Camera Self-Calibration by an Equilateral Triangle

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ABSTRACT

In this article, we present a technique for self-calibration of a CCD camera with constant focal distance using a planar scene. The particularity of our technique is the use of triangle equilateral which two vertices are defined from the matches detected in images taken by the camera. Using these vertices to estimate all the projection matrices which are operated with homographies between the images to estimate the intrinsic parameters of the camera. Experimental results show the robustness of our algorithms in terms of stability and convergence**.**

Keywords*:* Self-Calibration, Equilatéral Triangle, Absolute Conic , Homography.

1. INTRODUCTION

The calibration of a camera is to determine the intrinsic and extrinsic parameters using a known object called calibration pattern [8, 9, 10, 18]. The grid may be three-dimensional.(calibration 3D) or plane (calibration 2D). This constraint is not always present in the applications of computer vision, which gives rise to new methods called self-calibration which allow to calculate the intrinsic and extrinsic parameters without any prior knowledge on the stage. The latter method uses 3D scenes [1, 2, 3, 21, 22, 23] or planar scenes [4, 5, 6, 7, 9] to calculate the camera settings automatically, but they face many problems, such as the majority of methods encounters the problem of systems to solve non-linear, which must be initialized while adding constraints on the model of self-calibration, for example in [16] a constraint is added to the movement of camera that undergoes a translation and a small rotation, this constraint permits to estimate the homography of the plane at infinity and to calculate parameters of the camera.

In this paper we are interested on the camera self-calibration with fewer constraints by the use of the object of equilateral triangle any scene 2D and movement rigid camera in space. Our technique is to estimate the homography matrix between two images by the RANSAC algorithm [11] based on points of interest detected by Harris [12] which are matched by the correlation function $ZNCC$ [13, 14, 19]. Next, this matrix will be used with two matches (the projections of the two vertices of an equilateral triangle) to estimate the projection matrix and determined the system of equations.

Finally, three images are sufficient to determine the intrinsic parameters of camera by solving a system of nonlinear equation that needs initialization and optimization of a cost function associated to the parameters sought by the bais of 'Levenberg-Marquardt algorithm [15].

The paper is organized as follows: The second part presents the model of camera and equilateral triangle. The projection of the triangle is described in third part. Self-calibration of cameras presented in fourth part. The experiments are presented in the fifth and concluding part in the sixth game.

2. CAMERA AND EQUILATERAL TRIANGLE

A. Model of camera

We consider the pinhole camera model to transform a point in the scene in his image, this model is defined by the perspective projection matrix P of 3×4 given by the following formula : $P=K(R\ t)$ with :

- $(R\ \mathrm{t})$ is the extrinsic parameter matrix, with R is the rotation matrix and t the translation vector of camera in space.
- K is the intrinsic parameter matrix defined by:

$$K=\begin{pmatrix} f & \tau=0 & u_0 \\ 0 & \varepsilon f & v_0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (1)$$

f is the focal length, ε is the scale factors, (u_0, v_0) are the image coordinates of the principal point ant τ is the obliquity factor.

B. Triangle équilatéral

For any two points A and B in the plane of the stage, there is a point C so that ABC is an equilateral triangle set by the length of its side (Figure 1).

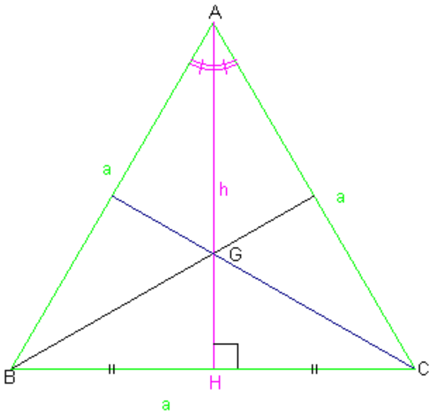

Figure 1. Equilateral triangle

a few properties of the equilateral triangle:

- The three heights (median, bisector and mediator) are the same measure. They are simultaneously equal $a\dfrac{\sqrt{3}}{2}$ with a is length of one side of an equilateral triangle.
- The three angles are the same as: $60°$.

3. EQUILATERAL TRIANGLE PROJECTION

A. *Interest points*

Harris Detector [12, 17]: To extract the high points in terms of information, we used the Harris detector is based on the following function:

$$E(x, y) = (x.y).M(x, y)^t, \qquad (2)$$

with :

(x, y) : The coordinates of pixel in question.

$$M = \begin{pmatrix} A & C \\ C & B \end{pmatrix} \qquad A = \frac{\partial^2 I}{\partial x^2} \otimes w$$

$$B = \frac{\partial^2 I}{\partial y^2} \otimes w \qquad C = \left(\frac{\partial^2 I}{\partial x \partial y}\right) \otimes w$$

$$w(x, y) = \frac{1}{2\pi\sigma^2}\ e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

The matrix M characterizes the local behavior of the function E. To detect points of interest, we evaluated the following measure:

$$\begin{cases} R = Det(M) - k \times Trace^2(M) \\ \text{with : } Det(M) = AB - C^2 \\ Trace(M) = A + B \text{ and } k = 0.04 \end{cases} \qquad (3)$$

- $R < 0$: at the vicinity of an edge
- $R = 0$: in a homogeneous region
- $R > seuil$: near a point of interest

B. *Correlation Measure*

We use the correlation measure $ZNCC(Zero\ mean\ Normalised\ Cross\ Correlation)$ to match the bridges of interest detected by the Harris algorithm, this measure is characterized by the invariance to changes in local luminance linear and defined by the following formula :

$$ZNCC(q_i, q_j) = \frac{\sum_n x_n y_n}{\sqrt{\sum_n x_n^2 \sum_n y_n^2}} \qquad (4)$$

with :

q_i and q_j two points of Harris detected in the two images i and j.

$$x_n = I(q_i + n) - \bar{I}(q_i)$$

$$y_n = I'(q_j + n) - \bar{I}'(q_j)$$

$\bar{I}(q_i)$ and $\bar{I}'(q_j)$ are means of pixel luminance on a window centered respectively in q_i and q_j.

C. *Homography between images*

RANSAC [11] : is an algorithm that estimates geometric entities (homography between images in our situation) from a dataset (matched points), whose error with respect to the entity is found above a threshold .

Homography btween images : is a $3{\times}3$ matrix transformation linking two points belonging to image i and j by the following equation:

$$q_j : H_{ij} q_i \qquad (5)$$

with q_i and q_j are respectively the projections of the same point in the scene in the picture i and j.

H_{ij} is the homography matrix between two images i and j, estimated by the knowledge of at least four matches by applying the RANSAC algorithm.

D. *Projection matrix of the equilateral triangle*

Projection Matrixes : to estimate the projection matrix we will use two references : reference Affine (B, X_a, Y_a, Z_a) and reference Euclidean (B, X_e, Y_e, Z_e) with Z_e and Z_a are perpendicular to the plane of the triangle ABC.

Table 1 presents the coordinates of the vertices of an equilateral triangle in two references Affine and Euclidean.

Table 1: Homogeneous Coordinates Of Vertices Of The Triangle In The Two References Affine And Euclidean

Point	Affine plan	Euclidean plan
A	$Q_1 = (0,\ 1,\ 1)^T$	$Q'_1 = (\frac{a}{2},\ \frac{\sqrt{3}}{2}a,\ 1)^T$
B	$Q_2 = (0,\ 0,\ 1)^T$	$Q'_2 = (0,\ 0,\ 1)^T$
C	$Q_3 = (1,\ 0,\ 1)^T$	$Q'_3 = (a,\ 0,\ 1)^T$

The triangle ABC, expressed in the Affine plan, is projected in the image i by a matrix L_i (Figure 2) as:

$$(u_{id},\ \mathrm{v}_{id},\ 1)^T \ :\ L_i Q_d \tag{6}$$

With $1 \le d \le 3$, $q_{id} = (u_{id}, v_{id})^T$ is a point in the 'image i represents the projection of a vertices of the equilateral triangle and L_i a 3×3 matrix that can be defined, up to a scale factor, by:

$$L_i \ :\ KR_i \begin{pmatrix} 1 & 0 & \\ 0 & 1 & R_i^T t_i \\ 0 & 0 & \end{pmatrix} S \tag{7}$$

R_i, t_i represent, respectively, orientation and position of camera in space to project the scene in the image i and $S = \begin{pmatrix} a & \frac{a}{2} & 0 \\ 0 & \frac{\sqrt{3}}{2}a & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the passage matrix, between the reference Affine and Euclidean, of vertices of the triangle as:

$$Q'_d = S\ Q_d \tag{8}$$

In practice there is not an automatic method to determine the triangle in the image i. But we can always determine which two vertices of the triangle can be deduced four linear equations from the equation (6). herefore we need other equations to calculate matrix L_i.

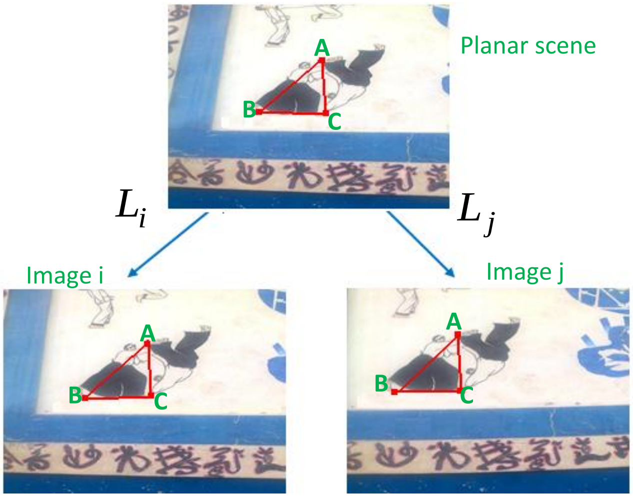

Figure 2. Projection of triangle ABC in the two images i and j by the two matrixes L_i and L_j

The matrix $H_i \ :\ KR_i \begin{pmatrix} 1 & 0 & \\ 0 & 1 & R_i^T t_i \\ 0 & 0 & \end{pmatrix}$ is the homography that permits to project the plan of the scene in the image i, therefor formul (7) becomes:

$$L_i \ :\ H_i S \tag{9}$$

And for the image j we can write:

$$L_j \ :\ H_j S \tag{10}$$

From equations (9) and (10) we deduce that:

$$L_j \ :\ H_{ij} L_i \tag{11}$$

with H_{ij} is the homography betwin image i and j as:

$$H_{ij} \ :\ H_j H_i^{-1} \tag{12}$$

The matrix H_{ij} is determined by the method explained in the part C. in the two images i and j the projections of the two vertices are given by:

$$(u_{id},\ \mathrm{v}_{id},\ 1)^T \ :\ L_i Q_d \tag{13}$$

$$(u_{jd},\ \mathrm{v}_{jd},\ 1)^T \ :\ L_j Q_d \tag{14}$$

with $1 \le d \le 3$. We devlop the two relations (11) and (14) we find that:

$$(u_{jd},\ \mathrm{v}_{jd},\ 1)^T \ :\ H_{ij}\ L_i Q_d \tag{15}$$

Equations (13) and (15) are sufficient to determine the matrix L_i, for this we use a method of singular value decomposition. the L_j matrix is determined by equation (11).

4. CAMERA SELF-CALIBRATION

A. Self-calibration equations

After the development of the Equation (7) we obtain:

$$K^{-1}L_i : R_i\left(S^{'} R_i^T t_i\right) \quad (16)$$

with $S' = \begin{pmatrix} a & \frac{a}{2} \\ 0 & \frac{\sqrt{3}}{2}a \\ 0 & 0 \end{pmatrix}$. the matrix R_i is orthogonal $(R_i^T R_i = I_3)$, So the relation (16) can be written as follows:

$$L_i^T \omega L_i : \begin{pmatrix} S'^T S' & S'^T R_i^T t_i \\ t_i^T R_i S' & t_i^T t_i \end{pmatrix} \quad (17)$$

with $\omega = \left(KK^T\right)^{-1}$ is the projection of the absolute conic $\Omega : I_3$ in all images.

We note by $M_i = \begin{pmatrix} z_{1i} & z_{3i} \\ z_{3i} & z_{2i} \end{pmatrix}$ the matrix contains the first two lines and columns of the matrix $L_i^T \omega L_i$, therfor from equation (17) we find that:

$$M_i : S'^T S^{'} \quad (18)$$

This formula expresses the relation between the intrinsic camera parameters and those of a triangle. Therefore for two images i and j, relation (18) can be written :

$$M_i : M_j \quad (19)$$

From the relation (19) we deduce the following equalities between image i and j :

$$\frac{z_{1i}}{z_{2i}} = \frac{z_{1j}}{z_{2j}}, \quad \frac{z_{3i}}{z_{1i}} = \frac{z_{3j}}{z_{1j}}, \quad \frac{z_{3i}}{z_{2i}} = \frac{z_{3j}}{z_{2j}} \quad (20)$$

These equations (20) brought us to a system of three nonlinear equations:

$$\begin{cases} z_{1i}z_{2j} - z_{2i}z_{1j} = 0 \\ z_{3i}z_{1j} - z_{1i}z_{3j} = 0 \\ z_{3i}z_{2j} - z_{2i}z_{3j} = 0 \end{cases} \quad (21)$$

Cost function : To solve the system (21). We minimize the following cost function:

$$\min_{\omega} \sum_{j=i+1}^{n} \sum_{i=1}^{n-1} \left(\delta_{ij}^2 + 1_{ij}^2 + \partial_{ij}^2\right) \quad (22)$$

with

$$\delta_{ij}^2 = z_{1i}z_{2j} - z_{2i}z_{1j}$$
$$1_{ij}^2 = z_{3i}z_{1j} - z_{1i}z_{3j}$$
$$\partial_{ij}^2 = z_{3i}z_{2j} - z_{2i}z_{3j}$$

n represents the number of images used. To solve the function (22) using the Levenberg-Marquardt algorithm [15] which requires a very important step initialization.

Initialization : to initialize the function (22), we assume that certain conditions are satisfied on the vision system.

- The principal point is to the center of the image, therefore u_0 and v_0 are known.
- Pixels are squared therefore $\varepsilon = 1$.
- by replacing these data in the system of equations (21), we find the following system between image i and j :

$$\Sigma \ \Gamma = \Psi \quad (23)$$

with $\Gamma = \left(f^4 \ f^2\right)^T$, Σ is a $3n \times 2$ matrix and Ψ is a $3n$ vector such as Σ and Ψ elements are expressed in function of u_0, v_0, ε, L_i et L_j.

5. EXPERIMENTATIONS

To experiment with this technique and demonstrate its effectiveness, we took three images of 512 × 512 of two 2D scenes unknown by an camera whose intrinsic parameters are kept constant (Figure 3).

Image 1 Image 2 Image 3

Figure 3.The three images of a scene unknown 2D used for self-calibration of the camera

To detect the rich points in terms of information, we used the Harris algorithm that eliminates the noise by a Gaussian filter and gives better results deal with the transformations of the image related to the rotation, scaling, change of views, change brightness and noise related to the sensor [20, 16, 17]. the Harris points are shown in figure 4:

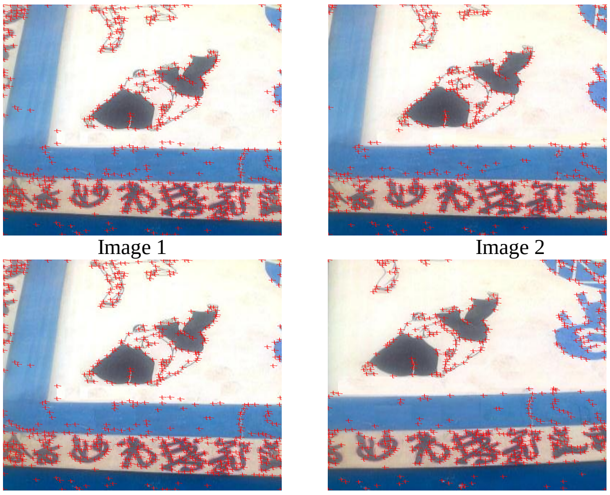

Image 1 Image 2

Image 1 Image 3

Figure 4. The Harris points

For matching points detected by Harris in the three images, we used ZNCC correlation measure which is invariant to linear changes in local luminance, these are presented in Figure 5:

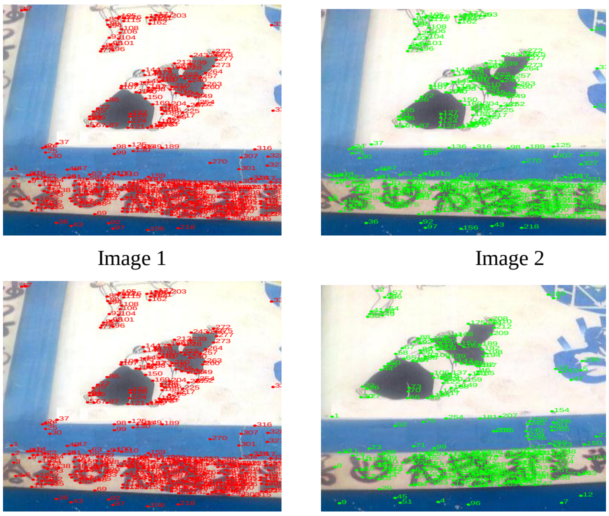

Image 1 Image 2

Image 1 Image 3

Figure 5. The Matched points

The homographies between images and projections matrixes of a triangle (define by two vertices from interest points, the third vertices is not used) in all images are calculated, in short the resolution of a non-linear equation system permits to estimate elements of the image of the absolute conic and to calculate the intrinsic camera parameters.

Table 2 presents the initial result and optimal intrinsic parameters of the camera of two scenes:

Table 2: Initial And Optimal Solution Of The Intrinsic Camera Parameters.

	f	ε	u_0	v_0
Initial solution	*1125*	*1*	*256*	*256*
Optimal solution	*1175*	*0.92*	*261*	*263*

6. CONCLUSION

In these papers, we treated the problem of camera self-calibration plan, using only two points of interest, which is the projection of the vertices of a triangle in each image. These points will be used to estimate the homography matrix between images and the projection matrix for each pair of image, leading to the end to a system of nonlinear equations for determining the intrinsic parameters of camera. Our technique therefore allows the camera self-calibration with a 2D scene known, with a simple, reliable and robust compared to other methods.

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