Unit Commitment in Composite Generation & Transmission Systems using Dynamic Programming

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ABSTRACT
In this paper, a unit commitment problem is being described & its solution using dynamic programming for 5 unit system over 24 hour time horizon is being presented. This also means that it is desirable to find the optimal generating unit commitment (UC) in the power system for the next H hours. The main objective of this paper is to reduce the total production cost includes fuel cost, maintenance cost etc. The 3 versions of DP are presented and their results are compared.

General Terms
Conventional DP, Sequential DP, Truncation DP.

Keywords
Unit commitment, Dynamic programming, Start-up cost, Economic dispatch.

1. INTRODUCTION
Unit Commitment (UC) is a non-linear, mixed integer combinatorial optimization problem in which the number of generators is being scheduled satisfying number of load and other equality and inequality constraints such that the total system production cost over the scheduled time horizon is minimized i.e. unit commitment is the turning on and turning off the generators over the time horizon. Since the human activities follows the cycle of electric power consumption which creates the problem for generating system because it's too expensive to run so many generating units during the off peak hours i.e. during the load is low. These cycles can be daily, weekly and seasonal. Generally the time horizon is taken to be 24 hours. The great amount of money can be saved by turning units off when it is not needed.

The global optimal solution can be obtained by complete enumeration under dynamic programming, which is not applicable to large power systems due to its excessive computational time requirements [1]. Hence, the UC problem is quite difficult due to its inherent high-dimensional, non-convex, discrete and non-linear nature. The UC problem can be considered as two linked optimization problems, namely the unit-scheduled problem, which is a combinatorial optimization problem and the economic dispatch (ED) problem, which is a non-linear programming optimization problem [2]. There are many UC methods such as the dynamic programming which is introduced in this paper, lagrangian relaxation [3], priority list method, branch and bound method [4] and mixed integer linear programming (MILP) [5].

In this paper, we will illustrate the components of UC and introduce the solution of UC problem using Conventional dynamic programming, Sequential combination (SC)-DP and Truncation Combination (TC)-DP for 5 unit system over 24 hours time interval.

The organization of the paper is as follows: Section II introduces the UC formulation. Section III shows the method to solve UC based on DP. In Section IV, Simulation results on a 5-unit power generation system is presented and the results are compared and Section V concludes the paper.

2. UC PROBLEM FORMULATION
The objective of UC problem is to minimize the production cost over the scheduled time horizon (e.g., 24 h) under the generator operational and spinning reserve constraints.

Mathematically, the objective function to be minimized is

$$F(P_i^t, U_{i,t}) = \sum_{t=1}^{T}\sum_{i=1}^{N}\left[F_i(P_i^t) + ST_{i,t}(1 - U_{i,t-1})\right]U_{i,t} \qquad (1)$$

subject to following constraints

(a) power balance constraint

$$P_{load}^t - \sum_{i=1}^{N} P_i^t U_{i,t} = 0 \qquad (2)$$

(b) spinning reserve constraint

$$P_{load}^t + R^t - \sum_{i=1}^{N} P_{i,max} U_{i,t} \leq 0 \qquad (3)$$

(c) generation limit constraints

$$P_{i,min} U_{i,t} \leq P_i^t \leq P_{i,max} U_{i,t}, \qquad i = 1,2,\ldots N \qquad (4)$$

(d) start-up cost

$$ST_{i,t} = \begin{cases} HST_i, & \text{if } T_{i,down} \leq T_{i,off} \leq T_{i,cold} + T_{i,down}, \\ CST_i, & \text{if } T_{i,off} > T_{i,cold} + T_{i,down}, \end{cases} \qquad (5)$$

where,
$F_i(P_i^t)$ – Fuel cost function of the ith unit with generation output P_i^t, at hour t. Usually, it is aquadratic polynomial with coefficients a_i, b_i and c_i as follows:

$$F_i(P_i^t) = a_i + b_i P_i^t + c_i (P_i^t)^2$$

N – Number of units

T – Number of hours

P_i^t – The generation output of the i-th unit at hour t

ST_i – Start-up cost of i-th unit

$U_{i,t}$ – The on/off status of thei-th unit at hour t, and $U_{i,t} = 0$ when off, $U_{i,t} = 1$ when on.

P_{load}^t - load demand at hour t (in MW)

R^t - spinning reserve at hour t (in MW)

$P_{i,max}$ - Maximum real power generation of unit i (in MW)

$P_{i,min}$ - Minimum real power generation of unit i (in MW)

HST_i - Hot start-up cost of unit i (in dollars)

CST_i - Cold start-up cost of unit i (in dollars)

$T_{i,down}$ - Minimum down time of unit i (in hours)

$T_{i,off}$ - Continuously off time of unit i (in hours)

$T_{i,cold}$ - Cold start hours of unit i (in hours)

3. DYNAMIC PROGRAMING APPROACH

First, Dynamic programming is a methodical procedure which systematically evaluates a large number of possible decisions in a multi-step problem. When we utilize the existing conventional dynamic programming method, although its solution is correct and has the optimal value, it takes a lot of memory and spends much time in getting an optimal solution [6]-[10]. For example, assume that there are 4 units which can supply the 24 hour load. So, the total maximum path to satisfy the 24 hour load curve is calculated by:

$$\text{Total Paths} = (2^4 - 1)^{24}$$

Because of this disadvantage, the SC-DP and TC-DP is used to solve the UC problem. The chief advantage of these two methods is the reduction of dimensionality of the problem. Also the calculation of production cost lies near the optimal solution. In SC-DP, the strict priority order of units is imposed. For example, assume the same 4 units, there would be only four combinations to try:

Priority 1 Unit
Priority 1 Unit + Priority 2 Unit
Priority 1 Unit + Priority 2 Unit + Priority 3 Unit
Priority 1 Unit + Priority 2 Unit + Priority 3 Unit + Priority 4 Unit

In TC-DP fixed number of units is allowed to run to satisfy the load demand for each hour.

Recursive algorithm to compute the minimum cost in Kth hour with Ith Combination is,

$$F_{cost}(J,K) = \min_{\{L\}}[P_{cost}(J,K) + S_{cost}(J-1,L:J,K) + F_{cost}(J-1,L)] \quad (6)$$

where,

$F_{cost}(J,K)$ – Least total cost to arrive at state (J, K)

$P_{cost}(J,K)$ – Production cost for state (J, K)

$S_{cost}(J-1,L:J,K)$ – Transition cost from state (J-1, L) to State (J, K)

State (J, K) – Kth Combination in Jth hour

4. SIMULATION RESULTS

The results for DP are presented and tested on 5 unit system with a 24-hour time horizon. The program was written in MATLAB. The input data for 5 unit system and load demands for 24 hours are shown in Tables I and Table II respectively. In this section, a 5-generator, 24-hour unit commitment schedule is determined with the help of Conventional DP, SC-DP and TC-DP and their results consist of production cost and CPU time are compared.

Table 1: Data for 5-Unit System [11]

Parameter	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5
Pmax (MW)	455	130	130	80	55
Pmin (MW)	150	20	20	20	10
a ($/h)	1000	700	680	370	660
b ($/MWh)	16.19	16.6	16.5	22.26	25.92
c ($/MW2-h)	0.00048	0.002	0.00211	0.00712	0.00413
min up (h)	8	6	6	4	1
min down (h)	8	6	6	4	1
hot start cost ($)	4500	550	560	170	30
cold start cost ($)	9000	1100	1120	340	60
cold start hours (h)	5	4	4	2	0
initial status (h)	8	-6	-6	-4	-1

Table 2: Load Demand for 24-hour [11]

Hour	P_{load}	Hour	P_{load}	Hour	P_{load}	Hour	P_{load}
1	330	**7**	730	**13**	810	**19**	790
2	450	**8**	780	**14**	820	**20**	750
3	480	**9**	620	**15**	750	**21**	770
4	360	**10**	650	**16**	800	**22**	610
5	520	**11**	680	**17**	650	**23**	520
6	590	**12**	630	**18**	670	**24**	360

4.1 Results for Conventional DP (Complete Enumeration)

In this, Table III shows the UC schedule of 5 units over a 24 hours period. Table IV shows the production cost of generators for each hour(P_{cost}). Fig. 1 shows the trajectory for best optimum production cost in each hour.

Table 3: Unit Commitment Schedule for Conventional DP

	hours →											
Unit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	0	1	1	1	1	1	1	1
3	0	0	0	0	1	1	1	1	1	1	1	1
4	0	0	1	0	0	0	0	1	0	0	0	0
5	0	0	0	0	0	0	1	0	0	0	0	0

	hours →											
Unit	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	0	0
3	1	1	1	1	1	1	1	1	1	1	1	0
4	1	1	1	1	0	0	1	1	1	0	0	0
5	1	1	0	1	0	0	0	0	0	0	0	0

Table 4: Production Cost for each hour

Hours	**1**	**2**	**3**	**4**	**5**	**6**
P_{cost}	6395.00	8382.70	9396.80	6890.60	10227.00	12098.00
Hours	**7**	**8**	**9**	**10**	**11**	**12**
P_{cost}	15268.00	16165.00	12604.00	13112.00	13621.00	12773.00
Hours	**13**	**14**	**15**	**16**	**17**	**18**
P_{cost}	17464.00	17725.00	15667.00	17217.00	13112.00	13451.00
Hours	**19**	**20**	**21**	**22**	**23**	**24**
P_{cost}	16332.00	15667.00	15999.00	12435.00	10227.00	6890.60

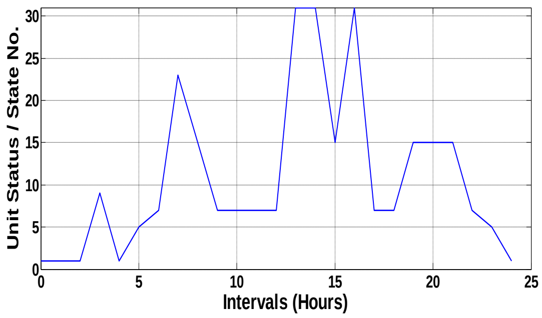

Fig 1: UC Trajectory for 5 Unit Over 24 Hour Period

4.2 Results for SC-DP

In this, the strict priority order of generating units is followed. The strict priority order of units is decided according to the full load average production cost of each unit. Table V shows the strict priority order of 5 units based on the full load average production cost. Table VI shows the UC schedule of 5 units over a 24 hours period and Table VII shows the production cost of generators for each hour (P_{cost}). Fig. 2 shows the trajectory for best optimum production cost in each hour.

Table 5: Priority Unit list

Units	**1**	**2**	**3**	**4**	**5**
FLAPC	18.39	21.99	21.73	26.89	37.92

Table 6: UC Schedule for SC-DP

	hours →											
Unit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	1	1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	1	1	1	1	1	1	1
4	0	0	0	0	0	0	1	1	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0

	hours →											
Unit	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1	1	0	0
4	1	1	1	1	0	0	1	1	1	0	0	0
5	1	1	0	1	0	0	0	0	0	0	0	0

Table 7: Production Cost for each hour

Hour	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
P_{cost}	6395.00	8382.70	9559.60	7570.9	10227.00	12098.00	15336.00	16165.00
Hour	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
P_{cost}	12604.00	13112.00	13621.00	12773.00	17464.00	17725.00	15667.00	17217.00
Hour	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
P_{cost}	13112.00	13451.00	16332.00	15667.00	15999.00	12435.00	10227.00	6890.60

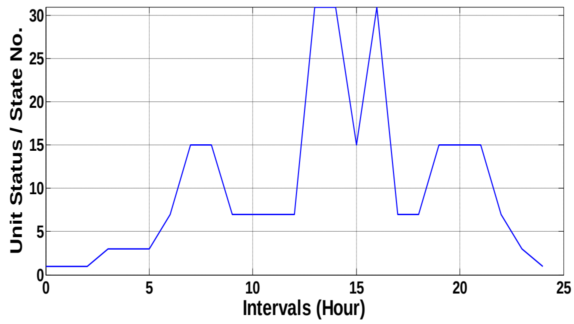

Fig 2: UC Trajectory for 5 Unit Over 24 Hour Period

4.3 Results for TC-DP

In this also the strict priority order is imposed and based on this fixed number of schedulable units are selected to satisfy the load demand for each hour.

Number of Units considered = 5

Table 8: UC Schedule for TC-DP

	Hours →											
Unit	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	0	0	1	1	1	1	1	1	1	1
3	0	0	0	0	0	1	1	1	1	1	1	1
4	0	0	1	0	0	0	0	1	0	0	0	0
5	0	0	0	0	0	0	1	0	0	0	0	0

	Hours →											
Unit	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	0
3	1	1	1	1	1	1	1	1	1	1	0	0
4	1	1	1	1	0	0	1	1	1	0	0	0
5	1	1	0	1	0	0	0	0	0	0	0	0

Table 9: Production Cost for each hour

Hours	**1**	**2**	**3**	**4**	**5**	**6**
P_{cost}	6395.00	8382.70	9396.80	6890.6	10227.00	12098.00
Hours	**7**	**8**	**9**	**10**	**11**	**12**
P_{cost}	15268.00	16165.00	12604.00	13112.00	13621.00	12773.00
Hours	**13**	**14**	**15**	**16**	**17**	**18**
P_{cost}	17464.00	17725.00	15667.00	17217.00	13112.00	13451.00
Hours	**19**	**20**	**21**	**22**	**23**	**24**
P_{cost}	16332.00	15667.00	15999.00	12435.00	10227.00	6890.60

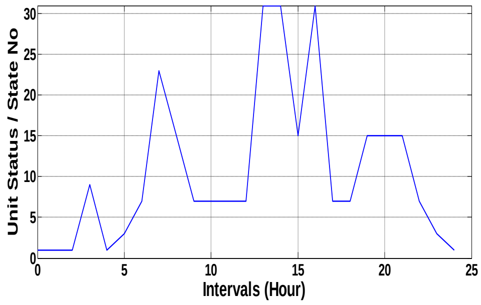

Fig 3: UC Trajectory for 5 Unit Over 24 Hour Period

The optimal results are obtained using conventional DP but the computational time taken is more than that of SC-DP and TC-DP. The comparison of production cost and CPU time between these methods are shown in Table X. Total Production cost for all versions of DP is less expensive than that of others and hence DP converges to the optimal solution. The Production cost for Conventional DP and TC-DP are same because in TC-DP all the 5 units are considered because to satisfy the maximum load during the load interval it is necessary to consider all 5 units.

Table 10: Results for different methods for 5-Unit System over 24 hour time period

Methods	**Overall Production Cost (in $)**	**CPU Time (in Sec)**
Conventional DP	312,880.00	12
SC-DP	313,390.00	6
TC-DP	312,880.00	12

5. CONCLUSION

There are a lot of methods for solving the Unit Commitment problem. Their advantages and disadvantages are studied and described. One of the main problems is that they do not get the optimal solution for performing the Unit Commitment. Therefore, we considered dynamic programming to get an optimal solution despite being impossible to utilize in a large scale power system. This paper presents the three versions of DP to solve UC problem. Easy implementation is main attractive feature of all versions of DP.

6. REFERENCES

[1] Wood, A. J., and Wollenberg, B. F. 2007. Power generation, operation & control. 2nd ed. New York: John Wiley & Sons. pp. 139.
[2] Pang, C. K., and Chen, H. C. 1976. Optimal short-term thermal unit commitment. IEEE Transaction on Power Apparatus and Systems. Vol. 95. No. 4. pp. 1336-1341.
[3] Virmani, S., Imhof, K., and Jee, S. M. 1989. Implementation of a Lagrangian relaxation based unit commitment problem. IEEE Transaction on Power System. Vol. 4. pp. 1373–1379.
[4] Cohen, A. I., and Yoshimura, M. 1983. A Branch-and-Bound Algorithm for Unit Commitment. IEEE Transaction on Power Apparatus and Systems. Vol. 102, no. 2. pp. 444-451.
[5] Williams, A. C. 1989. Marginal Values in Mixed Integer Linear Programming. Mathematical Programming 44. pp. 67-75., North-Holland Publishing Company.
[6] Snyder Jr., W. L., Powell Jr., H. D., and Rayburn, J. C. 1987. Dynamic Programming Approach to Unit Commitment. IEEE Transactions on Power Systems. Vol. PWRS-2., No. 2. pp. 339-350.
[7] Pang, K., Sheble, G. B., and Albuyeh, F. 1981. Evaluation of Dynamic Programming Based Methods and Multiple Area Representation for Thermal Unit Commitments. IEEE Transactions on Power Apparatus and Systems. Vol. PAS-100. No. 3. pp. 1212-1218.
[8] Ouyang, Z., and Shahidehpour, S. M. 1990. An Intelligent Dynamic Programming for Unit Commitment Applications. IEEE Transactions on Power Systems. Paper # 90 SM 468-9 PWRS.
[9] Momoh, J. A., and Zhang, Yi. 2005. Unit Commitment using Adaptive dynamic programming.
[10] Park, J. H., Kim, S. K., Park, G. P., Yoon Y. T., and Lee, S. S. 2010. Modified Dynamic Programming Based Unit Commitment Technique.
[11] Kazarlis, S. A., Bakirtzis, A. G., and Petridis, V. 1996. A genetic algorithm solution to the unit commitment problem. IEEE Transaction on Power System. vol. 11. pp. 83–92.