Cumulative Distribution of the Peak to Average Power Ratio in an OFDM System

Lokesh C
Professor
Dept of E&EE
VVCE,Mysore

Mamatha C.G
Asst Professor
Dept of E&EE
GSSSIETW, Mysore

Rekha K.R, PhD
Asst Professor
Dept of E&CE
SJBIT, Bangalore

ABSTRACT

An OFDM signal consists of a number of independently modulated SCs, which can give a large peak-to-average power (PAP) ratio when added up coherently. The peak power is defined as the power of a sine wave with amplitude equal to the maximum envelope value. Hence, an unmodulated carrier has a PAP ratio of 0 dB. An alternative measure of the envelope variation of a signal is the Crest factor, which is defined as the maximum signal value divided by the RMS signal value. For an unmodulated carrier, the Crest factor is 3 dB. This 3-dB difference between the PAP ratio and Crest factor also holds for other signals, provided that the center frequency is large in comparison with the signal bandwidth.

Keywords

SubCarriers (SCs), Peak-to-Average Power (PAP), Orthogonal Frequency Division Multiplexing (OFDM), Forward Error Correction (FEC), Quadrature Phase Shift Keying (QPSK), Cumulative Distribution Function (CDF), Signal-to-Noise Ratio (SNR), Coded OFDM (COFDM), Parallel-to-Serial (P/S).

1. INTRODUCTION

A large PAP ratio brings disadvantages like an increased complexity of the analog-to-digital and digital-to-analog converters and a reduced efficiency of the RF power amplifier. To reduce the PAP ratio[3], several techniques have been proposed, which basically can be divided in three categories. First, there are signal distortion techniques, which reduce the peak amplitudes simply by nonlinearly distorting the OFDM signal at or around the peaks. Examples of distortion techniques are clipping, peak windowing, and peak cancellation. Second, there are coding techniques that use a special FEC code set that excludes OFDM symbols with a large PAP ratio. The third technique scrambles each OFDM symbol with different scrambling sequences and selecting the sequence that gives the smallest PAP ratio. This paper discusses all of these techniques, but first analyzes the PAP ratio distribution function. This will give a better insight in the PAP problem and will explain why PAP reduction techniques can be quite effective.

2. DISTRIBUTION OF THE PAP RATIO

For one OFDM symbol with N SCs, the complex baseband signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} a_n \, exp(j\omega_n t) \qquad (1)$$

Here, a_n are the modulating symbols. For QPSK, for instance $a_n \in \{-1, 1, j, -j\}$. From the central limit theorem it follows that for large values of N, the real and imaginary values of $x(t)$ become Gaussian distributed, each with a mean of zero and a variance of $1/2$. The amplitude of the OFDM signal therefore has a Rayleigh distribution, while the power distribution becomes a central chi-square distribution with two degrees of freedom and zero mean with a cumulative distribution given by

$$F(z) = 1 - e^{-z} \qquad (2)$$

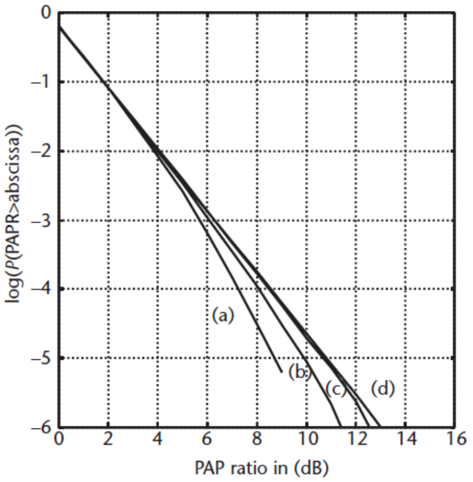

Fig 1 PAP distribution of an OFDM signal with (a) 12, (b) 24, (c) 48, and (d) an infinite number of SCs (Pure Gaussian noise). Four times oversampling used in simulation; total number of simulated samples is 12 million.

Figure 1 show the probability that the PAP ratio exceeds a certain value. We can see that the curves for various numbers of SCs are close to the Gaussian distribution shown in Figure 6.2(d) until the PAP value comes within a few decibels of the maximum PAP level of 10log*N*, where *N* is the number of SCs[7].Now, we want to derive the CDF for the peak power per OFDM symbol. Assuming the samples are mutually uncorrelated—which is true for non oversampling— the probability that the PAP ratio is below some threshold level can be written as

$$P(PAPR \le z) = F(z)^N = (1 - \exp(-z))^N \qquad (3)$$

This theoretical derivation is plotted against simulated values in Figure 2 for different values of *N*.

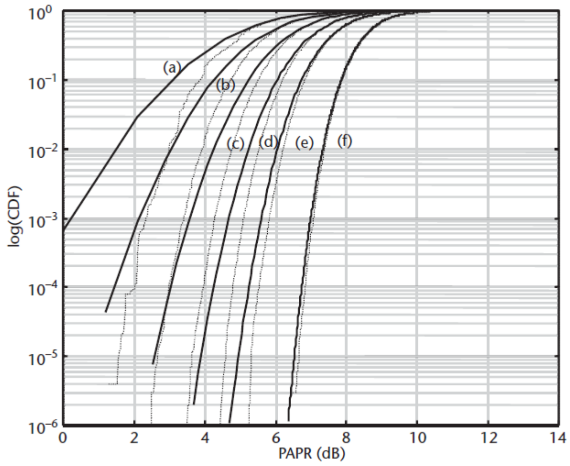

Fig 2 PAP distribution without oversampling for a number of SCs of (a) 16, (b) 32, (c) 64, (d) 128, (e) 256, and (f) 1,024. Dotted lines are simulated.

The assumption made in deriving (3) that the samples should be mutually uncorrelated no longer holds when oversampling is applied. Because it seems quite difficult to come up with an exact solution for the peak power distribution[4], we propose an approximation by assuming that the distribution for αN SCs and oversampling can be approximated by the distribution for αN SCs without oversampling, with $\alpha > 1$. Hence, the effect of oversampling is approximated by adding a certain number of extra independent samples.

The distribution of the PAP ratio is then given by

$$P(PAPR \leq z) = F(z)^N = (1-\exp(-z))^{\alpha - N} \qquad (4)$$

Figure 3 gives the PAP distribution for different amounts of carriers for $\alpha = 2.8$. The dotted lines are simulated curves. We see in Figure 3 that Eq. 4 is quite accurate for $N > 64$. For large values of the CDF close to one (> 0.5), however, (3) is actually more accurate.

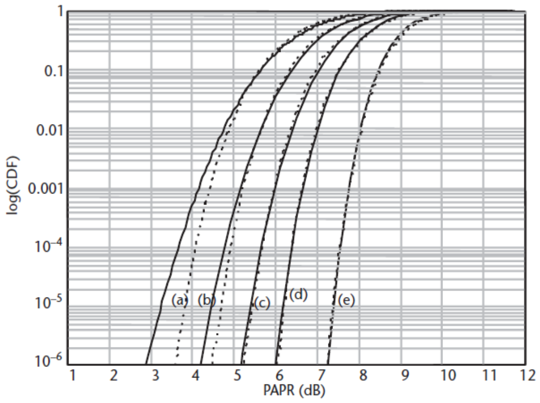

Figure 3 CDF of the PAP ratio (PAPR) for a number of SCs of (a) 32, (b) 64, (c) 128, (d) 256, and (e) 1,024. Solid lines are calculated; dotted lines are simulated.

From Figure 3, we can deduce that coding technique to reduce the PAP ratio may be a viable option as reasonable coding rates are possible for a PAP ratio around 4 dB. For 64 SCs, for instance, about 10^{-6} of all possible QPSK symbols have a PAP ratio of less than 4.2 dB. This means that only 20 out of a total of 128 bits would be lost if only the symbols with a low PAP ratio were transmitted. However, the main problem with this approach is finding a coding scheme with a reasonable coding rate ($\geq 1/2$) that produces only these low PAP ratio symbols and that also has reasonable error-correcting properties.

A different approach to the PAP problem is to use the fact that because large PAP ratios occur only infrequently, it is possible to remove these peaks at the cost of a slight amount of self-interference. Now, the challenge is to keep the spectral pollution of this self-interference as small as possible. Clipping is one example of a PAP reduction technique creating self- interference. In the next sections, two other techniques are described which have better spectral properties than clipping.

3. CLIPPING AND PEAK WINDOWING

The simplest way to reduce the PAP ratio is to clip the signal, such that the peak amplitude becomes limited to some desired maximum level[9]. Although clipping is definitely the simplest solution, there are a few problems associated with it. First, by distorting the OFDM signal amplitude, a kind of self-interference is introduced that degrades the BER. Second, the nonlinear distortion of the OFDM signal significantly increases the level of the out-of-band radiation. The latter effect can be understood easily by viewing the clipping operation as a multiplication of the OFDM signal by a rectangular window function that equals one if the OFDM amplitude is below a threshold and less than one if the amplitude needs to be clipped. The spectrum of the clipped OFDM signal is found as the input OFDM spectrum convolved with the spectrum of the window function. The out-of-band spectral properties are mainly determined by the wider spectrum of the two, which is the spectrum of the rectangular window function[2]. This spectrum has a very slow roll off that is inversely proportional to the frequency.

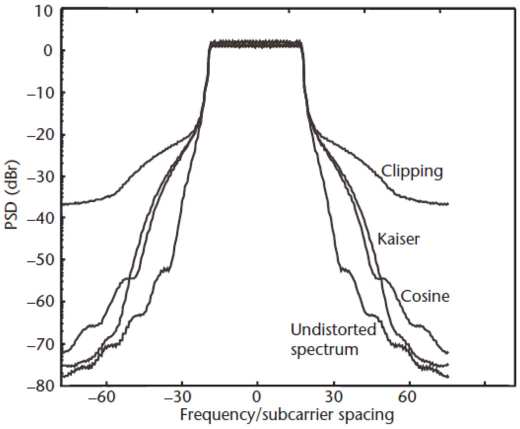

Figure 4 Frequency spectrum of an OFDM signal with 32 SCs with clipping and peak windowing at a threshold level of 3 dB above the RMS amplitude.

In Figure 4, the difference between clipping the signal and windowing the signal can be seen. Figure 5 shows how increasing the window width can decrease the spectral distortion [3].

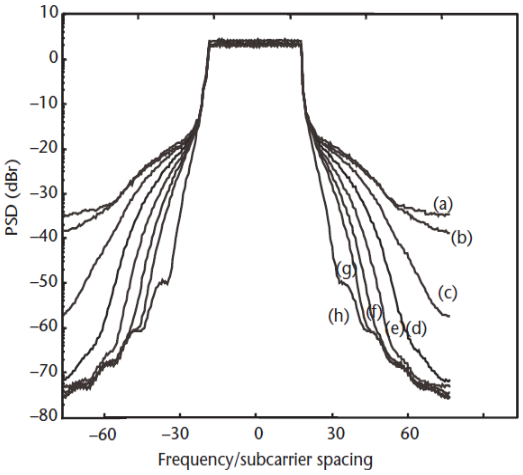

Figure 5 Frequency spectrum of an OFDM signal with 32 SCs with peak windowing at a threshold level of 3 dB above the RMS amplitude. Symbol length is 128 samples (4 times oversampled) and window length is (a) 3, (b) 5, (c) 7, (d) 9, (e) 11, (f) 13, and (g) 15 samples. Curve (h) represents the ideal OFDM spectrum.

Figure 6 shows packet-error ratio (PER) curves with and without clipping, using a rate $1/2$ convolutional code with constraint length 7. The simulated OFDM signal uses 48 SCs with 16-QAM. The plots demonstrate that nonlinear distortion has only a minor effect on the PER; the loss in SNR is about 0.25 dB when the PAP ratio is decreased to 6 dB. When peak windowing is applied, the results are slightly worse because peak windowing distorts a larger part of the signal than clipping for the same PAP ratio.

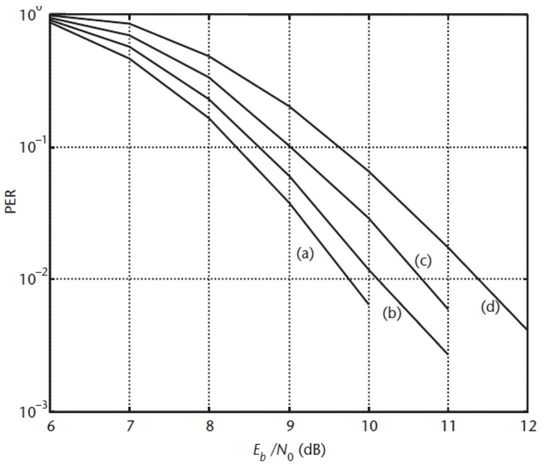

Figure 6 PER versus Eb /N0 for 64-byte packets in AWGN. OFDM signal is clipped to a PAP ratio of

(a) 16 (no distortion), (b) 6, (c) 5, and (d) 4 dB.

4. CODING AND SCRAMBLING

A disadvantage of distortion techniques is that symbols with a large PAP ratio suffer more degradation, so they are more vulnerable to errors. To reduce this effect, FEC coding can be applied across several OFDM symbols. By doing so, errors caused by symbols with a large degradation can be corrected by the surrounding symbols. In a COFDM system, the error probability is no longer dependent on the power of individual symbols, but rather on the power of a number of consecutive symbols. For example, assume that the FEC code produces an error if more than 4 out of every 10 symbols have a PAP ratio exceeding 10 dB. Further, assume that the probability of having a PAP ratio larger than 10 dB is 10^{-3}. Then, the error probability of the peak cancellation technique is

$$1-\sum_{i=0}^{3}\binom{10}{1}\,(10^{-3})^{i}(1-10^{-3})^{10-i} \;\cong\; 2.10^{-10}$$

less than the 10^{-3} in the case where no FEC coding is used. Although such a low symbol-error probability may be good enough for realtime circuit-switched traffic, such as voice, it may still cause problems for packet data. A packet with too many large PAP ratio symbols will have a large probability of error. Such packets occur only very infrequently, as shown above, but when they occur, they may never come through because every retransmission of the packet has the same large error probability[6,9].

5. PEAK CANCELLATION

The key element of all distortion techniques is to reduce the amplitude of samples whose power exceeds a certain threshold. In the case of clipping and peak windowing, this was done by a nonlinear distortion of the OFDM signal, which resulted in a certain amount of out-of-band radiation. This undesirable effect can be avoided by performing a linear peak-cancellation technique, whereby a time-shifted and scaled-reference function is subtracted from the signal; such that each subtracted reference function reduces the peak power of at least one signal sample. By selecting an appropriate reference function with approximately the same bandwidth as the transmitted signal, it can be assured that the peak power reduction will not cause any out-of-band interference[7,3,9].

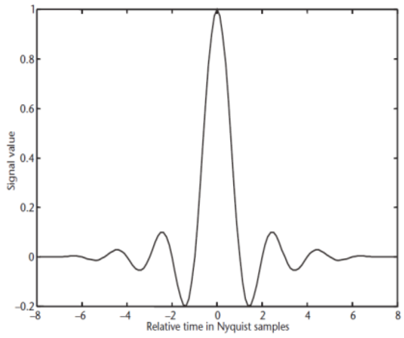

Figure 7 Sinc reference function, windowed with a raised cosine window.

Figure 7 shows an example of a reference function, obtained by multiplication of a *sinc* function and a raised cosine window. If the windowing function is the same as that used for the windowing of the OFDM symbols, then it is assured that the reference function has the same bandwidth as the regular OFDM signals. Hence, peak cancellation will not degrade the out-of-band spectrum properties. By making the reference signal window narrower, a trade-off can be made between less complexity in the peak-cancellation calculations and some increase in the out-of-band power.

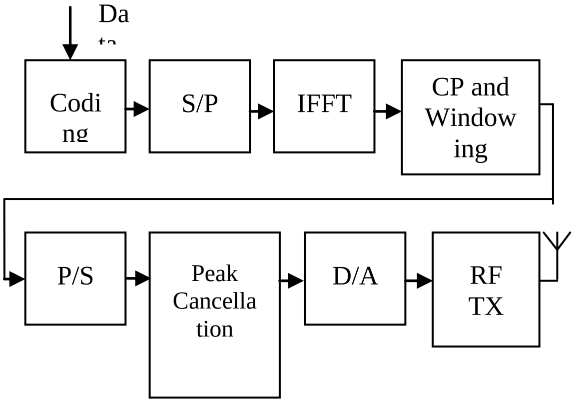

Figure 8 OFDM transmitter with peak cancellation.

Peak cancellation can be performed digitally after generation of the digital OFDM symbols. It involves a peak power (or peak amplitude) detector, a comparator to see if the peak power exceeds some threshold, and a scaling of the peak and surrounding samples. Figure 8 shows the block diagram of an OFDM transmitter with peak cancellation. Incoming data is first coded and converted from a serial bit stream to blocks of N complex signal samples. On each of these blocks, an IFFT is performed. Then, a cyclic prefix is added, extending the symbol size to $N + NG$ samples. After parallel-to-serial (P/S) conversion, the peak cancellation procedure is applied to reduce the PAP ratio. It is also possible to do peak cancellation immediately after the IFFT and before the cyclic prefix and windowing. Except for the peak cancellation block, there is further no difference from a standard OFDM transmitter. For the receiver, there is no difference at all, so any standard OFDM receiver can be used [11,12].

6. CONCLUSION

It is possible to remove these peaks at the cost of a slight amount of self-interference. Now, the challenge is to keep the spectral pollution of this self-interference as small as possible. When peak windowing is applied, the results are slightly worse because peak windowing distorts a larger part of the signal than clipping for the same PAP ratio. The AM/PM conversion of a solid-state power amplifier is small enough to be neglected. Peak cancellation seems to be a fundamentally different approach than clipping or peak windowing. It can be shown that peak cancellation is in fact almost identical to clipping followed by filtering.

7. REFERENCES

[1] Pauli, M., and H. P. Kuchenbecker, "Minimization of the Intermodulation Distortion of a Nonlinearly Amplified OFDM Signal," *Wireless Personal Communications,* Vol. 4, No. 1, January 1997, pp. 93–101.

[2] Rapp, C., "Effects of HPA-Nonlinearity on a 4-PSK/OFDM Signal for a Digital Sound Broadcasting System," *Proc. of 2nd European Conference on Satellite Communications,* Liège, Belgium, October 22–24, 1991, pp. 179–184.

[3] De Wild, A., "The Peak-to-Average Power Ratio of OFDM," .Sc. thesis, Delft University of Technology, Delft, the Netherlands, September 1997.

[4] May, T., and H. Rohling, "Reducing the Peak-to-Average Power Ratio in OFDM Radio Transmission Systems," Proceedings of IEEE VTC'98, Ottawa, Canada, May 18–21, 1998, pp. 2774–2778.

[5] Li, X., and L. J. Cimini, "Effects of Clipping and Filtering on the Performance of OFDM," *Proc. of IEEE VTC'97,* 1997, pp. 1634–1638.

[6] Wilkinson, T. A., and A. E. Jones, "Minimization of the Peak-to-Mean Envelope Power Ratio of Multicarrier Transmission Schemes by Block Coding," *Proc. of IEEE Vehicular Technology Conference,* Chicago, IL, July 1995, pp. 825–829.

[7] Golay, M. J. E., "Complementary Series," *IRE* Transactions on Information Theory, Vol. IT-7, April 1961, pp. 82–87.

[8] Sivaswamy, R., "Multiphase Complementary Codes," *IEEE Trans. on Information Theory,* Vol. IT-24, No. 5, September 1978, pp. 546–552.

[9] Frank, R. L., "Polyphase Complementary Codes," *IEEE* Trans. on Information Theory, Vol. IT-26, No. 6, November 1980, pp. 641–647.

[10] Popovic, B. M., "Synthesis of Power Efficient Multitone Signals with Flat Amplitude Spectrum," *IEEE Trans. on Communications,* Vol. 39, No. 7, July 1991, pp. 1031–1033.

[11] Van Nee, R. D. J., "OFDM Codes for Peak-to-Average Power Reduction and Error Correction," IEEE Global Telecommunications Conference, London, England, November 18–22, 1996, pp. 740–744.

[12] Van Nee, R. D. J., "An OFDM Modem for Wireless ATM," *IEEE* Symposium on Communications and Vehicular Technology, Ghent, Belgium, October 7–8, 1996.

8. AUTHOUR PROFILE

Mr. Lokesh C is a research student and Assistant Professor in the department of Electrical and Electronics Engineering from Vidyavardhaka College of Engineering, Mysore (India). His research mainly encompasses in the area of wireless communication, application of communication system in Electrical Engineering. He has published 9 research papers in international journals. And two papers in International conference.