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Reliability Measures of a Two-Unit Cold Standby Repairable System with Priority to Operation over Preventive Maintenance

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ABSTRACT
The main objective of present work is to carry out various important reliability measures of a two-unit cold standby repairable system with priority to operation over preventive maintenance by using regenerative points technique and semi-Markov process. A single repair facility is provided to conduct repair and maintenance of the system as and when required. All times distributions are exponential except the repair and preventive maintenance time distributions which are considered as general.

General Terms
Reliability Analysis

Keywords
Cold Standby System, Preventive Maintenance, Priority and Profit Function

1. INTRODUCTION
Malik [1] studied some stochastic models with priority to operation over repair with arrival time of the server and derived several measures of reliability for the system. Malik et al. [2] derived the expressions for economic analysis of repairable systems by using the concept of random appearance and disappearance of the server. In Ref. [1] and Ref. [2] the authors did not take the concepts of preventive maintenance after a maximum operation time and priority to operation over preventive maintenance.

Many papers [3-5] dealt with repairable standby systems subject to different repair policies and preventive maintenance, but they considered that the operating unit may go for preventive maintenance when other unit is also under repair or maintenance. Practically this is not true at all.

The goal of the present study is to discuss a two-unit cold standby repairable system with priority to operation over preventive maintenance. The following measures of reliability for the system are obtained:

- Transition probabilities
- Mean sojourn times
- Mean time to system failure
- Steady State Availability
- Busy period of the server due to repair
- Busy period of the server due to preventive maintenance
- Expected number of repairs
- Expected number of preventive maintenances
- Expected number of visits by the server
- Expected profit earned by the system in (0, t).

2. MODEL DESCRIPTION
(i) Initially system consists of two identical units- one operative and other is kept as cold standby.

(ii) Both units have three modes- normal, under repair due to failure and under preventive maintenance.

(iii) The failure and maximum operation time distribution are exponentially distributed while repair and preventive maintenance times are distributed arbitrarily.

(iv) There is a single server who visits the system immediately as and when required.

(v) The switch devices, repairs and preventive maintenance are perfect.

(vi) All random variables are statistically independent.

3. METHODOLOGY
The system has been analyzed using well known semi-Markov process and regenerative point technique which are briefly described as:

3.1 Markov Process:
If $\{X(t),\ t\in T\}$ is a stochastic process such that, given the value of X(s), the value of X(t), t>s do not depend on the values of X(u), u<s Then the process $\{X(t),\ t\in T\}$ is a Markov process.

3.2 Semi-Markov Process:
A semi-Markov process is a stochastic process in which changes of state occur according to a Markov chain and in which the time interval between two successive transitions is a random variable, whose distribution may depend on the state from which the transition take place as well as on the state to which the next transition take place.

3.3 Regenerative Process:
Regenerative stochastic process was defined by Smith (1955) and has been crucial in the analysis of complex system. In this, we take time points at which the system history prior to the time points is irrelevant to the system conditions. These points are called regenerative points. Let X(t) be the state of the system of epoch. If t_1, t_2,… are the epochs at which the process probabilistically restarts, then these epochs are called regenerative epochs and the process $\{X(t),\ t = t_1, t_2\ldots\ldots\}$ is called regenerative process. The state in which regenerative points occur is known as regenerative state.

4. NOTATIONS

O	:	The unit is operative and in normal mode.
Cs	:	The unit is in cold standby.
Λ	:	Constant failure rate of the unit.
α_0	:	Constant rate of Maximum Operation Time.
m(t)/M(t)	:	pdf / cdf of preventive maintenance of the unit after maximum operation time.
Pm/PM	:	The unit is under preventive Maintenance/ under preventive maintenance continuously from previous state.
FUr/FUR	:	The unit is failed and is under repair / under repair continuously from previous state
FWr / FWR	:	The unit is failed and is waiting for repair/ waiting for repair from previous state
g(t) / G(t)	:	pdf / cdf of repair time of the failed unit
$q_{ij}(t)$/ $Q_{ij}(t)$	:	pdf/cdf of direct transition time from a regenerative state i to a regenerative state j without visiting any other regenerative state.
$q_{ij.k}(t)$ / $Q_{ij.k}(t)$	:	pdf/cdf of first passage time from a regenerative state i to a regenerative state j or to a failed state j visiting state k once in (0,t].
$M_i(t)$	:	Probability that the system is up initially in state $S_i \in E$ is up at time 't' without visiting to any other regenerative state.
pdf / cdf	:	Probability density function/ Cumulative density function
$W_i(t)$	:	Probability that the server is busy in state S_i up to time t without making transition to any other regenerative state or returning to the same via one or more non regenerative states.
m_{ij}	:	Contribution to mean sojourn time in state S_i when system transits directly to state S_j (S_i ,S_j $\in E$) so that $\mu_i = \sum_j m_{ij}$ where $m_{ij} = \int t\, dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$ and μ_i is the mean sojourn time in state $S_i \in E$
μ_i	:	The mean Sojourn time in state S_i this is given by $\mu_i = E(T_i) = \int_0^\infty P(T_i > t)dt = \sum_j m_{ij}$, where T_i is the sojourn time in state S_i.
Ⓢ / ©	:	Symbol for Stieltjes convolution / Laplace convolution
~ / *	:	Symbol for Laplace Stieltjes Transform (LST) / Laplace Transform (LT).

The system may be in one of the following states:

$S_0 = (o, Cs)$, $S_1 = (o, Fur)$, $S_2 = (o, Pm)$, $S_3 = (PM, Fwr)$ and $S_4 = (FUR, Fwr)$

Up states are: S_0 , S_1 and S_2 . Down States are: S_3 and S_4 .Regenerative states: S_0 , S_1 and S_2 .Non-regenerative states: S_3 and S_4 .

5. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt \quad \text{as} \tag{1}$$

$P_{01} = \frac{\gamma_1}{\gamma_1 + \alpha_0}$, $P_{02} = \frac{\alpha_0}{\gamma_1 + \alpha_0}$, $P_{14} = 1 - g^*(\gamma_1)$, $P_{10} = g^*(\gamma_1)$, $P_{23} = 1 - f^*(\gamma_1)$,

$P_{20} = f^*(\gamma_1)$, $P_{31} = 1$, $P_{41} = 1$, $P_{11.4} = [1 - g^*(\gamma_1)]$, $P_{21.3} = 1 - f^*(\gamma_1)$, (2)

It can be easily verified that: $P_{01} + P_{02} = P_{14} + P_{10} = P_{23} + P_{20} = P_{31} = P_{41} = P_{11.4} + P_{21.3} = 1$ (3)

The mean sojourn times (μ_i) of the state S_i are

$$\mu_0 = \frac{1}{\alpha_0 + \gamma_1},\ \mu_1 = \frac{1}{\theta + \gamma_1},\ \mu_2 = \frac{1}{\alpha + \gamma_1},\ \mu_1' = \frac{1}{\theta},\ \mu_2' = \frac{1}{\alpha} \tag{4}$$

6. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE (MTSF)

Let $\varphi_i(t)$ be the cdf of first passage time from the regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for $\varphi_i(t)$;

$$\varphi_i(t) = \sum_j Q_{i,j}(t) \circledR \varphi_j(t) + \sum_k Q_{i,k}(t) \tag{5}$$

Where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is a failed state to which the state S_i can transit directly. Taking LST of above relation (5) and solving for $\tilde{\phi}_0(s)$ .We have

$$R^*(s) = \frac{1 - \tilde{\phi}_0(s)}{s} \tag{6}$$

The reliability of the system model can be obtained by taking inverse Laplace transform of (6).

The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s \to o} \frac{1 - \tilde{\phi}_0(s)}{s} = \frac{N_1}{D_1} \text{ where} \tag{7}$$

$N_1 = p_{01}p_{14} + p_{02}p_{23}$ *and* $D_1 = 1 - p_{01}p_{10} + p_{02}p_{20}$

7. STEADY STATE AVAILABILITY

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state S_i at t = 0. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{i,j}^{(n)}(t) © A_j(t) \quad (8)$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $M_i(t)$ is the probability that the system is up initially in state $S_i \in E$ up at time t without visiting to any other regenerative state, we have is

$$M_0(t) = e^{-(\lambda+\alpha_0)t}, M_1(t) = e^{-(\lambda+\alpha_0)t}\overline{F(t)},$$
$$M_2(t) = e^{-(\lambda+\alpha_0)t}\overline{H(t)}, M_4(t) = e^{-(\lambda+\alpha_0)t}\overline{G(t)}$$
$$M_3(t) = e^{-(\lambda+\alpha_0)t}\overline{M(t)}$$

Taking LT of above relations (8) and solving for $A_0^*(s)$. The steady state availability is given by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}, \text{where}$$

$N_2 =$
$$(\mu_o)[1-p_{11.4}] + (\mu_1)[p_{01} + p_{02}p_{21.3}] + (\mu_2)[p_{02}(1-p_{11.4})]$$

And

$D_2 =$
$$(\mu_0)[1-p_{11.4}] + (\mu_1')[p_{01} + p_{01}p_{21.3}] + (\mu_2')[p_{02}(1-p_{11.4})]$$

8. BUSY PERIOD ANALYSIS OF THE SERVER

8.1. Due to Preventive Maintenance (PM)

Let $B_i^P(t)$ be the probability that the server is busy in preventive maintenance of the system (unit) at an instant 't' given that the system entered state S_i at t = 0. The recursive relations for $B_i^P(t)$ are as follows:

$$B_i^p(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^p(t) \quad (9)$$

Where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to preventive maintenance up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_1(t) = e^{-(\lambda+\alpha_0)t}\overline{F}(t) + (\alpha_0 e^{-(\lambda+\alpha_0)t} © 1)\overline{F}(t) + (\lambda e^{-(\lambda+\alpha_0)t} © 1)\overline{F}(t)$$

8.2. Due to Repair

Let $B_i^R(t)$ be the probability that the server is busy in repairing the unit due to failure at an instant 't' given that the system entered state S_i at t = 0. The recursive relations for $B_i^R(t)$ are as follows:

$$B_i^R(t) = W_i(t) + \sum_j q_{i,j}^{(n)}(t) © B_j^R(t) \quad (10)$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. $W_i(t)$ be the probability that the server is busy in state S_i due to repair of the unit up to time t without making any transition to any other regenerative state or returning to the same via one or more non-regenerative states and so

$$W_2 = e^{-(\lambda+\alpha_0)t}\overline{G}(t) + (\alpha_0 e^{-(\lambda+\alpha_0)t} © 1)\overline{G}(t) + (\lambda e^{-(\lambda+\alpha_0)t} © 1)\overline{G}(t) \quad (9)$$

Taking LT of above relations and solving for B_0^{*P} (s) & B_0^{*R} (s). The time for which server is busy due to preventive maintenance and repair is

$$B_0^P = \lim_{s\to 0} sB_0^{*P}(s) = \frac{N_3^P}{D_2}, \quad B_0^R = \lim_{s\to 0} sB_0^{*R}(s) = \frac{N_3^R}{D_2}$$

Where $N_3^P = W_2^*(0)[p_{02}(1-p_{11.4})$,

$N_3^R = W_1^*(0)[p_{01} + p_{02}p_{21.3}]$ and D_2 is already defined.

9. EXPECTED NUMBER OF PREVENTIVE MAINTENANCES

Let $R_i^p(t)$ be the expected number of preventive maintenances conducted by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $R_i^p(t)$ are given as

$$R_i^p(t) = \sum_j Q_{i,j}^{(n)}(t) ® \left[\delta_j + R_j^p(t)\right] \quad (11)$$

where S_j is any regenerative state to which the given regenerative state S_i transits and δj =1, if S_j is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relations (11) and solving for $\tilde{R}_0^p(s)$. The expected numbers of preventive maintenances per unit time are given by

$$R_0^p(\infty) = \lim_{s\to 0} s\tilde{R}_0^p(s) = \frac{N_4^p}{D_2}$$

Where $N_4^p = [p_{02} - p_{11.4}p_{02}]$ and D_2 is already mentioned.

10. EXPECTED NUMBERS OF REPAIRS

Let $R_i^r(t)$ be the expected number of repairs by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $R_i^r(t)$ are given as

$$R_i^r(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + R_j^r(t)\right] \quad (12)$$

Where S_j is any regenerative state to which the given regenerative state S_i transits and $\delta j = 1$, if S_j is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relations (12) and solving for $\tilde{R}_0^r(s)$. The expected numbers of repairs per unit time are given by

$$R_0^r(\infty) = \lim_{s\to 0} s\tilde{R}_0^r(s) = \frac{N_4^r}{D_2}$$

Where $N_4^r = [p_{01} + p_{02}p_{21.3}]$ and D_2 is already mentioned.

11. EXPECTED NUMBERS OF VISITS BY THE SERVER

Let $N_i(t)$ be the expected number of visits by the server in (0, t] given that the system entered the regenerative state S_i at t = 0. The recursive relations for $N_i(t)$ are given as ;

$$N_i(t) = \sum_j Q_{i,j}^{(n)}(t) \circledR \left[\delta_j + N_i(t)\right] \quad (13)$$

where S_j is any regenerative state to which the given regenerative state S_i transits and δj =1, if j is the regenerative state where the server does job afresh, otherwise δj = 0. Taking LST of relations (13) and solving for $N_0(s)$. The expected numbers of replacements per unit time are given by

$$N_0(\infty) = \lim_{s\to 0} s\tilde{N}_0(s) = \frac{N_5}{D_2} \quad (14)$$

where $N_0 = [1 - p_{11.4}]$ and D_2 is already mentioned.

12. PROFIT ANALYSIS

The profit incurred to the system model in steady state can be obtained as

$$P = K_0A_0 - K_1B_0^R - K_2B_0^P - K_3R_0^R - K_4R_0^P - K_5N_0$$

K0 = Revenue per unit up-time of the system

K1 = Cost per unit time for which server is busy due to repair

K2 = Cost per unit time for which server is busy due to preventive maintenance

K3 = Cost per unit time repair of the unit

K4 = Cost per unit time preventive maintenance of the unit

K5 = Cost per unit time visit by the server

13. CONCLUSION

The numerical results for some reliability measures are obtained in tables 1-3 by considering the particular case $g(t) = \theta e^{-\theta t}$ and $f(t) = \alpha e^{-\alpha t}$. From table 1-2, It is revealed that MTSF, profit and availability decrease with the increase of failure rate (λ_1) and maximum operation time (α_0), but the value of availability and MTSF increase with the increase of repair rate (θ) and preventive maintenance rate (α). The profit of the system declines with the increases of preventive maintenance rate (α). Thus finally it is concluded that a system in which priority to operation is given over preventive maintenance can be made more reliable and profitable to use

(i) By taking one more unit in cold standby.

(ii) By ignoring the concept of priority to operation over preventive maintenance.

(iii) By increasing the repair rate.

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15. NUMERICAL ANALYSIS

λ_1	$\theta = 0.2, \alpha_0 = 5, \alpha = 2$	$\theta = 1.2, \alpha_0 = 5, \alpha = 2$	$\theta = 0.2, \alpha_0 = 5, \alpha = 5$	$\theta = 0.2, \alpha_0 = 10, \alpha = 2$
0.010	139.4449	140.0669	195.6368	119.9094
0.011	126.7218	127.3396	177.4943	109.0012
0.012	116.1199	116.7336	162.3815	99.9112
0.013	107.1496	107.7593	149.5990	92.2198
0.014	99.4613	100.0670	138.6473	85.6272
0.015	92.7986	93.4003	129.1602	79.9138
0.016	86.9692	87.5670	120.8630	74.9146
0.017	81.8260	82.4200	113.5455	70.5037
0.018	77.2547	77.8448	107.0444	66.5829
0.019	73.1649	73.7513	101.2308	63.0750
0.020	69.4844	70.0671	96.0013	59.9179

Table 1: MTSF vs. Failure Rate (λ_1)

λ_1	$\theta = 0.2, \alpha_0 = 5, \alpha = 2$	$\theta = 1.2, \alpha_0 = 5, \alpha = 2$	$\theta = 0.2, \alpha_0 = 5, \alpha = 5$	$\theta = 0.2, \alpha_0 = 10, \alpha = 2$
0.010	0.9943	0.9964	0.9967	0.9937
0.011	0.9935	0.9960	0.9961	0.9928
0.012	0.9926	0.9957	0.9955	0.9920
0.013	0.9918	0.9953	0.9948	0.9910
0.014	0.9908	0.9950	0.9941	0.9901
0.015	0.9899	0.9946	0.9934	0.9891
0.016	0.9889	0.9942	0.9926	0.9881
0.017	0.9879	0.9939	0.9918	0.9870
0.018	0.9869	0.9935	0.9910	0.9859
0.019	0.9858	0.9931	0.9901	0.9848
0.020	0.9847	0.9928	0.9892	0.9836

Table 2: Availability vs. Failure Rate (λ_1)

λ_1	$\theta = 0.2, \alpha_0 = 5, \alpha = 2$	$\theta = 1.2, \alpha_0 = 5, \alpha = 2$	$\theta = 0.2, \alpha_0 = 5, \alpha = 5$	$\theta = 0.2, \alpha_0 = 10, \alpha = 2$
0.010	4.3945	4.4065	4.1417	4.3014
0.011	4.3900	4.4042	4.1396	4.2971
0.012	4.3853	4.4020	4.1374	4.2926
0.013	4.3804	4.3998	4.1349	4.2880
0.014	4.3754	4.3976	4.1323	4.2832
0.015	4.3701	4.3953	4.1295	4.2781
0.016	4.3647	4.3931	4.1264	4.2730
0.017	4.3592	4.3909	4.1232	4.2676
0.018	4.3534	4.3886	4.1198	4.2621
0.019	4.3475	4.3864	4.1163	4.2565
0.020	4.3415	4.3842	4.1125	4.2506

Table 3: Profit vs. Failure Rate (λ_1)