Variations in Delaunay Model Representation for Soft Tissue Modeling

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ABSTRACT
Soft tissue modeling is the process of representing soft tissues present in human skin. Modeling and simulation in the field of medical applications related to virtual surgery has increased the need for creating high quality meshes using various techniques. This paper focuses on Delaunay model representation because it has many desirable qualities suitable for practical applications. Most commonly triangular and tetrahedral mesh techniques utilizes Delaunay criterion. This paper compares the accuracy of triangular and tetrahedral meshes. Some surgery requires minute information and some might not, so accuracy depends on the model. We hope that this comparison methodology used here makes it easier for others to choose and build the model for proper applications.

Keywords
Two Dimension (2D), Three Dimension (3D), Visualization Tool Kit (VTK), Central Processing Unit (CPU).

1. INTRODUCTION
Soft tissues are white membranes located under the skin that connects muscles, bones, blood vessels of the body. Soft tissue modeling is the geometrical description of human soft tissues. Based on the anatomy various mesh models have been explained. The basic theme is to study about mesh and its variations. Mesh is a set of closely connected polygons. A mesh could be closed or may not be closed. Set of vertices, faces etc represents a mesh which is not closed. Set of triangles usually represents closed mesh. Mesh generation algorithms are important methods to produce good quality meshes linked with cell associatively. These are also called as Grid generation algorithms. Aly A.Farag [11] explains that mesh generation algorithms is the practice of generating a mesh that consists of polygon or polyhedral with a geometric domain. There are varieties of algorithm techniques that exists such as 2D mesh generation, 3D mesh generation, surface mesh generation,etc., However mesh generation is broadly classified as structured and unstructured meshes as explained by Yvinec.M[7]. The usual representation of any mesh model starts with triangular mesh, but the same mesh could take a variation representing tetra mesh leading to complex model representations. Lizier.M.et.al. [5] explains that triangular meshes are simple compared to tetrahedral meshes which are quite complex. But when dealing with complex geometries, tetrahedral meshes have proven to be a powerful tool because its elements are of varying sizes, angles and orientations. Papadomanolakis et.al, [4] states that to obtain variable resolution especially in case of earthquake modeling and mechanical engineering tetrahedral meshes are more popular. The next part discusses about the various classification and methods to generate mesh models.

2. MESH GENERATION METHODS
Mesh models are based on meshing algorithms which are broadly classified into structured and unstructured meshes. Structured meshes have regular distribution of points. In structured meshes the geometry can be traced because vertices are numbered, one can easily find out the vertex shared with selected vertex. They have uniform topological structure but don't have a strongly graded mesh. Structured meshes are suitable for numerical grid generation using differential equations. However algorithms for structured meshes consist of complex iterative smoothing. Unstructured meshes have irregular distribution of points. They exhibit neighborhood requirement i.e. each vertex stores the information about neighboring vertices. Due to this property, meshes with good element shapes can be generated with varying sizes scalable from very small to very large. Unstructured meshes are more flexible to generate complex geometries but require much more complex data structure. These meshes are recognized as efficient and powerful method for improving accuracy of the solution as proposed by Saurez JP Et.al., [3]. Unstructured meshes are versatile and algorithms involve iterative smoothing. Figure1 shows classification of mesh generation methods. Each of which uses Triangular or tetrahedral method and Quad or hex method.

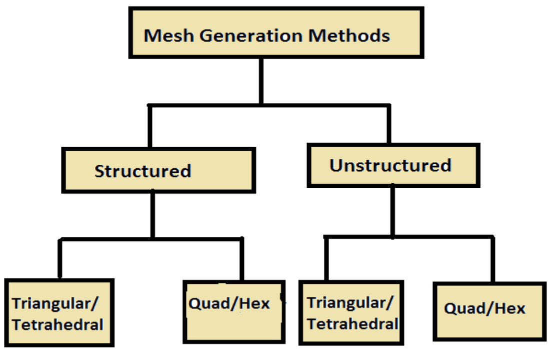

Figure 1. Classification of Mesh Generation Methods

Triangular or tetrahedral method uses three representation techniques namely octree, Delaunay or advancing front. Quad or hex method uses geometrical representation. This paper discusses about how the Delaunay representation technique differs in triangular and tetrahedral methods. Bakker [9] and Robert Putnam [10] discuss the fact that based on the dimensionality and shapes of elements, meshes are categorized as triangular, tetrahedral, quadrilateral and hexahedral element types. Different types of meshes with element/cell shapes are shown in figure2.

1) Triangular Meshes: Triangle elements are simple polygons with three vertices and three edges. They serve in modeling two dimensional domains and

also in generation of surface meshes. Benzley et.al. [1] proposes that it is generally accepted fact that triangular elements with higher order displacement provide acceptable accuracy and convergence characteristics.

2) Tetrahedral Meshes: Tetrahedral elements are simple polyhedral with four vertices and four triangular faces.

3) Quadrilateral meshes: Quadrilateral elements are polygons with four sides such that their sides need not be parallel.

4) Hexahedral Meshes: hexahedral elements are polyhedral with brick like shape with six quadrilateral faces such that faces need not be planar or parallel.

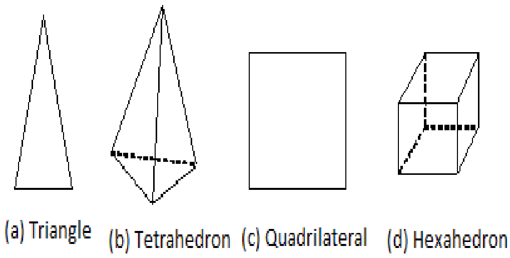

Figure 2. Different types of Meshes

The triangles and quadrilateral elements are useful in modeling two dimensional domains and also in generation of three dimensional surface meshes. However, due to accurate interpolation and approximation triangular and tetrahedral meshes are easier to generate than quadrilateral and hexahedral meshes. The next part discusses about the different results obtained using Delaunay criteria.

3. EXPERIMENTAL SETUP AND RESULTS

Delaunay triangulation is a geometric structure with remarkable mathematical properties uniquely well suited for creation of good triangular and tetrahedral meshes. They have proven to be more powerful and versatile than grid and octree algorithms. Maur.P [2] states that the optimal running time of Delaunay triangulation is O(nlogn). To achieve optimal expected running time vertices insertion order should be in random as explained d by Lagae and Dutre [6]. But this randomization results in elimination of special locality in a vertex set. Therefore a balance between randomization and spatial locality has to be set in order to achieve good performance. Triangulation in 3D space is called 3D triangulation or tetrahedralization. Triangular mesh consists entirely of triangular elements with three vertices and edges. Tetrahedral mesh consists of tetrahedral elements with four triangular faces at each vertex. Figure3 shows pictorial representation representations of Delaunay triangulation and tetrahedralization.

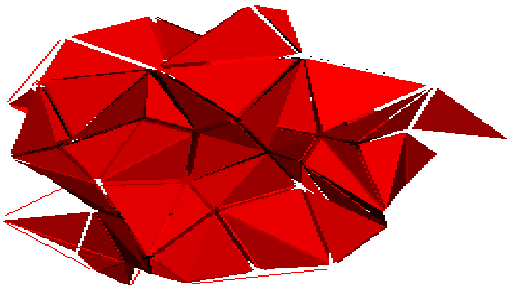

Figure 3(a). Delaunay Triangulation

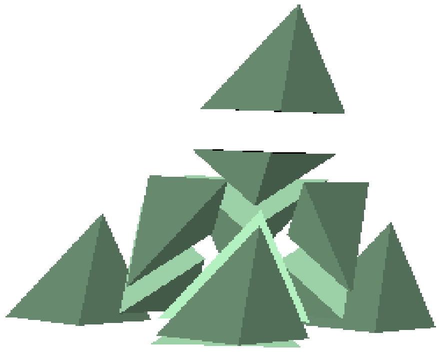

Figure 3(b). Delaunay Tetrahedralization

In virtual surgery it is very essential to model three layers of soft tissues of skin. So an experiment is conducted to obtain three layers of soft tissues based on the algorithms given below. First algorithm is for triangular mesh construction and the second one is for tetrahedral mesh construction. Both the algorithms are self explanatory. The only difference among both the algorithms lies in setting the number of points. If the number of points are three, a triangular mesh is obtained, if set to four a tetrahedral mesh.

Algorithm for Triangulation:

While (input set is not empty) do

Insert a new point Vi from the input set

If (the number of points = = 3)

For i= 1 to 8 do

Vi = Vi + 1

Result = in cube (V1 to V8)

End for

End if

Algorithm for Tetrahedralization:

While (input set is not empty) do

Insert a new point Vi from the input set

If (the number of points = = 4)

For i= 1 to 8 do

Vi = Vi + 1

Result = in cube (V1 to V8)

End for

End if

The following results are obtained which depicts the geometrical description of three layers of soft tissues using Delaunay triangulation and tetrahedralization. Both experimental datasets are developed using the same cube data structure shown in figure4 using VTK and Python with vertex-list and triangle-list and also tetra-list.

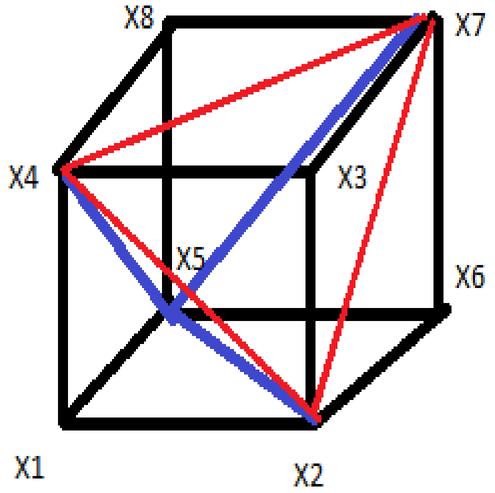

Figure 4.Cube Data structure

3D Delaunay Triangulation: The prototype of soft tissues is developed using cube data structure as shown in figure4 using 3D Delaunay triangulation technique. It consists of vertex-list, edge-list and triangle-list. The same is illustrated in table1 showing the triangle list of a cube. Set of three vertices form a triangle, hence there will be twelve triangles in a single cube.

1 Cube = 8 Vertices + 18 Edges + 12 Triangles.

Table 1. Triangle list of a cube

Triangle List	Vertex List		
T1	X1	X2	X3
T2	X1	X3	X4
T3	X5	X6	X7
T4	X5	X7	X8
T5	X2	X6	X7
T6	X2	X7	X3
T7	X1	X5	X8
T8	X1	X8	X4
T9	X1	X2	X6
T10	X1	X6	X5
T11	X4	X3	X7
T12	X4	X7	X8

Each cube consists of twelve triangles; this concept is used to simulate the prototype of skin using VTK and Python programming language. A single triangular cube with vertices and edges are shown in figure5 (a) and 5(b) in both solid and wireframe model. The quality of triangulation is estimated based on the shape of triangles. Common measures of triangle shapes are like maximum or minimum angle, length of minimum elevation, radius of circumcircle, radius-edge ratio, radius-radius ratio etc. Delaunay triangulation satisfies all these measures.

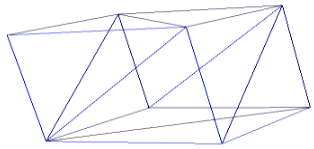

Figure 5(a). Single Triangular Cube in Solid model

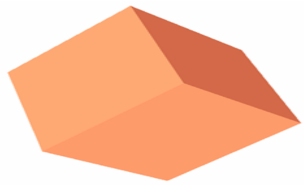

Figure 5(b). Single Triangular Cube in Wireframe model

Since skin is composed of multiple layers namely epidermis, dermis and subcutis, all these are modeled using the same cube data structure concept. The following figure depicts rendering of soft tissue layers that are layered using triangular cube data structure concept. Epidermis is the first layer which is smooth and soft in nature, modeled as four single cubical data structure (green color) that are adjacently placed. Dermis is the second layer which is fat and cushiony, modeled as double four single cubical data structure (rose color) adjacently placed. Subcutis is the third layer which is hard and connected to muscles, modeled again as double four single cubical data structure (magenta color) adjacently placed. The results are shown in figure6 (a) and 6(b) in both solid and wireframe models.

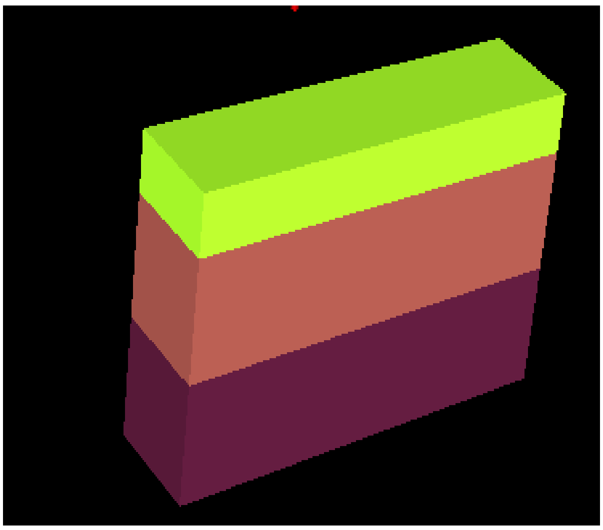

Figure 6(a). Layered Triangular cube in solid model

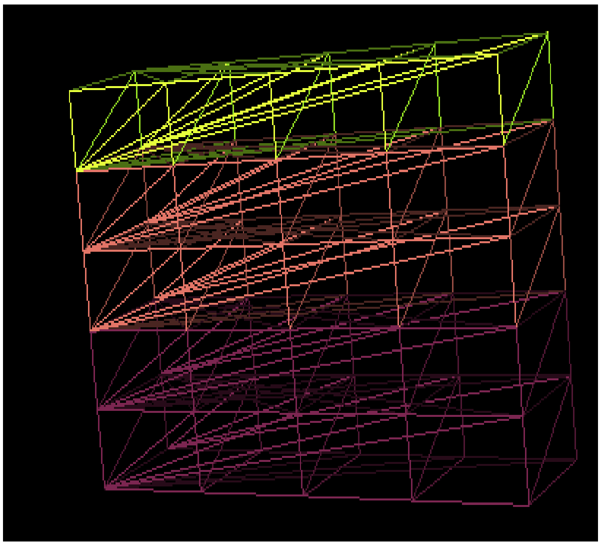

Figure 6(b). Layered Triangular cube in Wireframe model

3D Delaunay Tetrahedralization: The prototype soft tissues of skin can also be modeled using 3D Delaunay tetrahedralization. In this the data structure of cube is as shown in figure4 consists of vertex-list, edge-list and tetra-list. Table2 shows the tetra list of cube consisting of tetra-list and vertex-list. Collection of four vertices form a single tetra, so all together there will be five tetras in a single cube.

1 Cube = 8 Vertices + 18 Edges + 5 Tetras

Table 2. Tetra List of a cube

Tetra List	Vertex List			
T1	X5	X4	X1	X2
T2	X5	X4	X8	X7
T3	X5	X2	X6	X7
T4	X7	X4	X2	X8
T5	X5	X4	X7	X2

Here each cube consists of eight vertices, eighteen edges and five tetras. Tetrahedrons are all build using unstructured grid technique using VTK and python programming. Tetrahedral mesh are quite complex in nature and requires heavy programming. Figure7 (a) and 7(b) depicts a single tetrahedral cube in solid and wireframe model with each tetra colored differently (red, green, blue, yellow and rose).

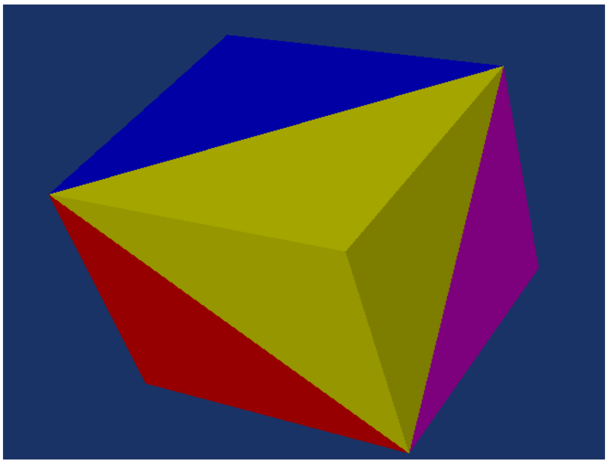

Figure 7(a). Single Tetrahedral cube in solid model

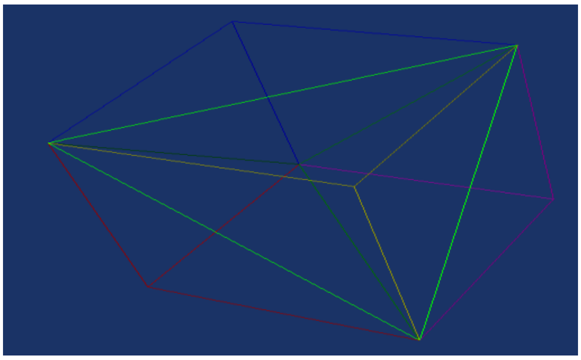

Figure 7(b). Single Tetrahedral cube in Wireframe model

Based on this tetrahedral cube data structure the three different layers of skin are modeled and the following pictures shown in figure8(a)&(b) shows rendering of soft tissue layers i.e. layered tetrahedral cubes in solid and wireframe model. Altogether there are five layers. First layer is the epidermal layer, next two layers are dermis layers and last two layers are subcutis layers. The only difference in the layers is about the thickness of each layer. First layer is smooth with four adjacent single tetra cubes. Second layer is little fleshy modeled as two layer four adjacent tetra cubes and third layer is hard with again two layer four adjacent tetra cubes.

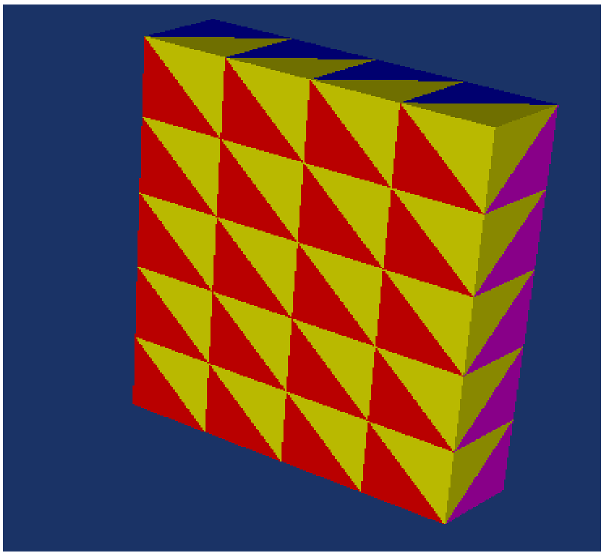

Figure 8(a).Layered Tetrahedral cube in solid model

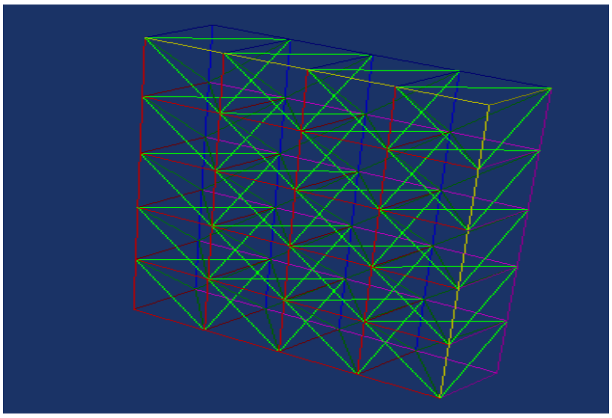

Figure 8(b). Layered Tetrahedral cube in wireframe model

Texture mapping is applied on the three layers irrespective of the model i.e. whether triangular/tetra model. Texture mapping is the process of wrapping an image on to the text. Since epidermis is the first layer which is smooth, it is texture mapped with skin image. Dermis is the second layer, cushion type hence it is texture mapped with flesh image. Subcutis is the third layer connected to muscles, texture mapped with bones image. Figure 9 shows texture mapping applied on all the three layers of soft tissues.

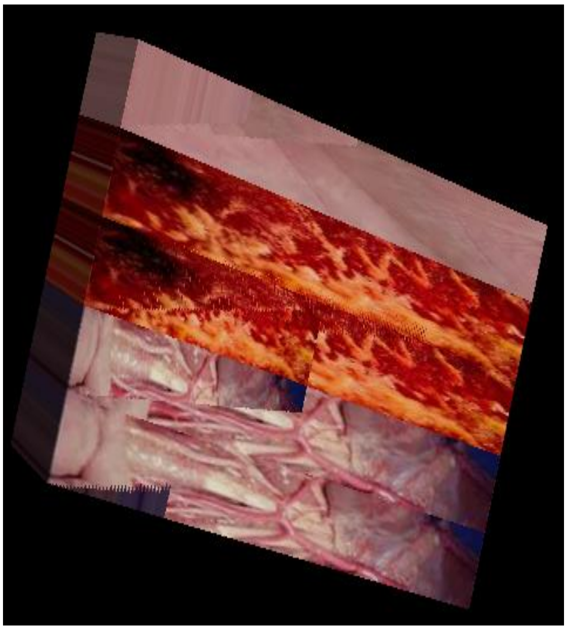

Figure 9.Three layers of skin with texture mapping

Henceforth based on the previous results[8] & observation and results obtained from experiments obtained from both kinds of models, a comparison table has been prepared table 3 to show clear cut difference between the models and its usage.

Table 3. Comparison of Models

Sl. No	Delaunay Triangulation	Delaunay Tetrahedralization
1.	Triangulation can be 2D & 3D	Tetrahedralization only in 3D
2.	Less complicated	More complicated
3.	Takes less time for execution	Takes longer time for execution
4.	More Lighter	More Heavier
5.	Less space complexity	More space complexity
6.	Effective at producing elements of good quality	Effective at producing meshes with complex geometries.
7.	Not Popular in mesh generation	Popular in mesh generation
8.	Cannot generalize optimality properties	Can generalize optimality principles
9.	They maximize the minimum angle(plane or dihedral)	They do not maximize the minimum angle(plane or dihedral)
10.	Delaunay triangulation in the plane generalize to higher dimensions	Delaunay tetrahedras in the plane do not generalize to higher dimensions.

However the comparisons of triangular and tetrahedral meshes are mainly based on three terms namely: storage, performance and simplicity. Delaunay triangulations takes less space to store its data compared to tetrahedrons. In the case of displacement interpolation functions used to calculate stress/strain behavior of meshes, the performance of triangular meshes are expected to be much better compared to tetrahedral meshes, this is because they need extremely fine mesh with location of elements. Generally the quality of meshes is measured on three terms namely: skewness, smoothness and aspect ratio.

I. **Skewness**: Skewness is the degree of asymmetry of a pixel distribution in the specified window around its mean. Skewness is a pure number that characterizes only the shape of the distribution. According to Andre Bakker[9], the skewness table consists of skeweness value and cell quality shown in table4.

Table 4.Skewness Table

Skewness value	0.0-0.25	0.25-0.50	0.50-0.80	0.80-0.95	0.95-0.99	0.99-1.00
Cell Quality	Excellent	Good	Acceptable	Poor	Silver	Degenerate

II. **Smoothness**: Smoothness refers to constant element size throughout the mesh. There should not be sudden change in the element size. More elements gives higher accuracy but drawback is increased memory and CPU time.

III. **Aspect ratio**: Aspect ratio is the ratio of longest edge length to shortest edge length. It is equal to one for equilateral triangle or square.

4. CONCLUSION

In this paper the prototype for soft tissue model for multiple layers is developed and analyzed using both triangular and tetrahedral meshes. The data sets pertaining to respective meshes are discussed. Also their strengths and weakness are compared accordingly. Based on time complexity, space complexity, mesh connectivity and its efficiency is independent of complexity of mesh geometry, it is relevent that triangular meshes are well suited and can give better performance for further enhancement of the model. In future the triangular model is chosen to show deformation effect on all the three layers i.e. multilayer model of soft tissues by applying forces on exterior of the mesh.

5. ACKNOWLEDGEMENTS
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