

IIR Band Pass and Band Stop Filter Design Employing Teaching-Learning based Optimization Technique

Damanpreet Singh
Sant Longowal Institute of Engineering and
Technology, Longowal, India

J.S. Dhillon
Sant Longowal Institute of Engineering and
Technology, Longowal, India

ABSTRACT

In this paper newly developed teaching-learning based optimization (TLBO) algorithm is applied for designing band pass (BP) and band stop (BS) digital IIR filters. TLBO is heuristic algorithm based on the social phenomenon of teaching-learning process. The effectiveness of proposed algorithm is validated by designing the BP and BS filters by approximating the magnitude response with L_p -norm error criterion, minimizing pass band and stop band ripples along with guaranteed stability. The results obtained employing TLBO are compared to those obtained by the well known evolutionary algorithms such as hierarchical genetic algorithm, hybrid taguchi genetic algorithm and immune algorithm. The results reveal that the proposed TLBO algorithm gives better optimal filter in terms of magnitude response and ripples in pass band and stop band.

Keywords

IIR filter, teaching-learning based optimization, magnitude response, band pass, band stop, stability, L_p -approximation error.

1. INTRODUCTION

Digital filters are used in number of application like communication, radar signal processing, speech and image processing, biomedicine, and seismic exploration. The main purpose of using digital filter is to extract the useful information from the signal, and to remove the unwanted part of the signal. On the basis of impulse response length, digital filters are classified in to two types: finite impulse response (FIR) filter and infinite impulse response (IIR) filter [1]. IIR filter gives better performance with lesser number of coefficients than FIR for the same design specification. IIR filters are used when sharp cut off and high throughput are needed. However, there are some disadvantages of IIR digital filter such as [2]: 1) instability of IIR filter 2) multi-modal error surface. The stability problem can be easily handled by imposing stability constraints on the denominator coefficients of IIR digital filter. Due to non-linear and multimodal error surface of IIR digital filter, the classical gradient based optimization techniques cannot find the global minimum [3, 4]. In recent years, many researcher have applied number of evolutionary algorithms for the design of IIR digital filters such as: GA [5-8], immune algorithm (TIA) [9], tabu search [10], particle swarm optimization (PSO) [11, 12], Seeker optimization algorithm (SOA) [13], two-stage ensemble evolutionary algorithm [14], Gravitation search algorithm [15] and many more.

All the evolutionary and swarm intelligence based algorithms are probabilistic algorithms and require common controlling parameters like number of generations, population size, elite size, etc. Besides the common control parameters, each algorithm requires its own algorithm-specific control parameters. The performance of the above mentioned

algorithms is immensely dependent upon proper tuning of the algorithm-specific parameters. The improper tuning of algorithm-specific parameters either increases the computational effort or yields the local optimal solution. To overcome the above discussed drawbacks, Rao et al. [16, 17] proposed a teaching-learning based optimization (TLBO) algorithm based on the natural phenomenon of teaching-learning. The implementation of TLBO does not require the determination of any algorithm specific controlling parameters which makes the algorithm robust and powerful. TLBO requires only common controlling parameters like population size and number of generations for its working.

The intent of this paper is to use the effectiveness of TLBO algorithm for designing band pass (BP) and band stop (BS) IIR filter employing L_p -norm error criterion, and pass band ripples and stop band ripples of digital IIR filter. The designed BP and BS IIR filters with TLBO algorithm are compared with hierarchical genetic algorithm (HGA) [7], hybrid taguchi genetic algorithm (HTGA) [8] and taguchi immune algorithm [TIA] [9] to find out the comparative effectiveness of the algorithm and determination of best optimal IIR filters. The paper is organized as follows: In Section 2, mathematical expression of an IIR filter and the objective function are formulated. In Section 3, TLBO algorithm is briefly discussed for the IIR filter design problem. In Section 4, comprehensive and demonstrative sets of results and illustrations are given to make a comparison of the study with existing algorithms. Finally, Section 5 concludes the paper.

2. FORMULATION OF IIR FILTER DESIGN PROBLEM

IIR digital filters are characterized by the following difference equation [2]:

$$y(n) = \sum_{k=0}^{\infty} h(k)x(n-k) \quad (1)$$

$$y(n) = \sum_{k=0}^N b_k x(n-k) - \sum_{k=1}^M a_k y(n-k) \quad (2)$$

where $h(k)$ is the impulse response of the filter, which is theoretically infinite in duration, b_k and a_k are the coefficients of the filter, $x(n)$ and $y(n)$ are the discrete input and output of the filter. The transfer function of IIR filter is defined as below:

$$H(z) = \frac{\sum_{k=0}^M a_k z^{-k}}{1 + \sum_{k=1}^N b_k z^{-k}} \quad (3)$$

Where M , N are numerator and denominator orders, respectively. An important part of the IIR filter design process is to find suitable values for the coefficients b_k and a_k such that some aspect of the filter characteristics, such as frequency response, behaves in a desired manner. An IIR digital filter

can be expressed by the cascading first and second order sections [18] stated as:

$$H(\omega, x) = A \left(\prod_{u=1}^M \frac{1 + p_{1u}e^{-j\omega}}{1 + q_{1u}e^{-j\omega}} \right) \times \left(\prod_{v=1}^N \frac{1 + g_{1v}e^{-j\omega} + g_{2v}e^{-2j\omega}}{1 + h_{1v}e^{-j\omega} + h_{2v}e^{-2j\omega}} \right) \quad (4)$$

where

$$x = [p_{11}, q_{11}, \dots, p_{1M}, q_{1M}, s_{11}, g_{21}, h_{11}, h_{21}, \dots, s_{1N}, g_{2N}, h_{1N}, h_{2N}, A]^T$$

and Vector x denotes the filter coefficients of dimension $S \times 1$ with $S = 2M + 4N + 1$ and A is the gain. The main goal of the design algorithm of digital IIR filter is to find a set of filter coefficients to minimize the magnitude approximation error function in L_p -norm [8, 9] and ripples in pass band and stop band.. The magnitude response is specified at K equally spaced discrete frequency samples in pass-band and stop-band. $E_1(x)$ denotes the absolute error L_1 -norm of magnitude response and $E_2(x)$ denotes the squared error L_2 -norm of magnitude response and are defined as given below:

$$E_1(x) = \sum_{i=0}^K |H_I(\omega_i) - |H(\omega_i, x)|| \quad (5)$$

$$E_2(x) = \sum_{i=0}^K (|H_I(\omega_i) - |H(\omega_i, x)||)^2 \quad (6)$$

Ideal magnitude response $H_I(\omega_i)$ of IIR filter is given as:

$$H_I(\omega_i) = \begin{cases} 1, & \text{for } \omega_i \in \text{passband} \\ 0, & \text{for } \omega_i \in \text{stopband} \end{cases} \quad (7)$$

The ripple magnitudes of pass-band and stop-band are to be minimized, which are denoted by $\delta_p(x)$ and $\delta_s(x)$ respectively. Ripple magnitudes are defined as:

$$\delta_p(x) = \max_{\omega_i} \{ |H(\omega_i, x)| \} - \min_{\omega_i} \{ |H(\omega_i, x)| \} \quad (8)$$

for $\omega_i \in \text{passband}$

and

$$\delta_s(x) = \max_{\omega_i} \{ |H(\omega_i, x)| \} \quad (9)$$

for $\omega_i \in \text{stopband}$

Aggregating all objectives and stability constraints, the multicriterion constrained optimization problem is stated as

$$\text{Minimize } O_1(x) = E_1(x) \quad (10a)$$

$$\text{Minimize } O_2(x) = E_2(x)$$

$$\text{Minimize } O_3(x) = \delta_p(x)$$

$$\text{Minimize } O_4(x) = \delta_s(x)$$

Subject to: the stability constraints

$$1 + q_{1u} \geq 0 \quad (u = 1, 2, \dots, M) \quad (10b)$$

$$1 - q_{1u} \geq 0 \quad (u = 1, 2, \dots, M) \quad (10c)$$

$$1 - h_{2v} \geq 0 \quad (v = 1, 2, \dots, N) \quad (10d)$$

$$1 + h_{1v} + h_{2v} \geq 0 \quad (v = 1, 2, \dots, N) \quad (10e)$$

$$1 - h_{1v} + h_{2v} \geq 0 \quad (v = 1, 2, \dots, N) \quad (10f)$$

Design of IIR filter is a multi-objective optimization problem (MOOP) as several objectives are optimized simultaneously as shown in Eq. (10a). The multiobjective constrained optimization problem for the design of digital IIR filter is converted into a scalar constrained optimization problem by using weighting method as defined below:

The objective function to be optimized is defined as:

$$\text{Minimize } f(x) = \sum_{a=1}^4 w_a O_a(x) \quad (11)$$

Subject to: The satisfaction of stability constraints given by equation (10b) to equation (10f). where w_a is nonnegative real number called weight. In this paper weights are taken same as given by [9].

All the poles of designed digital IIR filter should lie inside the unit circle for the filter to be stable. Therefore, the stability constraints by using the Jury method [19] have been imposed on the denominator coefficients as given by equation (10b) to equation (10f). To satisfy the stability conditions coefficients have been updated with random variation as given in equation (12a) to equation (12c). The variation is given as small so that the characteristic of population in TLBO should not be changed.

$$q_{1u} = \begin{cases} q_{1u}(1-r)^2 & ; (1+q_{1u}) < 0 \\ \text{or } (1-q_{1u}) < 0 \\ q_{1u} & ; \text{Otherwise} \end{cases} \quad (12a)$$

$$h_{2v} = \begin{cases} h_{2v}(1-r)^2 & ; (1-h_{2v}) < 0 \\ h_{2v} & ; \text{Otherwise} \end{cases} \quad (12b)$$

$$h_{1v} = \begin{cases} h_{1v}(1-r)^2 & ; (1+h_{1v} + h_{2v}) < 0 \\ \text{or } (1-h_{1v} + h_{2v}) < 0 \\ h_{1v} & ; \text{Otherwise} \end{cases} \quad (12c)$$

3. TLBO ALGORITHM FOR THE DESIGN OF IIR FILTER

In this paper a recently developed heuristic optimization algorithm namely Teaching-learning based optimization (TLBO) proposed by Rao et al. [16, 17] is applied for the design of digital IIR filters. The useful attribute of TLBO is that it require few control parameters which remain fixed throughout the optimization process and need minimum tuning.

In the noble profession of Teaching, a teacher who is the most knowledgeable person in the class always motivates the students to acquire supreme knowledge by which they can improve their academic performances. A teacher works hard to increase the average result of the students / learners from initial level to his own level. However, in spite of the best effort of the teacher, the students not only gain knowledge on the basis of the teacher delivered but also based on their own quality. There are other means in which learners can also gain knowledge by discussing and sharing ideas among other learners in form of tutorials and seminars.

In the implementation of TLBO for the design of digital IIR filter the population is analogous to L number of learners in a class and each learner is assigned S subjects (filter coefficients). The i^{th} learner is represented as

$X_i = [x_{i1}, x_{i2}, \dots, x_{iS}]$ and $f(X_i)$ represent the fitness function for i^{th} learner. The functioning of TLBO based upon two main phases namely Teacher phase and Learner phase.

• *Teacher Phase*

A learner having the minimum value of fitness function value calculated using Eq. (9) is designated as teacher x_j^t for current iteration t . A teacher tries to increase / improve the mean score of all the learners in each of the subject allotted towards its own mean score. So, the mean fitness of the class is increased by the teacher according to his / her own capability. The mean (τ_j) for S subjects allotted to the students is evaluated and a randomly weighted differential vector ($Diff_j^t$) from current mean and various desired mean vectors [20] is calculated as shown below:

$$\tau_j^t = \frac{1}{L} \sum_{i=1}^L (x_{i,j}^t) \quad (j=1,2,\dots,S) \quad (13)$$

$$Diff_j^t = (x_j^t - (TF \times \tau_j^t)) \times R() \quad (j=1,2,\dots,S) \quad (14)$$

where

τ_j^t is mean of j^{th} subject for all learners of a class; x_j^t is the score of the teacher in j^{th} subject; TF is the teaching factor; R is a uniform generated random number between (0,1).

The teaching factor (TF) is one of the vital aspect that facilitates the convergence of TLBO. The value of TF decides about the volume of effect a teacher has on the output of a learner. In this paper the value of TF is heuristically selected as 1 or 2 as shown below:

$$TF = ROUND(1.0 + R()) \quad (15)$$

The weighted differential vector ($Diff_j^t$) generated using Eq. (14) is added to current score of learners in different subjects to generate new learners:

$$x_{new_{ij}}^t = x_{ij}^t + Diff_j^t \quad (j=1,2,\dots,S) \quad (16)$$

The newly generated learner with a better fitness value replaces the existing learner in the class.

• *Learner Phase*

The concept of tutorials and seminars of class room teaching system is followed in the learner phase as the knowledge acquired by the learners / students in teacher phase is further disseminated among learners themselves through discussions, sharing of notes and presentations. Two target learners namely i and m are selected randomly such that $i \neq m$. The

resultant new learners after sharing / exchange of know-how are generated as follows:

$$x_{new_{ij}}^t = \begin{cases} x_{ij}^t + R() \times (x_{ij}^t - x_{mj}^t); & f(X_i) < f(X_m) \\ x_{ij}^t + R() \times (x_{mj}^t - x_{ij}^t); & \text{Otherwise} \end{cases} \quad (17)$$

where

$$(j=1,2,\dots,S)$$

After one successful completion of Teacher and Learner phase, algorithm is made to update teacher value before the start of next iteration. In this TLBO algorithm maximum number of iterations is chosen as the stopping criterion. If the stopping criterion is not satisfied, the above procedure is repeated with incremented t value

4. DESIGN EXAMPLES AND COMPARISONS

For the purpose of comparison design conditions taken for designing BP and BS filters are same as [7] and are given in Table 1. The frequency range from 0 to π is divided in to 200 frequency sampling points. In the proposed TLBO approach the combination of four criteria, L_1 -norm approximation error, L_2 -norm approximation error, ripple magnitudes of pass-band and ripple magnitude of stop-band are considered for designing IIR filter. These four criteria are contrary to each other in most situations. The filter designer needs to adjust the weights of criteria to design the filter depending on the filter specifications. For the purpose of comparison the weights w_1, w_2, w_3 and w_4 are set same as in [9] for BP and BS filters respectively. The performance of filter designed with TLBO algorithm are presented and compared with the results obtained by [7], [8] and [9] in Table 2 and Table 3 for BP and BS filters respectively. The obtained magnitude response of BP and BS filters designed with TLBO and [7], [8] and [9] are presented in Figure 1 and Figure 2 for the purpose of comparison. The best optimized numerator coefficients and denominator coefficients obtained by the TLBO approach for BP and BS filters are given by Eq. (18) and Eq. (19) respectively. The obtained results reveal that the purposed approach gives the better performance in terms of magnitude response and ripples in pass band and stop band.

$$H_{BP}(z) = 0.02807718 \left(\frac{(z^2 - 0.1788042z - 0.9907626)(z^2 + 0.152085z - 0.8536902)}{(z^2 - 0.6290708z + 0.7749901)(z^2 + 0.0057888z + 0.5494546)} \right) \times \left(\frac{(z^2 - 0.08027664z - 0.9487353)}{(z^2 + 0.6330091z + 0.7733375)} \right) \quad (18)$$

$$H_{BS}(z) = 0.4394692 \frac{(z^2 + 0.4366674z + 1.000583)(z^2 - 0.436745z + 1.000652)}{(z^2 - 0.815700z + 0.5407802)(z^2 + 0.815624z + 0.540886)} \quad (19)$$

Table 1. Prescribed design conditions for BP and BS filters

Filter type	Pass band	Stop band	Order
BP	$0.4\pi \leq \omega \leq 0.6\pi$	$0 \leq \omega \leq 0.25\pi$ and $0.75 \leq \omega \leq \pi$	6
BS	$0 \leq \omega \leq 0.25\pi$ and $0.75 \leq \omega \leq \pi$	$0.4\pi \leq \omega \leq 0.6\pi$	4

Table 2. Design results for BP filter

Method	L ₁ -norm error	L ₂ -norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
TLBO Approach	1.2762	0.1789	$0.9886 \leq H(e^{j\omega}) \leq 1.005$ (0.0171)	$ H(e^{j\omega}) \leq 0.525$ (0.0525)
TIA Approach [9]	1.6119	0.2191	$0.9806 \leq H(e^{j\omega}) \leq 1.000$ (0.0194)	$ H(e^{j\omega}) \leq 0.0658$ (0.0658)
HTGA Approach [8]	1.9418	0.2350	$0.9760 \leq H(e^{j\omega}) \leq 1.000$ (0.0234)	$ H(e^{j\omega}) \leq 0.0711$ (0.0711)
TIA Approach [7]	5.2165	0.6949	$0.8956 \leq H(e^{j\omega}) \leq 1.000$ (0.1044)	$ H(e^{j\omega}) \leq 0.1772$ (0.1772)

Table 3. Design results for BS filter

Method	L ₁ -norm error	L ₂ -norm error	Pass-band performance (Ripple magnitude)	Stop-band performance (Ripple magnitude)
TLBO Approach	3.6444	0.4619	$0.9692 \leq H(e^{j\omega}) \leq 1.008$ (0.0407)	$ H(e^{j\omega}) \leq 0.1116$ (0.1116)
TIA Approach [9]	4.1275	0.4752	$0.9560 \leq H(e^{j\omega}) \leq 1.000$ (0.0440)	$ H(e^{j\omega}) \leq 0.1171$ (0.1171)
HTGA Approach [8]	4.5504	0.4824	$0.9563 \leq H(e^{j\omega}) \leq 1.000$ (0.0437)	$ H(e^{j\omega}) \leq 0.1013$ (0.1013)
TIA Approach [7]	6.6072	0.7903	$0.8920 \leq H(e^{j\omega}) \leq 1.000$ (0.1080)	$ H(e^{j\omega}) \leq 0.1726$ (0.1726)

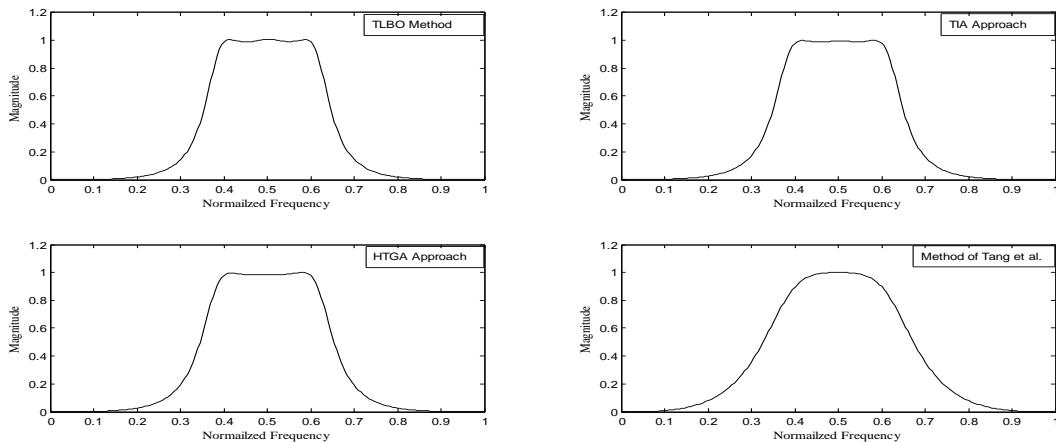


Fig 1: Magnitude responses of BP filter using TLBO approach and the method given in [9], [8] and [7]

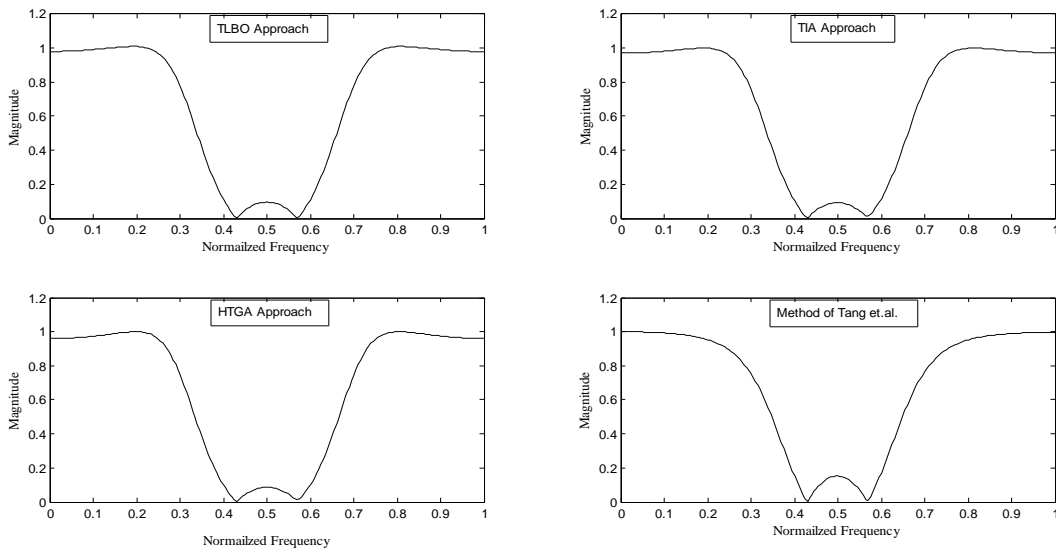


Fig 2: Magnitude responses of BS filter using TLBO approach and the method given in [9], [8] and [7]

5. CONCLUSION

A heuristic optimization algorithm namely TLBO is successfully applied to design BP and BS digital IIR filter. The designed optimal filters obtained by employing TLBO meet the stability criterion and gives better performances in terms of L_p -approximation error for magnitude response and ripples in pass band and stop band in comparison to GA based methods. The main characteristics of the TLBO algorithm over other GA methods are its simplified numerical structure and its independence on a number of parameters to define the algorithm's performance. TLBO is a powerful search and applicable optimization method for the problem of digital filter design problems.

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