

A ZCMT Pre-coding based PTS Technique for PAPR Reduction in OFDM System

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ABSTRACT

Orthogonal frequency division multiplexing (OFDM) has been considered as a promising technique to achieve high data rate transmission in mobile environment. However, they are very sensitive to nonlinear effects due to high peak to average power ratio (PAPR). This paper proposes an efficient technique for reducing the PAPR of OFDM signals. The reduction in PAPR of the OFDM signal is obtained through Zadoff-Chu matrix transform (ZCMT) precoding based partial transmit sequence (PTS). The obtained results show that this precoding scheme is an attractive solution of the PAPR problem of OFDM signals.

Keywords

Peak to average power ratio (PAPR), Zadoff-Chu matrix transform (ZCMT), Orthogonal frequency division multiplexing (OFDM), PTS-OFDM.

1. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is a very attractive technique for high bit rate transmission due to its various advantages such as high spectral efficiency, robustness to channel fading, immunity to impulse interference, and capability of handling very strong multi-path fading and frequency selective fading without having to provide powerful channel equalization [1].

Besides a lot of advantages, a major obstacle in OFDM system is that the multiplex signal exhibits a very high peak-to-average power ratio (PAPR). Therefore, nonlinearities may get overloaded by high signal peaks, causing inter-modulation among subcarriers and undesired out-of-band radiation. If RF power amplifiers are operated without large power back-offs, it is impossible to keep the out-of-band power below specified limits and leads to inefficient amplification and expensive transmitters. So, it is highly desirable to reduce the PAPR [2]. Several PAPR reduction techniques have

shaping [3], phase optimization [4], nonlinear commanding transforms [5], Tone Reservation (TR) and Tone Injection (TI) [6]-[7], clipping and filtering [8], Partial Transmit Sequence (PTS) [9], Precoding based techniques [10] and Precoding based Selected Mapping (PSLM) [11] are popular. According to [15] the precoding based techniques show great promise as they are simple linear techniques to implement without the need of any side information. Also, the precoding based techniques take advantage of frequency variations of the communication channel and offers substantial performance gain in fading multipath channels. These techniques can reduce the PAPR up to the PAPR of single carrier systems [15]. Walsh-Hadamard transform (WHT) precoding based techniques, discrete cosine transform (DCT) precoding based techniques, Discrete Hartley Transform (DHT) precoding based techniques are common examples of precoding based PAPR reduction techniques.

This paper presents a new PAPR reduction technique, namely Zadoff-Chu matrix transform (ZCMT) precoding based partial transmit sequence (PTS) technique for PAPR reduction in OFDM systems. Zadoff-Chu sequences are the class of polyphase sequences having optimum correlation properties. These sequences have an ideal periodic autocorrelation, constant magnitude and circular auto-orthogonality.

2. OFDM SYSTEM WITH CONVENTIONAL PARTIAL TRANSMIT SEQUENCE

2.1 OFDM systems and PAPR definition

In an OFDM system with N subcarriers, the discrete-time transmitted signal is represented as:

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j 2\pi nk / N}, k = 0, 1, 2, \dots, N-1 \quad (1)$$

Where $j = \sqrt{-1}$, X_n are the input symbols modulated by QPSK such that $X_n = j^j$, $j = 0, 1, 2, 3$

The PAPR of the transmitted signal in (1) is defined as the ratio of the maximum to the average power, can be expressed by (2)

$$PAPR = 10 \log_{10} \left(\frac{\max_k |x_k|^2}{E \sum_k |x_k|^2} \right) \quad (dB) \quad (2)$$

where $E[\cdot]$ denotes expectation operation.

2.2 OFDM with conventional PTS

The objective of the PTS scheme is to design a rotation phase vector α that minimizes the PAPR. The functional block diagram of an OFDM system with conventional PTS scheme is shown in Fig.1 as that in [12]. The data block X is partitioned into U disjoint sub blocks X_u , where $u = 1, 2, \dots, U$ such that

$$X = \begin{bmatrix} X_1 \\ \vdots \\ X_U \end{bmatrix} \quad (3)$$

Here, it is assumed that the sub blocks X_u consist of a set of subcarriers of equal size. The partitioned sub-blocks are converted from the frequency domain to the time domain using N point IFFT. After taking IFFT, the representation of the block in the time domain is given by

$$x_u = \text{IFFT}(X_u) = x_u \quad (4)$$

The goal of the PTS is to form a weighted combination

of the U time-domain partial sequences x_u by a vector b b_U to rotation $b_1 b_2$ minimize the PAPR, which is given by

$$x' = \begin{bmatrix} b_1 x_1 \\ \vdots \\ b_U x_U \end{bmatrix} \quad (5)$$

To minimize the peak power of x' , each partial sequence x_u should be properly rotated. Letting $b_u = e^{j\theta_u}$

$$\theta_u \in [0, 2\pi]$$

where θ_u can be chosen freely. Now equation (5) can be expressed as

$$x'_u = e^{j\theta_u} x_u \quad (6)$$

where θ_u is the rotation phase vector. PAPR reduction with the

PTS technique is related to the problem of minimizing $\max(x')$ subject to $0 \leq \theta_u \leq 2\pi, u = 1, 2, \dots, U$, but it requires an exhaustive search and an enormous amount of computations to search all possible rotation phase vectors.

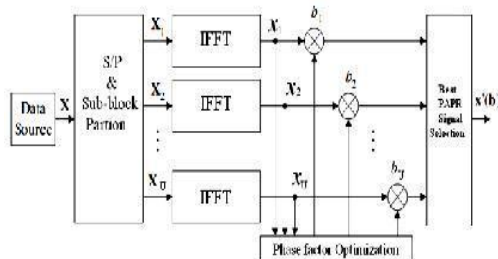


Fig 1: Block diagram of conventional PTS Technique

3. ZADOFF-CHU SEQUENCE AND PTS-OFDM WITH ZCMT PRECODING

3.1 Zadoff-Chu Sequence

Zadoff-Chu Sequences have the optimum correlation properties. These sequences have an ideal periodic autocorrelation, constant magnitude and circular auto-orthogonality. Due to the constant envelope feature of the Zadoff-Chu sequences, the peak-to-average power (PAPR) problem occurred in orthogonal frequency division multiplexing (OFDM) systems can easily be reduced [11]. According to [13] Zadoff-Chu sequences of

length N can be defined as follows:

$$z_n = e^{j2\pi r k \frac{n^2}{N}} \quad \text{for } N \text{ even} \quad (7)$$

$$z_n = e^{j2\pi r k \frac{n^2 + n}{N}} \quad \text{for } N \text{ odd}$$

Where $k = 0, 1, 2, \dots, N-1, q$ is an integer, r is

any integer relatively prime to N and $j = \sqrt{-1}$.

Zadoff-Chu matrix transform (ZCMT) is used to lower the correlation relationship of the IFFT input sequence. If ZCMT precoder is applied before the IFFT stage in PTS scheme, it lowers the correlation relationship of the input sequence, hence the PAPR is reduced.

3.2 PTS-OFDM with ZCMT Precoding

In this paper, we consider ZCMT precoding as a way of reducing the PAPR of OFDM signals. Fig.2 shows the block diagram of PTS-OFDM with ZCMT precoding. The precoding is applied before the IFFT stage of conventional PTS. Initially the data block X is partitioned into U disjoint sub blocks X_u , and then the ZCMT precoding is applied to each cluster which forms a new vector of length N and can be written as

$$Y = ZX = [Y_0, Y_1, Y_2, \dots, Y_{N-1}]^T \quad (8)$$

Where Z is ZCMT based precoding matrix of size

$N \times N$, which is obtained by reshaping the Zadoff-Chu sequence row-wise as in equation (9)

$$k = m + lN, \quad (9)$$

Where m is the row variable and l is the column variable.

Matrix Z can be written as:

$$Z = \begin{bmatrix} z_{00} & z_{01} & \dots & z_{0(N-1)} \\ z_{10} & z_{11} & \dots & z_{1(N-1)} \\ \vdots & \vdots & \ddots & \vdots \\ z_{(N-1)0} & z_{(N-1)1} & \dots & z_{(N-1)(N-1)} \end{bmatrix}$$

Now after passing the signal through Z CT precoder, the resultant signal can be expressed as:

$$Y_{m,l} = \sum_{l=0}^{N-1} z_{m,l} X_l^u \quad (11)$$

where $u \in \{1, 2, \dots, U\}$ and $m \in \{0, 1, \dots, N-1\}$
And $Z_{m,l}$ is m^{th} row and l^{th} column of the precoder matrix.

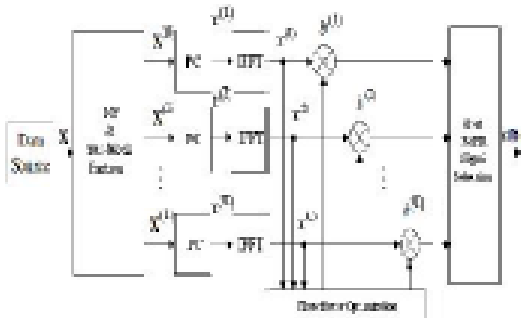


Fig2: Block diagram of PTS-OFDM with ZCT Precoding.

The signal in Eq. (11) after performing the IFFT can be written as:

Now this U time domain signal is weighted by rotation vector $\{b_1, b_2, b_3, \dots, b_U\}$ to further minimize the PAPR, which is given by:

The ZCMT precoding based PTS-OFDM technique needs U IFFT operations. This technique is applicable for any number of subcarriers and all types of modulation techniques. This technique can not reproduce more PAPR if we increase the value of U , but with the increase in the value of U the computational complexity is also increased.

4. SIMULATION RESULTS

In this section, the conventional PTS scheme and the proposed ZCMT precoding based PTS scheme for different no. of subcarriers is compared using MATLAB simulation. To show the PAPR performance of the proposed system the data is generated randomly and then modulated by QPSK. We evaluate the PAPR by using complementary cumulative distribution function (CCDF), which is used to express the probability of exceeding a given threshold.

Table 1 Simulation Parameter

Parameters	Values
Channel Bandwidth	5 MHz
Subcarriers(N)	64,128,256
Modulation	QPSK
Pulse Shaping	Root raised cosine(RRC)
RRC Roll-off factor	0.22
CCDF clip rate	10^{-3}

All the simulations have been performed on 104 random data blocks for various sub carriers $N= 64, 128$ and 256 . The simulation parameters are given in Table.

Fig.3 shows the CCDF comparison curves for conventional OFDM, conventional PTS with OFDM for $U=4$ and ZCMT-PTS at $U=4, 8$ and 16 for $N=64$. At clip rate of 10^{-3} , the PAPR gain of 1.6 dB and 2.7 dB is achieved when we compare ZCMT precoding based PTS with conventional PTS-OFDM system and conventional OFDM systems respectively for $U=4$. Fig.4 shows the CCDF comparison curves for conventional OFDM, conventional PTS with OFDM M for $U=4$ and ZCMT-PTS at $U=4, 8$ and 16 for $N=128$. At clip rate of 10^{-3} , the PAPR gain of 1.9 dB and 2.8 dB is achieved when we compare ZCMT precoding based PTS with conventional PTS-OFDM system and conventional OFDM systems respectively for $U=4$.

Similarly Fig.5 shows the CCDF comparison curves for conventional OFDM, conventional PTS with OFDM M for $U=4$ and ZCMT-PTS at $U=4, 8$ and 16 for $N=256$. At clip rate of 10^{-3} , the PAPR gain of 1.6 dB and 3.1 dB is achieved when we compare ZCMT precoding based PTS with conventional PTS-OFDM system and conventional all OFDM systems respectively. Table2 presents the PAPR analysis of conventional OFDM, conventional PTS with OFDM and ZCMT-PTS at clip rate of 10^{-3} .

Table 2 PAPR Analysis at clip rate of 10^{-3} .

System	PAPR in dB		
	N=64	N=128	N=256
Conventional- OFDM	10.7	10.9	11.5
Conventional- PTS,U=4	9.6	10	10.2
ZCMT- PTS,U=4	8.0	8.1	8.4
ZCMT- PTS,U=8	7.1	7.2	7.4
ZCMT- PTS,U=16	6.6	6.7	6.8

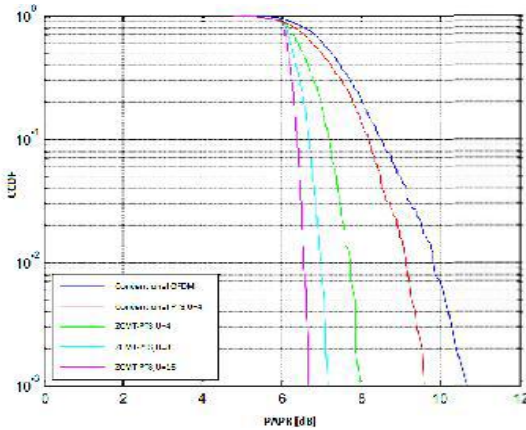


Fig3: PAPR performance of conventional OFDM, PTS-OFDM and ZCMT based PTS-OFDM system with N=64.

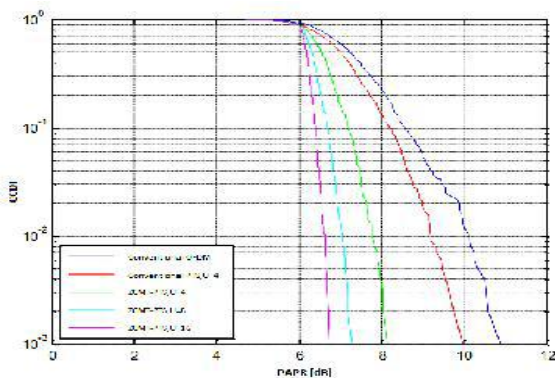


Fig4: PAPR performance of conventional OFDM, PTS-OFDM and ZCMT based PTS-OFDM system with N=128.

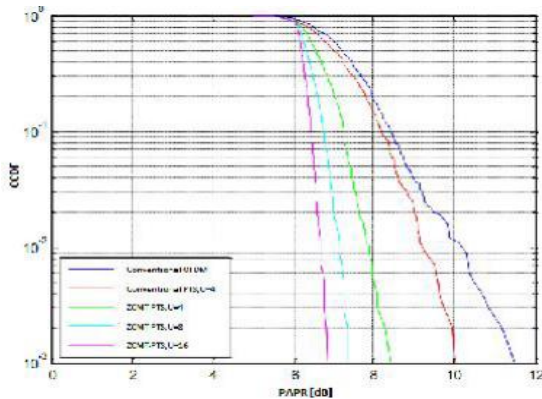


Fig5: PAPR performance of conventional OFDM, PTS-OFDM and ZCMT based PPTS-OFDM system with N=256

5. CONCLUSION

In this paper a novel ZCMT precoded PTS scheme is presented for PAPR reduction. Simulation results show that it has good performance in reducing the PAPR of OFDM

system as compare to conventional PTS scheme. Proposed method can reduce more PAPR as U is increased. The proposed method is efficient and distortion less and no side information is required, so this method can be an attractive approach for future OFDM system.

6. REFERENCES

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