iii) semi-totally continuous [17] if $f^{-1}(V)$ is clopen in $X$ for every semi-open set $V$ in $Y$.

## 3. SEMI*α-CONTINUOUS FUNCTIONS

In this section we define the semi*α-continuous and contra-semi*α-continuous functions and investigate their fundamental properties.

**Definition 3.1:** A function $f:X\longrightarrow Y$ is said to be semi*α-continuous at $x\in X$ if for each open set $V$ of $Y$ containing $f(x)$, there is a semi*α-open set $U$ in $X$ such that $x\in U$ and $f(U)\subseteq V$.

**Definition 3.2:** A function $f:X\longrightarrow Y$ is said to be semi*α-continuous if $f^{-1}(V)$ is semi*α-open in $X$ for every open set $V$ in $Y$.

**Theorem 3.3:** Let $f:X\longrightarrow Y$ be a function. Then the following statements are equivalent:
(i) $f$ is semi*α-continuous.
(ii) $f$ is semi*α-continuous at each point $x\in X$.
(iii)$f^{-1}(F)$ is semi*α-closed in $X$ for every closed set $F$ in $Y$.
(iv)$f(s^*\alpha Cl(A))\subseteq Cl(f(A))$ for every subset $A$ of $X$.
(v)$s^*\alpha Cl(f^{-1}(B))\subseteq f^{-1}(Cl(B))$ for every subset $B$ of $Y$.
(vi)$f^{-1}(Int(B))\subseteq s^*\alpha Int(f^{-1}(B))$ for every subset $B$ of $Y$.
(vii)$Int^*(\alpha Cl(f^{-1}(F)))=Int^*(f^{-1}(F))$ for every closed set $F$ in $Y$.
(viii)$Cl^*(\alpha Int(f^{-1}(V)))=Cl^*(f^{-1}(V))$ for every open set $V$ in $Y$.
**Proof:** **(i)⇒(ii):** Let $f:X\longrightarrow Y$ be semi*α-continuous. Let $x\in X$ and $V$ be an open set in $Y$ containing $f(x)$. Then $x\in f^{-1}(V)$. Since $f$ is semi*α-continuous, $U=f^{-1}(V)$ is a semi*α-open set in $X$ containing $x$ such that $f(U)\subseteq V$.
**(ii)⇒(i):** Let $f:X\longrightarrow Y$ be semi*α-continuous at each point of $X$. Let V be an open set in Y. Let $x\in f^{-1}(V)$. Then V is an open set in Y containing $f(x)$. By (ii), there is a semi*α-open set $U_x$ in $X$ containing $x$ such that $f(x)\in f(U_x)\subseteq V$. Therefore $U_x\subseteq f^{-1}(V)$. Hence $f^{-1}(V)=\cup\{U_x : x\in f^{-1}(V)\}$. By Theorem 2.10(i), $f^{-1}(V)$ is semi*α-open in $X$.
**(i)⇒(iii):** Let $F$ be a closed set in $Y$. Then $V=Y\setminus F$ is open in $Y$. Then $f^{-1}(V)$ is semi*α-open in X. Therefore $f^{-1}(F)=f^{-1}(Y\setminus V)=X\setminus f^{-1}(V)$ is semi*α-closed.
**(iii)⇒(i):** Let V be an open set in Y. Then F=Y\V is semi*α-closed. By (iii), $f^{-1}(F)$ is semi*α-closed. Hence $f^{-1}(V)=f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)$ is semi*α-open in $X$.
**(iii)⇒(iv):** Let $A\subseteq X$. Let $F$ be a closed set containing $f(A)$. Then by (iii), $f^{-1}(F)$ is a semi*α-closed set containing $A$. This implies that $s^*\alpha Cl(A)\subseteq f^{-1}(F)$ and hence $f(s^*\alpha Cl(A))\subseteq F$.
**(iv)⇒(v):** Let $B\subseteq Y$ and let $A=f^{-1}(B)$. By assumption, $f(s^*\alpha Cl(A))\subseteq Cl(f(A))\subseteq Cl(B)$. This implies that $s^*\alpha Cl(A)\subseteq f^{-1}(Cl(B))$.
**(v)⇒(iii):** Let F be closed in Y. Then $Cl(B)=B$. Therefore (v) implies $s^*\alpha Cl(f^{-1}(B))\subseteq f^{-1}(B)$.Hence $s^*\alpha Cl(f^{-1}(B))= f^{-1}(B)$. By Theorem 2.13(i), $f^{-1}(B)$ is semi*α-closed.
**(v)⇔(vi):** The equivalence of (v) and (vi) can be proved by taking the complements.
**(vii)⇔(iii):** Follows from Theorem 2.12.
**(viii)⇔(i):** Follows from Theorem 2.11.

**Theorem 3.4:** (i) Every α-continuous function is semi*α-continuous.
(ii) Every continuous function is semi*α-continuous.
(iii) Every constant function is semi*α-continuous.
(iv) Every semi*α-continuous function is semi α-continuous.
(v) Every semi*α-continuous function is semi*-precontinuous.
(vi) Every semi*α-continuous function is semi-precontinuous.
(vii) Every semi*-continuous function is semi*α-continuous.
(viii) Every semi*-continuous function is semi-continuous.
**Proof:** Follows from Theorem 2.8 and the fact that every constant function is continuous.

**Remark 3.5:** In general the converse of each of the statements in Theorem 3.4 is not true.

**Theorem 3.6:** If the topology of the space $Y$ is given by a basis B, then a function $f:X\longrightarrow Y$ is semi*α-continuous if and only if the inverse image of every basic open set in $Y$ under $f$ is semi*α-open in $X$.
**Proof:** Suppose $f:X\longrightarrow Y$ is semi*α-continuous. Then inverse image of every open set in $Y$ is semi*α-open in $X$. In particular, inverse image of every basic open set in $Y$ is semi*α-open in $X$. Conversely, let $V$ be an open set in $Y$. Then $V=\cup B_i$ where $B_i\in$B. Now $f^{-1}(V)=f^{-1}(\cup B_i)=\cup f^{-1}(B_i)$. By hypothesis, $f^{-1}(B_i)$ is semi*α-open for each i. By Theorem 2.10(i), $f^{-1}(V)=\cup f^{-1}(B_i)$ is semi*α-open. Hence $f$ is semi*α-continuous.

**Theorem 3.7:** A function $f:X\longrightarrow Y$ is not semi*α-continuous at point $x\in X$ if and only if $x$ belongs to the semi*α-frontier of the inverse image of some open set in $Y$ containing $f(x)$.
**Proof:** Suppose $f$ is not semi*α-continuous at $x$. Then by Definition 3.1, there is an open set $V$ in $Y$ containing $f(x)$ such that $f(U)$ is not a subset of $V$ for every semi*α-open set $U$ in $X$ containing $x$. Hence $U\cap(X\setminus f^{-1}(V))\neq\phi$ for every semi*α-open set $U$ containing $x$. By Theorem 2.13(ii), we get $x\in s^*\alpha Cl(X\setminus f^{-1}(V))$. Also $x\in f^{-1}(V)\subseteq s^*\alpha Cl(f^{-1}(V))$. Hence $x\in s^*\alpha$Cl($f$

$^{1}(V))\cap s^*\alpha Cl(X\backslash f^{1}(V))$. By Theorem 2.15, $x\in s^*\alpha Fr(f^{-1}(V))$. On the other hand, let $f$ be semi*α-continuous at $x\in X$. Let $V$ be any open set in $Y$ containing $f(x)$. Then there exists a semi*α-open set $U$ in $X$ containing $x$ such that $f(U)\subseteq V$. That is, $U$ is a semi*α-open set in $X$ containing $x$ such that $U\subseteq f^{-1}(V)$. Hence $x\in s^*\alpha Int(f^{-1}(V))$. Therefore by Definition 2.14, $x\notin s^*\alpha Fr(f^{-1}(V))$.

**Theorem 3.8:** Let $f:X\longrightarrow \Pi X_\alpha$ be semi*α-continuous where $\Pi X_\alpha$ is given the product topology and $f(x)=(f_\alpha(x))$. Then each coordinate function $f_\alpha:X\longrightarrow X_\alpha$ is semi*α-continuous.

**Proof:** Let $V$ be an open set in $X_\alpha$. Then $f_\alpha^{-1}(V)=(\pi_\alpha\circ f)^{-1}(V)=f^{-1}(\pi_\alpha^{-1}(V))$, where $\pi_\alpha:\Pi X_\alpha\longrightarrow X_\alpha$ is the projection map. Since is $\pi_\alpha$ continuous, $\pi_\alpha^{-1}(V)$ is open in $\Pi X_\alpha$. By the semi*α-continuity of $f$, $f_\alpha^{-1}(V)=f^{1}(\pi_\alpha^{-1}(V))$ is semi*α-open in $X$. Therefore $f_\alpha$ is semi*α-continuous.

**Theorem 3.9:** Let $f:X\longrightarrow \Pi X_\alpha$ be defined by $f(x)=(f_\alpha(x))$ and $\Pi X_\alpha$ be given the product topology. Suppose $S^*\alpha O(X)$ is closed under finite intersection. Then $f$ is semi*α-continuous if each coordinate function $f_\alpha:X\longrightarrow X_\alpha$ is semi*α-continuous,

**Proof:** Let $V$ be a basic open set in $\Pi X_\alpha$. Then $V=\cap\pi_\alpha^{-1}(V_\alpha)$ where each $V_\alpha$ is open in $X_\alpha$, the intersection being taken over finitely many α's. Now $f^{-1}(V)=$ $f^{-1}(\cap\pi_\alpha^{-1}(V_\alpha))=\cap(f^{-1}(\pi_\alpha^{-1}(V_\alpha)))=\cap(\pi_\alpha\circ f)^{-1}(V)=\cap f_\alpha^{-1}(V)$ is semi*α-open, by hypothesis. Hence by Theorem 3.6, $f$ is semi*α-continuous.

**Theorem 3.10:** Let $f:X\longrightarrow Y$ be continuous and $g:X\longrightarrow Z$ be semi*α-continuous. Let $h:X\longrightarrow Y\times Z$ be defined by $h(x)=(f(x), g(x))$ and $Y\times Z$ be given the product topology. Then $h$ is semi*α-continuous.

**Proof:** By virtue of Theorem 3.6, it is sufficient to show that inverse image under $h$ of every basic open set in $Y\times Z$ is semi*α-open in $X$. Let $U\times V$ be a basic open set in $Y\times Z$. Then $h^{-1}(U\times V) = f^{-1}(U)\cap g^{-1}(V)$. By continuity of $f$, $f^{-1}(U)$ is open in $X$ and by semi*α-continuity of $g$, $g^{-1}(V)$ is semi*α-open in $X$. By invoking Theorem 2.10(ii), we get $h^{-1}(U\times V) = f^{-1}(U)\cap g^{-1}(V)$ is semi*α-open.

**Remark 3.11:** The above theorem is true even if $f$ is semi*α-continuous and $g$ is continuous.

**Theorem 3.12:** Let $f:X\longrightarrow Y$ be semi*α-continuous and $g:Y\longrightarrow Z$ be continuous.
Then $g\circ f:X\longrightarrow Z$ is semi*α-continuous.

**Proof:** Let $V$ be an open set in $Z$. Since $g$ is continuous, $g^{-1}(V)$ is open in $Y$. By semi*α-continuity of $f$, $(g\circ f)^{-1}(V)= f^{1}(g^{-1}(V))$ is semi*α-open in $X$. Hence $g\circ f$ is semi*α-continuous.

**Remark 3.13:** Composition of two semi*α-continuous functions need not be semi*α-continuous.

**Definition 3.14:** A function $f:X\longrightarrow Y$ is called contra-semi*α-continuous if $f^{1}(V)$ is semi*α- closed in $X$ for every open set $V$ in $Y$.

**Theorem 3.15:** For a function $f:X\longrightarrow Y$, the following are equivalent:

(i) $f$ is contra-semi*α-continuous.

(ii) For each $x\in X$ and each closed set $F$ in $Y$ containing $f(x)$, there exists a semi*α-open set $U$ in $X$ containing $x$ such that $f(U)\subseteq F$.

(iii)The inverse image of each closed set in $Y$ is semi*α-open in $X$.

(iv)$Cl^*(\alpha Int(f^{-1}(F)))=Cl^*(f^{-1}(F))$ for every closed set $F$ in $Y$.

(v)$Int^*(\alpha Cl(f^{1}(V)))=Int^*(f^{1}(V))$ for every open set $V$ in $Y$.

**Proof: (i)⇒(ii):** Let $f:X\longrightarrow Y$ be contra-semi*α-continuous. Let $x\in X$ and $F$ be a closed set in $Y$ containing $f(x)$. Then $V=Y\backslash F$ is an open set in $Y$ not containing $f(x)$. Since $f$ is contra-semi*α-continuous, $f^{-1}(V)$ is a semi*α-closed set in $X$ not containing $x$. That is, $f^{-1}(V)= X\backslash f^{-1}(F)$ is a semi*α-closed set in $X$ not containing $x$. Therefore
$U=f^{1}(F)$ is a semi*α-open set in $X$ containing $x$ such that $f(U)\subseteq F$.

**(ii)⇒(iii):** Let $F$ be a closed set in $Y$. Let $x\in f^{-1}(F)$, then $f(x)\in F$. By (ii), there is a semi*α-open set $U_x$ in $X$ containing $x$ such that $f(x)\in f(U_x)\subseteq F$. That is, $x\in U_x\subseteq$ $f^{-1}(F)$. Therefore $f^{-1}(F)= \cup\{U_x:x\in f^{-1}(F)\}$. By Theorem 2.10(i), $f^{-1}(F)$ is semi*α-open in $X$. **(iii)⇒(iv):** Let $F$ be a closed set in $Y$. By (iii), $f^{-1}(F)$ is a semi*α-open set in $X$. By Theorem 2.11, $Cl^*(\alpha Int(f^{-1}(F)))= Cl^*(f^{-1}(F))$.

**(iv)⇒(v):** If $V$ is any open set in $Y$, then $Y\backslash V$ is closed in $Y$. By (iv), we have $Cl^*(\alpha Int(f^{-1}(Y\backslash V))) =Cl^*(f^{-1}(Y\backslash V))$. Taking the complements, we get $Int^*(\alpha Cl(f^{-1}(V)))=Int^*(f^{-1}(V))$.

**(v)⇒(i):** Let $V$ be any open set in $Y$. Then by assumption, $Int^*(\alpha Cl(f^{-1}(V)))=Int^*(f^{-1}(V))$. By Theorem 2.12, $f^{-1}(V)$ is semi*α-closed .

**Theorem 3.16:** Every contra-α-continuous function is contra-semi*α-continuous.

**Proof:** Follows from Remark 2.9.

**Remark 3.17:** It can be seen that the converse of the above theorem is not true.

**Theorem 3.18:** Every contra-semi*α-continuous function is contra-semi α-continuous.

**Proof:** Let $f:X\longrightarrow Y$ be contra-semi*α-continuous. Let $V$ be an open set in $Y$. Since $f$ is contra-semi*α-continuous, $f^{-1}(V)$ is semi*α-closed in $X$. By Remark 2.9, $f^{-1}(V)$ is semi-α-closed in $X$. Hence $f$ is contra-semi-α-continuous.

**Remark 3.19:** It can be easily seen that the converse of the above theorem is not true.
Composition of two contra-semi*α-continuous functions need not be contra-semi*α-continuous.

# 4. SEMI*α-IRRESOLUTE FUNCTIONS

In this section we define the semi*α-irresolute and contra-semi*α-irresolute functions and investigate their fundamental properties.

**Definition 4.1:** A function $f:X\longrightarrow Y$ is said to be semi*α-irresolute at $x\in X$ if for each semi*α-open set $V$ of $Y$ containing $f(x)$, there is a semi*α-open set $U$ of $X$ such that $x\in U$ and $f(U)\subseteq V$.

**Definition 4.2:** A function $f:X\longrightarrow Y$ is said to be semi*α-irresolute if $f^{-1}(V)$ is semi*α-open in $X$ for every semi*α-open set $V$ in $Y$.

**Definition 4.3:** A function $f:X\longrightarrow Y$ is said to be contra-semi*α-irresolute if $f^{-1}(V)$ is semi*α-closed in $X$ for every semi*α-open set $V$ in $Y$.

**Definition 4.4:** A function $f:X\longrightarrow Y$ is said to be strongly semi*α-irresolute if $f^{-1}(V)$ is open in $X$ for every semi*α-open set $V$ in $Y$.

**Definition 4.5:** A function $f:X\longrightarrow Y$ is said to be contra-strongly semi*α-irresolute if $f^{-1}(V)$ is closed in $X$ for every semi*α-open set $V$ in $Y$.

**Theorem 4.6:** Every semi*α-irresolute function is semi*α-continuous.
**Proof:** Let $f:X\longrightarrow Y$ be semi*α-irresolute. Let $V$ be open in $Y$. Then by Theorem 2.8(ii), $V$ is semi*α-open. Since $f$ is semi*α-irresolute, $f^{1}(V)$ is semi*α-open in $X$. Thus $f$ is semi*α-continuous.

**Theorem 4.7:** Every constant function is semi*α-irresolute.
**Proof:** Let $f:X\longrightarrow Y$ be a constant function defined by $f(x)=y_0$ for all $x$ in $X$, where $y_0$ is a fixed point in $Y$. Let $V$ be a semi*α-open set in $Y$. Then $f^{-1}(V)=X$ or $\phi$ according as $y_0\in V$ or $y_0\notin V$. Thus $f^{-1}(V)$ is semi*α-open in $X$. Hence $f$ is semi*α-irresolute.

**Theorem 4.8:** Let $f:X\longrightarrow Y$ be a function. Then the following are equivalent:
(i) $f$ is semi*α-irresolute.
(ii) $f$ is semi*α-irresolute at each point of $X$.
(iii) $f^{-1}(F)$ is semi*α-closed in $X$ for every semi*α-closed set $F$ in $Y$.
(iv) $f(s*\alpha Cl(A))\subseteq s*\alpha Cl(f(A))$ for every subset $A$ of $X$.
(v) $s*\alpha Cl(f^{1}(B))\subseteq f^{1}(s*\alpha Cl(B))$ for every subset $B$ of $Y$.
(vi) $f^{-1}(s*\alpha Int(B))\subseteq s*\alpha Int(f^{-1}(B))$ for every subset $B$ of $Y$.
(vii) $Int*(\alpha Cl(f^{-1}(F)))=Int*(f^{-1}(F))$ for every semi*α-closed set $F$ in $Y$.
(viii) $Cl*(\alpha Int(f^{-1}(V)))=Cl*(f^{-1}(V))$ for every semi*α-open set $V$ in $Y$.
**Proof: (i)⇒(ii):** Let $f:X\longrightarrow Y$ be semi*α-irresolute. Let $x\in X$ and $V$ be a semi*α-open set in $Y$ containing $f(x)$. Then $x\in f^{-1}(V)$. Since $f$ is semi*α-irresolute, $U=f^{-1}(V)$ is a semi*α-open set in $X$ containing $x$ such that $f(U)\subseteq V$.
**(ii)⇒(i):** Let $f:X\longrightarrow Y$ be semi*α-irresolute at each point of $X$. Let V be a semi*α-open set in Y. Let $x\in f^{-1}(V)$. Then V is a semi*α-open set in Y containing $f(x)$. By (ii), there is a semi*α-open set $U_x$ in $X$ containing $x$ such that $f(x)\in f(U_x)\subseteq V$. Therefore $U_x\subseteq f^{-1}(V)$. Hence $f^{-1}(V)=\cup\{U_x : x\in f^{-1}(V)\}$. By Theorem 2.10(i), $f^{-1}(V)$ is semi*α-open in $X$.
**(i)⇒(iii):** Let $F$ be a semi*α-closed set in $Y$. Then $V=Y\backslash F$ is semi*α-open in $Y$. Then $f^{-1}(V)$ is semi*α-open in X. Therefore $f^{-1}(F)=f^{-1}(Y\backslash V)=X\backslash f^{-1}(V)$ is semi*α-closed.
**(iii)⇒(i):** Let V be a semi*α-open set in Y. Then F=Y\V is semi*α-closed. By (iii), $f^{-1}(F)$ is semi*α-closed. Hence $f^{-1}(V)=f^{-1}(Y\backslash F)=X\backslash f^{-1}(F)$ is semi*α-open in $X$.
**(iii)⇒(iv):** Let $A\subseteq X$. Let $F$ be a semi*α-closed set containing $f(A)$. Then by (iii), $f^{-1}(F)$ is a semi*α-closed set containing $A$. This implies that $s*\alpha Cl(A)\subseteq f^{1}(F)$ and hence $f(s*\alpha Cl(A))\subseteq F$. Therefore $f(s*\alpha Cl(A))\subseteq Cl(f(A))$.
**(iv)⇒(v):** Let $B\subseteq Y$ and let $A=f^{-1}(B)$. By assumption, $f(s*\alpha Cl(A))\subseteq s*\alpha Cl(f(A))\subseteq s*\alpha Cl(B)$. This implies that $s*\alpha Cl(A)\subseteq f^{-1}(s*\alpha Cl(B))$. Hence $s*\alpha Cl(f^{-1}(B))\subseteq f^{-1}(s*\alpha Cl(B))$.
**(v)⇒(iii):** Let F be semi*α-closed in Y. Then $s*\alpha Cl(F)=F$. Therefore (v) implies $s*\alpha Cl(f^{-1}(F))\subseteq f^{-1}(F)$. Hence $s*\alpha Cl(f^{-1}(F))=f^{-1}(F)$. By Theorem 2.13(i), $f^{-1}(F)$ is semi*α-closed.
**(v)⇔(vi):** The equivalence of (v) and (vi) can be proved by taking the complements.
**(vii)⇔(iii):** Follows from Theorem 2.12.
**(viii)⇔(i):** Follows from Theorem 2.11.

**Theorem 4.9:** Let $f:X\longrightarrow Y$ be a function. Then $f$ is not semi*α-irresolute at a point $x$ in $X$ if and only if $x$ belongs to the semi*α-frontier of the inverse image of some semi*α-open set in $Y$ containing $f(x)$.
**Proof:** Suppose $f$ is not semi*α-irresolute at $x$. Then by Definition 4.1, there is a semi*α-open set $V$ in $Y$ containing $f(x)$ such that $f(U)$ is not a subset of $V$ for every semi*α-open set $U$ in $X$ containing $x$. Hence $U\cap(X\backslash f^{-1}(V))\neq\phi$ for every semi*α-open set $U$ containing $x$. Thus $x\in s*\alpha Cl(X\backslash f^{1}(V))$. Since $x\in f^{1}(V)\subseteq s*\alpha Cl(f^{-1}(V))$, we have $x\in s*\alpha Cl(f^{-1}(V))\cap s*\alpha Cl(X\backslash f^{-1}(V))$. Hence by Theorem 2.15, $x\in s*\alpha Fr(f^{-1}(V))$. On the other hand, let $f$ be semi*α-irresolute at $x$. Let $V$ be a semi*α-open set in $Y$ containing $f(x)$. Then there is a semi*α-open set $U$ in $X$ containing $x$ such that $f(x)\in f(U)\subseteq V$. Therefore $U\subseteq f^{-1}(V)$. Hence $x\in s*\alpha Int(f^{-1}(V))$. Therefore by Definition 2.14, $x\notin s*\alpha Fr(f^{-1}(V))$ for every open set $V$ containing $f(x)$.

**Theorem 4.10:** Every contra-semi*α-irresolute function is contra-semi*α-continuous.
**Proof:** Let $f:X\longrightarrow Y$ be a contra-semi*α-irresolute function. Let $V$ be an open set in $Y$. Then by Theorem 2.8(ii), $V$ is semi*α-open in $Y$. Since $f$ is contra-semi*α-irresolute, $f^{-1}(V)$ is semi*α-closed in $X$. Hence $f$ is contra-semi*α-continuous.

**Theorem 4.11:** For a function $f:X\longrightarrow Y$, the following are equivalent:
(i) $f$ is contra-semi*α-irresolute.
(ii) The inverse image of each semi*α-closed set in $Y$ is semi*α-open in $X$.
(iii) For each $x\in X$ and each semi*α-closed set $F$ in $Y$ with $f(x)\in F$, there exists a semi*α-open set $U$ in $X$ such that $x\in U$ and $f(U)\subseteq F$.
(iv) $Cl*(\alpha Int(f^{-1}(F)))=Cl*(f^{-1}(F))$ for every semi*α-closed set $F$ in $Y$.
(v) $Int*(\alpha Cl(f^{-1}(V)))=Int*(f^{-1}(V))$ for every semi*α-open set $V$ in $Y$.
**Proof:(i)⇒(ii):** Let $F$ be a semi*α-closed set in $Y$. Then $Y\backslash F$ is semi*α-open in $Y$. Since $f$ is contra-semi*α-irresolute, $f^{-1}(Y\backslash F)=X\backslash f^{-1}(F)$ is semi*α-closed in $X$.
**(ii)⇒(iii):** Let $F$ be a semi*α-closed set in $Y$ containing $f(x)$. Then $U=f^{-1}(F)$ is a semi*α-open set containing $x$ such that $f(U)\subseteq F$.
**(iii)⇒(iv):** Let $F$ be a semi*α-closed set in $Y$ and $x\in f^{-1}(F)$, then $f(x)\in F$. By assumption, there is a semi*α-open set $U_x$ in $X$ containing $x$ such that $f(x)\in f(U_x)\subseteq F$ which implies that $x\in U_x\subseteq f^{-1}(F)$. This follows that $f^{-1}(F)=\cup\{U_x : x\in f^{-1}(F)\}$. By Theorem 2.10(i), $f^{-1}(F)$ is semi*α-open in $X$. By Theorem 2.11, $Cl*(\alpha Int(f^{-1}(F)))=Cl*(f^{-1}(F))$.
**(iv)⇒(v):** Let $V$ be a semi*α-open set in $Y$. Then $Y\backslash V$ is semi*α-closed in $Y$. By assumption, $Cl*(\alpha Int(f^{-1}(Y\backslash V)))=Cl*(f^{-1}(Y\backslash V))$. Taking the complements we get, $Int*(\alpha Cl(f^{-1}(V)))=Int*(f^{-1}(V))$.
**(v)⇒(i):** Let $V$ be any semi*α-open set in $Y$. Then by assumption, $Int*(\alpha Cl(f^{1}(V)))=Int*(f^{-1}(V))$. By Theorem 2.12, $f^{-1}(V)$ is semi*α-closed in $X$.

**Theorem 4.12:** (i) Every strongly semi*α-irresolute function is semi*α-irresolute and hence semi*α-continuous.
(ii) Every semi α**-continuous function is strongly semi*α-irresolute.
**Proof:** Let $f:X\longrightarrow Y$ be strongly semi*α-irresolute. Let $V$ be semi*α-open in $Y$. Since $f$ is strongly semi*α-irresolute, $f^{1}(V)$ is open in $X$. Then by Theorem 2.8(ii), $f^{1}(V)$ is semi*α-open. Therefore $f$ is semi*α-irresolute. Hence by Theorem 4.6, $f$ is semi*α-continuous.

**Theorem 4.13:** Every constant function is strongly semi*α-irresolute.
**Proof:** Let $f:X\longrightarrow Y$ be a constant function defined by $f(x)=y_0$ for all $x$ in $X$, where $y_0$ is a fixed point in $Y$. Let $V$ be a semi*α-open set in $Y$. Then $f^{-1}(V)=X$ or $\phi$ according as $y_0\in V$ or $y_0\notin V$. Thus $f^{-1}(V)$ is open in $X$. Hence $f$ is strongly semi*α-irresolute.

**Theorem 4.14:** Let $f:X\longrightarrow Y$ be a function. Then the following are equivalent: (i)$f$ is strongly semi*α-irresolute.
(ii)$f^{-1}(F)$ is closed in $X$ for every semi*α-closed set $F$ in $Y$.
(iii) $f(Cl(A))\subseteq s^*\alpha Cl(f(A))$ for every subset $A$ of $X$.
(iv)$Cl(f^{-1}(B))\subseteq f^{-1}(s^*\alpha Cl(B))$ for every subset $B$ of $Y$.
(v)$f^{-1}(s^*\alpha Int(B))\subseteq Int(f^{-1}(B))$ for every subset $B$ of $Y$.
**Proof: (i)⇒(ii):** Let $F$ be a semi*α-closed set in $Y$. Then $V=Y\setminus F$ is semi*α-open in $Y$. Then $f^{-1}(V)$ is open in X. Therefore $f^{-1}(F)=f^{-1}(Y\setminus V)=X\setminus f^{-1}(V)$ is closed.
**(ii)⇒(i):** Let V be a semi*α-open set in Y. Then F=Y\V is semi*α-closed. By (ii), $f^{-1}(F)$ is closed. Hence $f^{-1}(V)= f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)$ is open in $X$. Therefore $f$ is strongly semi*α-irresolute.
**(ii)⇒(iii):** Let $A\subseteq X$. Let $F$ be a semi*α-closed set containing $f(A)$. Then by (ii), $f^{-1}(F)$ is a closed set containing $A$. This implies that $Cl(A)\subseteq f^{-1}(F)$ and hence $f(Cl(A))\subseteq F$. Therefore $f(Cl(A))\subseteq s^*\alpha Cl(f(A))$.
**(iii)⇒(iv):** Let $B\subseteq Y$ and let $A= f^{-1}(B)$. By assumption, $(Cl(A))\subseteq s^*\alpha Cl(f(A))\subseteq s^*\alpha Cl(B)$. This implies that $Cl(A)\subseteq f^{-1}(s^*\alpha Cl(B))$.
**(iv)⇒(ii):** Let F be semi*α-closed in Y. Then by Theorem 2.13(i), $s^*\alpha Cl(F)= F$. Therefore (iv) implies $Cl(f^{-1}(F))\subseteq f^{-1}(F)$. Hence $Cl(f^{-1}(F))=f^{-1}(F)$. Therefore $f^{-1}(F)$ is closed.
**(iv)⇔(v):** The equivalence of (iv) and (v) follows from taking the complements.

**Theorem 4.15:** For a function $f:X\longrightarrow Y$, the following are equivalent:
(i) $f$ is contra-strongly semi*α-irresolute.
(ii)The inverse image of each semi*α-closed set in $Y$ is open in $X$.
(iii)For each $x\in X$ and each semi*α-closed set $F$ in $Y$ with $f(x)\in F$, there exists a open set $U$ in $X$ such that $x\in U$ and $f(U)\subseteq F$.
**Proof:(i)⇒(ii):** Let $F$ be a semi*α-closed set in $Y$. Then $Y\setminus F$ is semi*α-open in $Y$. Since $f$ is contra-strongly semi*α-irresolute, $f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)$ is closed in $X$. Hence $f^{-1}(F)$ is open in $X$. This proves (ii).
**(ii)⇒(i):** Let $U$ be a semi*α-open set in $Y$. Then $Y\setminus U$ is semi*α-closed in $Y$. By assumption, $f^{-1}(Y\setminus U)=X\setminus f^{-1}(U)$ is closed in $X$. Hence $f^{-1}(U)$ is open in $X$.
**(ii)⇒(iii):** Let $F$ be a semi*α-closed set in $Y$ containing $f(x)$.Then $U= f^{-1}(F)$ is an open set containing $x$ such that $f(U)\subseteq F$.
**(iii)⇒(ii):** Let $F$ be a semi*α-closed set in $Y$ and $x\in f^{-1}(F)$, then $f(x)\in F$. By assumption, there is an open set $U_x$ in $X$ containing $x$ such that $f(x)\in f(U_x)\subseteq F$ which implies that $x\in U_x\subseteq f^{-1}(F)$. Hence $f^{-1}(F)$ is open in $X$.
**Theorem 4.16: (i)** Composition of semi*α-irresolute functions is semi*α-irresolute.
(ii) Inverse of a bijective semi*α-irresolute function is also semi*α-irresolute.
**Proof:** Follows from definition and set theoretic results.

## 5. MORE FUNCTIONS ASSOCIATED WITH SEMI*α-OPEN SETS

**Definition 5.1:** A function f :X⟶Y is said to be semi*α-open if $f(U)$ is semi*α-open in $Y$ for every open set $U$ in $X$.

**Definition 5.2:** A function $f:X\longrightarrow Y$ is said to be contra-semi*α-open if $f(U)$ is semi*α-closed in $Y$ for every open set $U$ in $X$.

**Definition 5.3:** A function $f:X\longrightarrow Y$ is said to be pre-semi*α-open if $f(U)$ is semi*α-open in $Y$ for every semi*α-open set $U$ in $X$.

**Definition 5.4:** A function $f:X\longrightarrow Y$ is said to be contra-pre-semi*α-open if $f(U)$ is semi*α-closed in $Y$ for every semi*α-open set $U$ in $X$.

**Definition 5.5:** A function $f:X\longrightarrow Y$ is said to be semi*α-closed if $f(F)$ is semi*α-closed in $Y$ for every closed set $F$ in $X$.

**Definition 5.6:** A function $f:X\longrightarrow Y$ is said to be contra-semi*α-closed if $f(F)$ is semi*α-open in $Y$ for every closed set $F$ in $X$.

**Definition 5.7:**A function $f:X\longrightarrow Y$ is said to be pre-semi*α-closed if $f(F)$ is semi*α-closed in $Y$ for every semi*α-closed set $F$ in $X$.

**Definition 5.8:** A function $f:X\longrightarrow Y$ is said to be contra-pre-semi*α-closed if $f(F)$ is semi*α-open in $Y$ for every semi*α-closed set $F$ in $X$.

**Definition 5.9:** A bijection $f:X\longrightarrow Y$ is called a semi*α-homeomorphism if $f$ is both semi*α-irresolute and pre-semi*α-open. The set of all semi*α-homeomorphisms of $(X, \tau)$ into itself is denoted by $s^*\alpha H(X, \tau)$.

**Definition 5.10:** A function $f:X\longrightarrow Y$ is said to be semi*α-totally continuous if $f^{-1}(V)$ is clopen in $X$ for every semi*α-open set $V$ in $Y$.

**Definition 5.11:** A function $f:X\longrightarrow Y$ is said to be totally semi*α-continuous if $f^{-1}(V)$ is semi*α-regular in $X$ for every open set $V$ in $Y$.

**Theorem 5.12:** (i) Every pre-semi*α-open function is semi*α-open.
(ii) Every semi*α-open function is semi α-open.
(iii) Every contra-pre-semi*α-open function is contra-semi*α-open.
(iv) Every pre-semi*α-closed function is semi*α-closed.
(v) Every contra-pre-semi*α-closed function is contra-semi*α-closed.

**Proof:** Follows from definitions, Theorem 2.8 and Remark 2.9.

**Theorem 5.13:** Let $f:X\longrightarrow Y$ and be $g:Y\longrightarrow Z$ be functions. Then (i) $g\circ f$ is pre-semi*α-open if both f and g are pre-semi*α-open.
(ii) $g\circ f$ is semi*α-open if f is semi*α-open and g is pre-semi*α-open.
(iii) $g\circ f$ is pre-semi*α-closed if both f and g are pre-semi*α-closed.
(iv) $g\circ f$ is semi*α-closed if both f is semi*α-closed and g is pre-semi*α-closed.
**Proof:** Follows from definitions.

**Theorem 5.14:** Let $f:X\longrightarrow Y$ be a function where $X$ is an Alexandroff space and $Y$ is any topological space. Then the following are equivalent:
(i) $f$ *is* semi*α-totally continuous.
(ii) For each $x\in X$ and each semi*α-open set $V$ in $Y$ with $f(x)\in V$, there exists a clopen set $U$ in $X$ such that $x\in U$ and $f(U)\subseteq V$.
**Proof: (i)⇒(ii):** Suppose $f:X\longrightarrow Y$ is semi*α-totally continuous. Let $x\in X$ and let $V$ be a semi*α-open set containing $f(x)$. Then $U=f^{-1}(V)$ is a clopen set in $X$ containing $x$ and hence $f(U)\subseteq V$.
**(ii)⇒(i):** Let $V$ be a semi*α-open set in $Y$. Let $x\in f^{-1}(V)$. Then $V$ is a semi*α-open set containing $f(x)$. By hypothesis there exist a clopen set $U_x$ containing $x$ such that $f(U_x)\subseteq V$ which implies that $U_x\subseteq f^{-1}(V)$. Therefore we have $f^{-1}(V)=\cup\{U_x : x\in f^{-1}(V)\}$. Since each $U_x$ is open, $f^{-1}(V)$ is open. Since each $U_x$ is a closed set in the Alexandroff space $X$, $f^{-1}(V)$ is closed in $X$. Hence $f^{-1}(V)$ is clopen in $X$.
**Theorem 5.15:** A function $f:X\longrightarrow Y$ is semi*α-totally continuous if and only if $f^{-1}(F)$ is clopen in $X$ for every semi*α-closed set $F$ in $Y$.
**Proof:** Follows from definition.

**Theorem 5.16:** A function $f:X\longrightarrow Y$ is totally semi*α-continuous if and only if $f$ is both semi*α-continuous and contra-semi*α-continuous.
**Proof:** Follows from definitions.
**Theorem 5.17:** A function $f:X\longrightarrow Y$ is semi*α-totally continuous if and only if $f$ is both strongly semi*α-irresolute and contra-strongly semi*α-irresolute.
**Proof:** Follows from definitions.
**Theorem 5.18:** Let $f: X\longrightarrow Y$ be semi*α-totally continuous and $A$ is a subset of $Y$. Then the restriction $f_A : A\longrightarrow Y$ is semi*α-totally continuous.
**Proof:** Let $V$ be a semi*α-open set in $Y$. Then $f^{-1}(V)$ is clopen in $X$ and hence $(f|_A)^{-1}(V) =A\cap f^{-1}(V)$ is clopen in $A$. Hence the theorem follows.
**Theorem 5.19:** Let $f:X\longrightarrow Y$ be a bijection. Then the following are equivalent: (i) $f$ is semi*α-irresolute.
(ii) $f^{-1}$ is pre-semi*α-open.
(iii) $f^{-1}$ is pre-semi*α-closed.
**Proof:** Follows from definitions.
**Theorem 5.20:** A bijection $f:X\longrightarrow Y$ is a semi*α-homeomorphism if and only if $f$ and $f^{-1}$ are semi*α-irresolute.
**Proof:** Follows from definitions.
**Theorem 5.21:** (i) The composition of two semi*α-homeomorphisms is a semi*α-homeomorphism
(ii) The inverse of a semi*α-homeomorphism is also a semi*α-homeomorphism.

**Proof:** (i) Let $f:X\longrightarrow Y$ and $g:Y\longrightarrow Z$ be semi*α-homeomorphisms. By Theorem 4.16 and theorem 5.13(i), $g\circ f$ is a semi*α-homeomorphism.
(ii)Let $f:X\longrightarrow Y$ be a semi*α-homeomorphism. Then by Theorem 4.16(ii) and by Theorem 5.20, $f^{-1}:Y\longrightarrow X$ is also semi*α-homeomorphism.

**Theorem 5.22:** If (X, τ) is a topological space, then the set s*αH($X$, τ) of all semi*α-homeomorphisms of ($X$, τ) into itself forms a group.
**Proof:** Since the identity mapping I on X is a semi*α-homeomorphism, I∈s*αH($X$, τ) and hence s*αH($X$, τ) is non-empty and the theorem follows from Theorem 5.21.

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