Stress Intensity Factor for Internal Cracks in Thick Walled Pressure Vessels using Weight Function Technique

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ABSTRACT
In the present study, a thick walled cylinder with a semi elliptical crack located at the inner surface is considered. Weight functions for the surface and the deepest point of an internal semi elliptical crack in a thick-wall cylinder were derived from a general weight function and two reference
The weight functions were validated against finite element data given by Mettu and hybrid weight the paper are valid for cylinders with an inner radius to wall thickness ratio, Ri/t = 4.
complex stress fields. All stress intensity factor expressions given in several linear and nonlinear

Keywords
Weight Function, Stress Intensity factor, Thick-walled cylinder, stress intensity factors.

1. INTRODUCTION
The theory of thick cylinders (Lame's Theory) shows that longitudinal cracks located on the internal face of the cylinder are most dangerous. Semi elliptical surface cracks are occasionally developed in pressure vessels and pipes during service or production. Subsequent fracture and fatigue analysis of such cracks is of great practical interest, and requires the determination of stress intensity factors. Although several stress intensity factor handbooks have been published, the available solutions of stress intensity factors for pressure vessels are not always adequate for particular engineering applications. This is especially true for cracks subjected to non-uniform stress fields such as residual or thermal stresses. In such cases the weight function approach is the most useful tool. The unique feature of the weight function method is that once the weight function for a particular cracked body has been determined, the stress intensity factor for any loading system applied to that body can be calculated by simple integration.
Since the stress intensity factor is linearly dependent on the applied load, the contributions from multiple splitting forces S applied over the crack surface can be superposed and the resultant stress intensity factor can be calculated as the sum of all individual load contributions. In order to calculate stress intensity factors using the weight function technique the following tasks need to be carried out:

Determine stress distribution $\sigma(x)$ in the prospective crack plane using the linear elastic

- analysis of uncracked body, i.e., perform the stress analysis ignoring the crack and determine the stress distribution $\sigma(x)=\sigma_0 f(a,x)$
- Apply the ''uncracked'' stress distribution, $\sigma(x)$, to the crack surfaces (fig 1c) as traction.
- Choose an appropriate generic weight function.
- Integrate the product of the stress function $\sigma(x)$ and the weight function $m(x,a)$ over the entire crack length or crack surface.

2. REVIEW OF LITERATURE WORK
Jian- Feng Wen, Shan-Tung Tu, Jian- Ming Gong in 2011 has considered a semi-elliptical crack located at the inner surface of pressurized cylinder. C^*- integrals for this semi-elliptical crack has been calculated by finite element (FE) method. A total of 96 cases for widw practical ranges of geometry and material parameters are performed to obtain FE results of C^*- integral. Brahim El Khalil Hachi, Said Rechak, Yacien Belkacemi and Genard Maurice in 2005 has done the modeling of elliptical cracks in an infinite body and in a pressurized cylinder by a hybrid weight function approach. The idea of hybridization consists of dividing the ellipse in to two zones, and then to use each weight function in the area where it is more efficient. A, Kiciak, G. Glinka and D. J. Burns in 2003 proposed a paper for calculating the stress intensity factors and crack opening displacements for cracks subjected to complex stresses. A method based on generalized weight function of calculating stress intensity factors for cracks subjected to complex stresses fields have been discussed in this paper. M. Perl, C. levy and J. Pierola in 1996 has presented a paper in which for internal surface crack both in the radial and longitudinal direction are considered for pressurized thick-walled cylinders. The 3-D analysis is performed via the finite element with the sub modeling technique, employing singular element among the crack front. SIFs are evaluated for arrays up to n=180 cracks; for a wide range of crack depth to wall thickness ratios, a/t, from 0.05 to 0.6; and, for various ellipticities of the crack, i.e., the ratio of crack depth to semi-crack length, a/c from 0.2 to 2. The formulas, which are function of a/t and a/c, are of very good engineering accuracy. The results clearly indicate that the SIFs are considerably affected by the interaction among the cracks in the arrays as well as the three-dimensionality of the problem.
Zheng, X.J., Glinka G. in 1995 had suggested that weight functions for the surface and the deepest point of an internal longitudinal semi-elliptical crack in a thick walled cylinder (Ri/t =1) were derived from a general weight function and two reference stress intensity factors. For several linear and nonlinear crack face stress field, the weight functions were validated against finite element data. Stress intensity factors were also calculated for the Lame through the thickness stress distribution induced by internal pressure. The weight functions appear to be particularly suitable for fatigue and fracture analysis of surface semi-elliptical cracks in complex stress fields. All stress intensity factor expressions given in the paper are valid for cylinders with the inner radius to wall thickness ratio, Ri/t = 1. G. Shen and G. Glinka in 1992 has proposed a paper on determination of weight function from reference stress intensity factors. In this paper a method of deriving weight functions for cracks subjected to mode I loading is discussed. The method requires two reference stress intensity factors to be

known. The general weight function expression and its characteristic properties are used as the complementary information necessary for the determination of the unknown weight function parameters. In validation several weight functions derived by this method are compared to exact analytical weight function available in the literature. Differences are less than 2 %.

3. MATHEMATICAL MODELING

3.1 Weight function

The mathematical forms of the weight functions depend on the particular geometry of a cracked body and the mathematical approach. Shen and Glinka [8] found that a variety of existing weight functions have the same singular term and they can be accurately approximated by a general expression:

$$m\left(x,a,{}^{a}\!/_{t}\right)=\frac{2}{\sqrt{2\pi(a-x)}}\left[1+M_1\left(1-\frac{x}{a}\right)^{1/2}+M_2\left(1-\frac{x}{a}\right)+M_3\left(1-\frac{x}{a}\right)^{3/2}\right]$$

……… (3.1)

Knowing the general weight function expression (3.1), the derivation of the weight function for a particular geometrical configuration of cracked body can be reduced to the determination of parameters M_1, M_2 and M_3.

Shen and Glinka [8] have given the general form of the weight function for the deepest point A and the surface point B of a semi elliptical crack shown in Fig. 1 and Fig. 2.

For the deepest point A the weight function is given as:

$$m_A(x,a)=\frac{2}{\sqrt{\pi(a-x)}}\left[1+M_{1A}\left(1-\frac{x}{a}\right)^{1/2}+M_{2A}\left(1-\frac{x}{a}\right)+M_{3A}\left(1-\frac{x}{a}\right)^{3/2}\right]$$

…………..(3.2)

The weight function $m_B(x,a)$ associated with the surface point B has been given as:

$$m_B(x,a)=\frac{2}{\sqrt{\pi x}}\left[1+M_{1B}\left(\frac{x}{a}\right)^{1/2}+M_{2B}\left(\frac{x}{a}\right)+M_{3B}\left(\frac{x}{a}\right)^{3/2}\right]$$

………………………….(3.3)

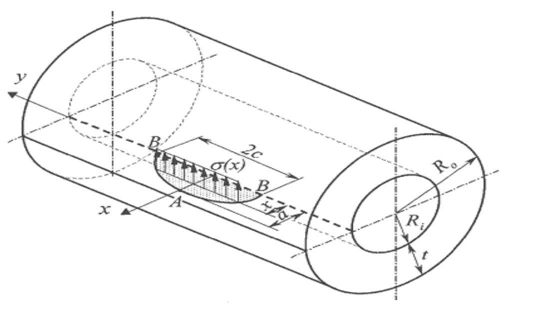


Fig. 1 Weight function notation for a semi-elliptical crack in a thick-walled cylinder

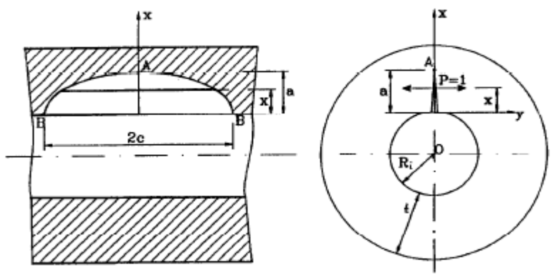


Fig. 2. Weight function notation for internal axial semi elliptical surface crack in a cylinder.

It should be noted that both weight functions have been derived for unit load P = 1 uniformly distributed along the line intersecting the crack front, as shown in Fig. 4. In the case of a continuously distributed stress field which is a function of coordinate x only, the stress intensity factors can be determined by the integration of the appropriate weight function multiplied

by the stress function [10] — :

$$K_A=\int_0^a m_A(x,a)\sigma(x)\,dx \quad \text{……… (3.4)}$$

$$K_B=\int_0^a m_B(x,a)\sigma(x)\,dx \quad \text{…… (3.5)}$$

The general weight functions (3.2) and (3.3) were successfully used earlier by Forman and Mettu [11] and coworkers to determine stress intensity factors for semi elliptical cracks in plates and thin-wall cylinders. However, in order to calculate stress intensity factors, it is necessary to determine parameters M_{iA}, and M_{iB}, in weight functions m_A and m_B, respectively. In addition the stress distribution a(x) normal to the prospective crack plane must be known. Moreover, the stress distribution should be determined for the untracked body. It is also important to note that the weight functions (3.2) and (3.3) are valid for one-dimensional stress distributions which are functions of only one coordinate x.

The parameters M_{iA} and M_{iB} can be derived using two reference stress intensity factor solutions and some properties of the weight functions. The reference stress intensity factors for internal semi elliptical axial cracks in thick cylinders were taken from Mettu who obtained them for a variety of stress distributions applied to the crack face surface by using the finite element method. The stress intensity factors obtained for the uniform and linearly varying stress field (Fig 3) chosen as the reference.

$\sigma(x)=\sigma_0$ ……….Uniform stress field……….. (3.6)

$\sigma(x)=\sigma\left(\frac{x}{a}\right)$ ..Linearly increasing stress field (3.7)

The stress intensity factors obtained for stress distributions (3.6) and (3.7) were given in the form of the geometry correction factors Y_i.

For the deepest point A:

$$K_0^A = \sigma_0 \sqrt{\frac{\pi a}{Q}} Y_0 \text{ …Uniform stress field} \quad (3.8)$$

$$K_1^A = \sigma_0 \sqrt{\frac{\pi a}{Q}} Y_1 \text{ ..Linearly increasing stress field} \quad (3.9)$$

For the surface point B:

$$K_0^B = \sigma_0 \sqrt{\frac{\pi a}{Q}} F_0 \text{ ……Uniform stress field} \quad (3.10)$$

$$K_1^B = \sigma_0 \sqrt{\frac{\pi a}{Q}} F_1 \text{ ..Linearly increasing stress field} \quad (3.11)$$

Where $Q = 1 + 1.464\left(a/c\right)^{1.65}$

The geometry correction factors Y_0, Y_1, F_0 and F_1 were given in a tabular form for a verity of crack depth a/t and crack aspect ratio a/c.

3.2 Derivation of stress intensity factors from weight function

In order to check the accuracy of the derived weight function (3.2) and (3.3) the stress intensity factors are calculated for points A and B of an internal semi elliptical axial crack by using equations (3.4) and (3.5). The stress intensity factors are calculated for several stress distributions applied to the crack surfaces: $\sigma(x) = \sigma_0 \left(\frac{x}{a}\right)^n$ where n=0,1,2,3…………….(3.12)

3.2.1 Stress intensity factor for the deepest point A

The stress intensity factors are calculated from equation (3.4) by integration of the product of the weight function (3.2) and the stress distribution given by equation (3.22)

(1) For uniform stress distribution, $\sigma(x) = \sigma_0$, n=0:

$$\frac{K_A}{\sigma_0\sqrt{\pi a/Q}} = \frac{\sqrt{2Q}}{\pi}\left[M_{1A} + \frac{2}{3}M_{2A} + \frac{M_{3A}}{2} + 2\right] = Y_0$$

(2) For linearly stress distribution, $\sigma(x) = \sigma_0$ (x/a), n =1

$$\frac{K_A}{\sigma_0\sqrt{\pi a/Q}} = \frac{\sqrt{2Q}}{\pi}\left[\frac{4}{3} + \frac{1}{2}M_{1A} + \frac{4}{15}M_{2A} + \frac{1}{6}M_{3A}\right] = \mathbf{Y_1}$$

(3) For quadratic stress distribution, $\sigma(x) = \sigma_0 \left(x/a\right)^2$, n = 2:

$$\frac{K_A}{\sigma_0\sqrt{\pi a/Q}} = \frac{\sqrt{2Q}}{\pi}\left[\frac{1}{3}M_{1A} + \frac{16}{105}M_{2A} + \frac{1}{12}M_{3A} + \frac{16}{15}\right]$$

(4) For cubic stress distribution, $\sigma(x) = \sigma_0 \left(x/a\right)^3$, n =3

$$\frac{K_A}{\sigma_0\sqrt{\pi a/Q}} = \frac{\sqrt{2Q}}{\pi}\left[\frac{1}{4}M_{1A} + \frac{32}{315}M_{2A} + \frac{1}{20}M_{3A} + \frac{32}{35}\right]$$

3.2.2 Stress intensity factor for the surface point B

(1) For uniform stress distribution, $\sigma(x) = \sigma_0$, n=0:

$$\frac{K_B}{\sigma_0\sqrt{\pi a/Q}} = \frac{2\sqrt{Q}}{\pi}\left[2 + M_{1B} + \frac{2}{3}M_{2B} + \frac{1}{2}M_{3B}\right]$$

(2) For linearly stress distribution, $\sigma(x) = \sigma_0$ (x/a), n =1

$$\frac{K_B}{\sigma_0\sqrt{\pi a/Q}} = \frac{\sqrt{Q}}{\pi}\left[\frac{4}{3} + M_{1B} + \frac{4}{5}M_{2B} + \frac{2}{3}M_{3B}\right]$$

(3) For quadratic stress distribution, $\sigma(x) = \sigma_0 \left(x/a\right)^2$, n = 2:

$$\frac{K_B}{\sigma_0\sqrt{\pi a/Q}} = \frac{\sqrt{Q}}{\pi}\left[\frac{4}{5} + \frac{2}{3}M_{1B} + \frac{4}{7}M_{2B} + \frac{1}{2}M_{3B}\right]$$

(4) For cubic stress distribution, $\sigma(x) = \sigma_0 \left(x/a\right)^3$, n =3

$$\frac{K_B}{\sigma_0\sqrt{\pi a/Q}} = \frac{\sqrt{Q}}{\pi}\left[\frac{4}{7} + \frac{1}{2}M_{1B} + \frac{4}{9}M_{2B} + \frac{2}{5}M_{3B}\right]$$

4. RESULTS AND DISCUSION

4.1 Stress Intensity Factor of semielliptical cracks in thick walled cylinder for deepest point A

(1) For uniform stress distribution, n=o

$$\frac{K_A}{\sigma\left(\pi a/Q\right)^{1/2}} = \left\{0.0998 \times e^{\left(-13.15\times\frac{a}{c}\right)} + 1.010\right\} + \left\{0.366 \times e^{\left(-31.17\times\frac{a}{c}\right)} + 0.055\right\} \times \left(\frac{a}{t}\right) + \left\{3.269 \times e^{\left(-3.859\times\frac{a}{c}\right)} - 0.057\right\}\left(\frac{a}{t}\right)^2 - \left\{0.06 \times e^{\left(1.334\times\frac{a}{c}\right)} - 0.149\right\}\left(\frac{a}{t}\right)^4$$

For linearly stress distribution, n=1

$$\frac{K_A}{\sigma\left(\pi a/Q\right)^{1/2}} = \left\{-5.994 \times e^{\left(-0.012\times\frac{a}{c}\right)} + 6.594\right\} + \left\{0.436 \times e^{\left(-8.663\times\frac{a}{c}\right)} - 0.136\right\} \times \left(\frac{a}{t}\right) + \left\{0.787 \times e^{\left(-4.562\times\frac{a}{c}\right)} + 0.269\right\}\left(\frac{a}{t}\right)^2 + \left\{-1.538 \times e^{\left(0.434\times\frac{a}{c}\right)} + 1.552\right\}\left(\frac{a}{t}\right)^4$$

(2) For quadratic stress distribution, n=2

$$\frac{K_A}{\sigma(\pi a/Q)^{1/2}} = \left\{-0.166\times e^{\left(-13.15\times\frac{a}{c}\right)}\right\} + \left\{0.061\times e^{\left(-31.17\times\frac{a}{c}\right)}\right\}\times\left(\frac{a}{t}\right) - \left\{0.544\times e^{\left(-3.859\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^2$$
$$-\left\{0.0102\times e^{\left(1.354\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^4 - \left\{5.944\times e^{\left(-0.012\times\frac{a}{c}\right)}\right\} + \left\{0.436\times e^{\left(-8.663\times\frac{a}{c}\right)}\right\}\times\left(\frac{a}{t}\right)$$
$$+\left\{0.787\times e^{\left(-4.562\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^2 - \left\{1.538\times e^{\left(0.0434\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^4 - 0.145\times\left(\frac{a}{t}\right) + 0.278\times\left(\frac{a}{t}\right)^2$$
$$+1.576\times\left(\frac{a}{t}\right)^4 + 6.426 + 0.0257\times\left\{1+1.464\times\left(\frac{a}{c}\right)^{1.65}\right\}^{1/2}$$

(4) For cubic stress distribution, n=3

$$\frac{K_A}{\sigma(\pi a/Q)^{1/2}} = \left\{-0.01996\times e^{\left(-13.15\times\frac{a}{c}\right)}\right\} - \left\{0.732\times e^{\left(-31.17\times\frac{a}{c}\right)}\right\}\times\left(\frac{a}{t}\right) - \left\{0.6538\times e^{\left(-3.859\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^2$$
$$-\left\{0.0122\times e^{\left(1.354\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^4 - \left\{5.35\times e^{\left(-0.012\times\frac{a}{c}\right)}\right\} + \left\{0.392\times e^{\left(-8.663\times\frac{a}{c}\right)}\right\}\times\left(\frac{a}{t}\right)$$
$$+\left\{0.708\times e^{\left(-4.562\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^2 - \left\{1.3842\times e^{\left(0.0434\times\frac{a}{c}\right)}\right\}\left(\frac{a}{t}\right)^4 - 0.2324\times\left(\frac{a}{t}\right) + 0.356\times\left(\frac{a}{t}\right)^2$$
$$+1.426\times\left(\frac{a}{t}\right)^4 + 5.7326 + 0.045\times\left\{1+1.464\times\left(\frac{a}{c}\right)^{1.65}\right\}^{1/2}$$

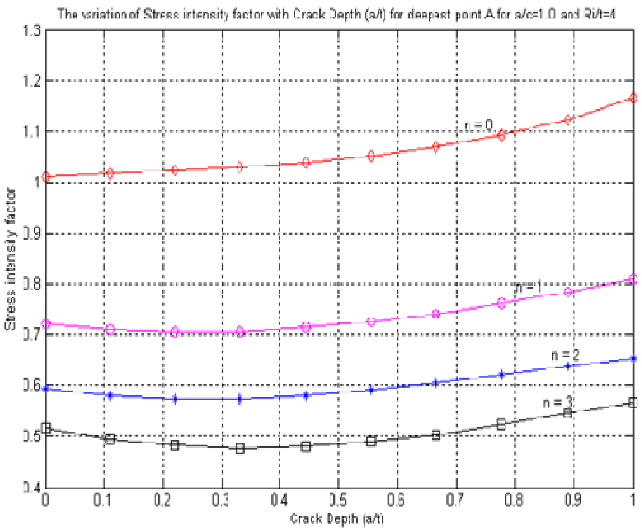


Fig. (3)

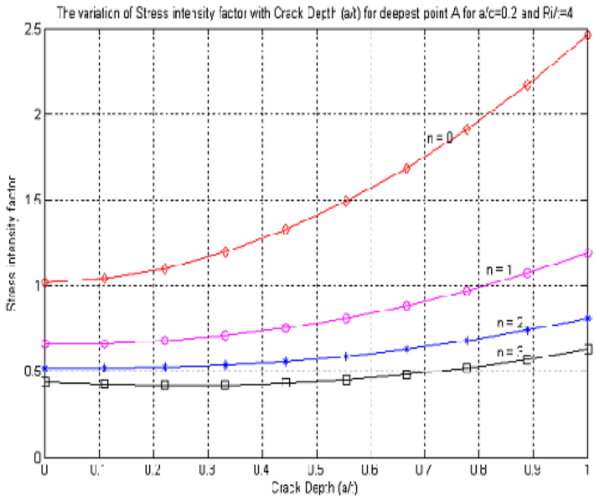


Fig (4)

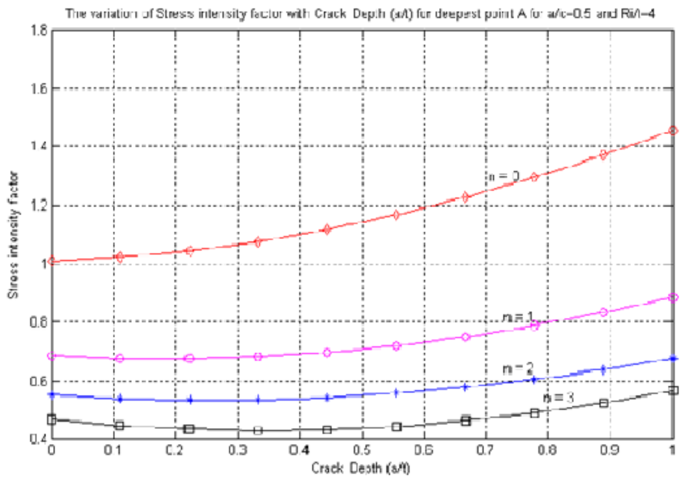


Fig. (5)

4.2 Stress intensity factor for semielliptical cracks in thick walled cylinders for surface point B

(1) For uniform stress distribution, n=0

$$\frac{K_A}{\sigma(\pi a/Q)^{1/2}} = \left[\begin{array}{l}\left\{-0.136+0.872\times\left(\frac{a}{c}\right)-1.92\times\left(\frac{a}{c}\right)^2+1.863\times\left(\frac{a}{c}\right)^3-0.664\times\left(\frac{a}{c}\right)^4\right\} \\ +\left\{-0.149+0.673\times\left(\frac{a}{c}\right)-0.593\times\left(\frac{a}{c}\right)^2-0.1995\times\left(\frac{a}{c}\right)^3+0.914\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right) \\ +\left\{1.218-4.7\times\left(\frac{a}{c}\right)+6.32\times\left(\frac{a}{c}\right)^2-3.09\times\left(\frac{a}{c}\right)^3+0.273\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^2 \\ +\left\{0.047-1.322\times\left(\frac{a}{c}\right)+5.57\times\left(\frac{a}{c}\right)^2-6.935\times\left(\frac{a}{c}\right)^3+3.05\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^4\end{array}\right]\times\left(\frac{a}{c}\right)$$

(2) For linearly stress distribution, n=1

$$\frac{K_A}{\sigma(\pi a/Q)^{1/2}}=\begin{bmatrix}\left\{0.486-0.879\times\left(\frac{a}{c}\right)+1.161\times\left(\frac{a}{c}\right)^2-0.793\times\left(\frac{a}{c}\right)^3+0.212\times\left(\frac{a}{c}\right)^4\right\}\\+\left\{-0.533+2.626\times\left(\frac{a}{c}\right)-3.412\times\left(\frac{a}{c}\right)^2+0.999\times\left(\frac{a}{c}\right)^3+0.333\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)\\+\left\{4.116-15.985\times\left(\frac{a}{c}\right)+22.358\times\left(\frac{a}{c}\right)^2-12.235\times\left(\frac{a}{c}\right)^3+1.862\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^2\\+\left\{0.569-6.605\times\left(\frac{a}{c}\right)+21.548\times\left(\frac{a}{c}\right)^2-26.37\times\left(\frac{a}{c}\right)^3+10.853\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^4\end{bmatrix}\times\left(\frac{a}{c}\right)$$

$$+0.012\times\left\{1+1.464\left(\frac{a}{c}\right)^{1.65}\right\}^{1/2}$$

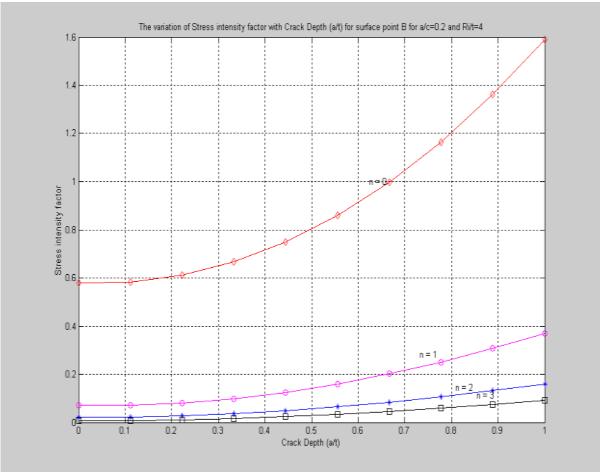
(3) For quadratic stress distribution, n=2

$$\frac{K_A}{\sigma(\pi a/Q)^{1/2}}=\begin{bmatrix}\left\{-0.05+0.766\times\left(\frac{a}{c}\right)-1.862\times\left(\frac{a}{c}\right)^2+1.83\times\left(\frac{a}{c}\right)^3-0.67\times\left(\frac{a}{c}\right)^4\right\}\\+\left\{-0.254+1.19\times\left(\frac{a}{c}\right)-1.24\times\left(\frac{a}{c}\right)^2-0.032\times\left(\frac{a}{c}\right)^3+0.35\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)\\+\left\{2.035-7.86\times\left(\frac{a}{c}\right)+10.72\times\left(\frac{a}{c}\right)^2-5.47\times\left(\frac{a}{c}\right)^3+0.61\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^2\\+\left\{0.156-2.59\times\left(\frac{a}{c}\right)+9.35\times\left(\frac{a}{c}\right)^2-12.1\times\left(\frac{a}{c}\right)^3+5.184\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^4\end{bmatrix}\times\left(\frac{a}{c}\right)$$

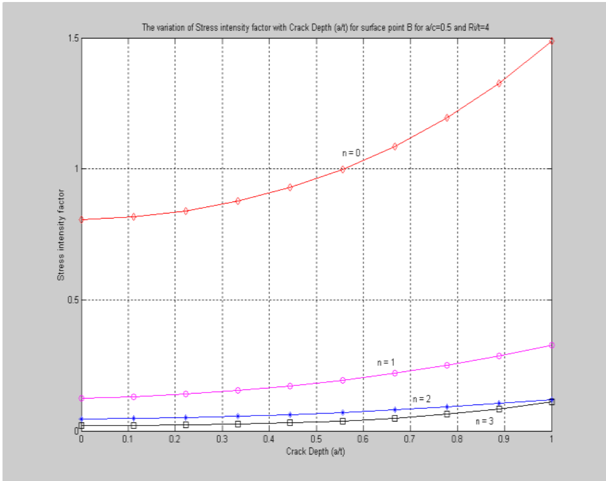
(4) For cubic stress distribution, n=3

$$\frac{K_A}{\sigma(\pi a/Q)^{1/2}}=\begin{bmatrix}\left\{-0.136+0.872\times\left(\frac{a}{c}\right)-1.92\times\left(\frac{a}{c}\right)^2+1.863\times\left(\frac{a}{c}\right)^3-0.664\times\left(\frac{a}{c}\right)^4\right\}\\+\left\{-0.149+0.673\times\left(\frac{a}{c}\right)-0.593\times\left(\frac{a}{c}\right)^2-0.1995\times\left(\frac{a}{c}\right)^3+0.914\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)\\+\left\{1.218-4.7\times\left(\frac{a}{c}\right)+6.32\times\left(\frac{a}{c}\right)^2-3.09\times\left(\frac{a}{c}\right)^3+0.273\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^2\\+\left\{0.047-1.322\times\left(\frac{a}{c}\right)+5.57\times\left(\frac{a}{c}\right)^2-6.935\times\left(\frac{a}{c}\right)^3+3.05\times\left(\frac{a}{c}\right)^4\right\}\times\left(\frac{a}{t}\right)^4\end{bmatrix}\times\left(\frac{a}{c}\right)$$

$$+0.012\times\left\{1+1.464\left(\frac{a}{c}\right)^{1.65}\right\}^{1/2}$$



Fig(6)



Fig(7)

The Fig. 8 and Fig. 9 shows the comparison of stress intensity factor calculated by the weight function technique with the finite element method given by Mettu [12].Here there is good accuracy between both the results.

The Fig. 10 and Fig.11 shows the comparison of stress intensity factor calculated by the weight function technique with the stress intensity factor calculated by hybrid weight function technique [7]. These results also show good accuracy between two techniques

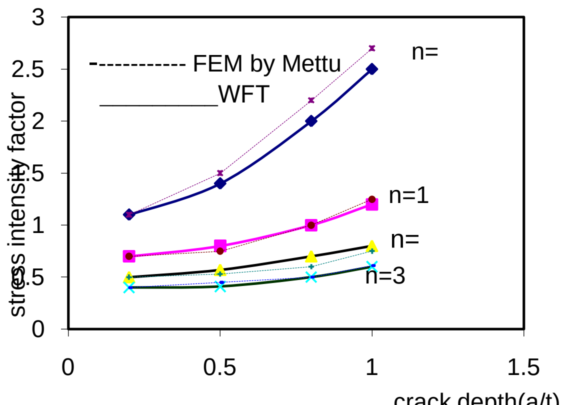


Fig. 8 comparision of SIF based on weight function technique with FEM given by Mettu for deepest point A and aspect ratio (a/c)=0.2

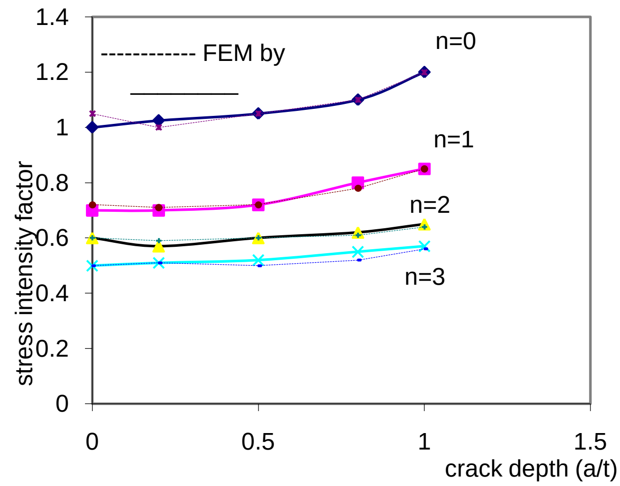


Fig. 10 Comparision of SIF based on weight function technique with FEM given by Mettu for deepest point A and aspect ratio (a/c)=1.0

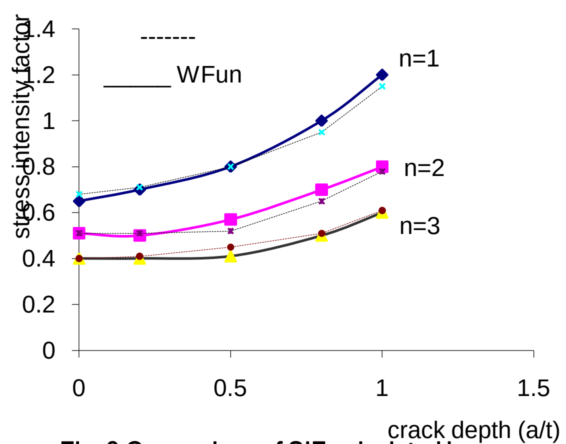


Fig. 9 Comparison of SIF calculated by weight function technique with hybrid weight function technique for crack aspect ratio (a/c)=0.2

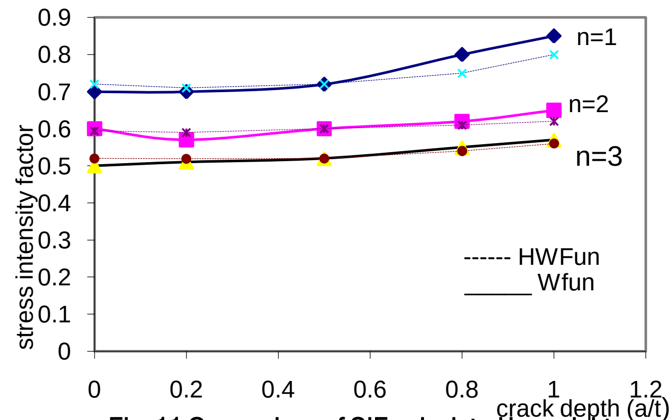


Fig. 11 Comparison of SIF calculated by weight function technique with hybrid weight function technique for (a/c)=1.0

5. CONCLUSIONS

Stress intensity factors for an internal semi elliptical crack in a thick cylinder with an inner radius to wall thickness ratio of R,/t = 4 have been derived using the weight function approach. The weight functions of the deepest and the surface point of the crack were derived using the general weight functions and two reference stress intensity factors supplied by Mettu [12]. Several stress intensity factors were subsequently calculated for a variety of crack face loadings, crack aspect ratios and crack depths. In the present study results from weight function technique has been compared with hybrid function technique and finite element method given by Mettu. The agreement with the available literature data was good over the entire range of crack aspect ratios $0.2 \leq a/c \leq 1$ and crack depths $0.1 \leq a/t \leq 0.8$. However, it should be noted that the use of the weight functions is justified for cases where the crack is fully open and the superposition principles apply. It has been shown that the simplified integration technique is very accurate and easy to perform. The weight functions and the stress intensity factors are particularly suitable for fatigue crack growth analysis.

Therefore, it is anticipated that equally good results will be obtained for any other stress distribution such as residual or thermal stresses

Scope of present work

In recent years, considerable progress has been done in calculating the stress intensity factor for various types of Mode I cracks in finite thickness plate[8] and thin walled cylinder[9]. Also for thick walled cylinder stress intensity has been calculated by finite element method [1] and by hybrid weight function technique[7]. In present work weight function are used for calculating stress intensity factor for semielliptical cracks in thick walled cylinder which gives results with great accuracy in comparison with above methods. More work is expected in this field for external cracks also. Also in recent years C -Integral and J-integral approach are also used to do the surface crack growth analysis.

6. REFERENCES

[1] Raju IS, Newman Jr JC. Stress-intensity factors for internal and external surface cracks in cylindrical vessels. Trans ASME J Press. Vessels Technol 1982, Vol. 104; 293-298.

[2] Newman Jr. JC, Raju IS. Stress-intensity factors for internal surface craks in cylindrical pressure vessels. J. pressure vessels Technology, ASME 1980, Vol. 102; 342-346.

[6] Kiciak A.,Glinka G., Burns D., J. Calculation of stress intensity factors and crack opening displacements for cracks subjected to complex stress fields. ASME vol.125, 2003; 260-266.

[7] Brahim EL Khalil Hachi, Said Rechak, Yacine Belkacemi, Gerard Maurice. Modeling of elliptical cracks in an infinite body and in a pressurized cylinder by hybrid weight function

approach, International journal of pressure vessels and piping 82, 2005; 917-924.

[8] G. Shen and G. Glinka, Weight functions for a surface semi-elliptical crack in a finite thickness plate, Theoretical and applied fracture mechanics 15, 1991; 237- 245.

[9] X. J. Zhen, G. Glinka and R. N. Dubey. Calculation of stress intensity factors for semielliptical cracks in a thin walled cylinder, International journal of press. Vessel and piping 62, 1995; 247-258.

[10] A.A. Moftakhar and G. Glinka, Calculation of stress intensity factor by efficient integration of weight function. Engineering fracture mechanics. 43, 1995: 149-156.

[11] Forman, R.G. Mettu, S.R. & Shivkumar, V. Fracture mechanics equation of pressure vessels and pipes in aerospace applications. In PVP, Vol. 241, fatigue fracture and Risk. ASME, Houston, 1992, pp 25-36.

[3] Glinka G. , Shen G. Determination of weight functions from reference stress intensity factors. Theoretical and applied fracture mechanics 15, 1991; 237-245.

[4] Mettu S.R. , Raju IS, Forman R.G. Stress intensity factors for part through surface cracks In hollow cylenders. NASA Technical report 1992.

[5] Perl M., Levy C. , Pierola J. Three-dimensional interaction effects in an internally multy-cracked pressurized thick-walled cylinder for radial and longitudinal crack arrays, ASME 1996, vol. 118; 357-368.

[12] Mettu, S.R. Raju. I.S. & forman, R.G. Stress intensity factors for part-through surface cracks in hollow cylinders. NASA technical report, No. JSC 25685 LESC 30124, 1992 ; pp 67-75.

[13] Jian- Feng Wen, Shan-Tung Tu, Jian- Ming Gong, Wei Sun. Creep fracture mechanics parameters for internal axial surface cracks in pressurized cylinders and creep crack growth analysis. Internationla journal of pressure vessel and piping 88, 2011: 452- 464.

[14] B. Mechab, B. Serier, B. Bachir Bouidjira, K. Kaddouri, X. Feaugas . Linear and non- linear analysis for semi-elliptical surface cracks in pipies under bending. Internationla journal of pressure vessel and piping 88, 2011: 57-63.

[15] Ali O. Ayhan, Ugar Yucel. Stress intensity factor equations for mixed mode surface and corner cracks in finite thickness plates subjected to tension loads. Internationla journal of pressure vessel and piping 88, 2011: 181- 188.

[16] S. LAczk, J. Rys, A.P. Zielinski. Load capacity of a thick walled cylinder with a radial hole. Internationla journal of pressure vessel and piping 87, 2010: 433- 439.

[17] M. R. Ayatollahi, H. Khoramishad. Stress entensity factors for an axially oriented internal crack embedded in a buried pipe. Internationla journal of pressure vessel and piping 87, 2010: 165- 169.