Discrete Cosine transform And Discrete Fourier Transform of RGB image

Hari Kumar Singh
Deptt. of ECE
I.E.T, M.J.P Rohilkhand University,
Bareilly,(U.P) INDIA

Prashant Kr. Maurya
Deptt. of ECE
I.E.T, M.J.P Rohilkhand University,
Bareilly,(U.P) INDIA

Khushboo Singh
Deptt. of ECE
I.E.T, M.J.P Rohilkhand University,
Bareilly,(U.P) INDIA

Pooja Singh
Deptt. of ECE
R.B.M.I, Bareilly
(UP),India

ABSTRACT
In this paper the RGB image is analyzed through DFT and DCT using MatLab tool. The DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image. The image in the spatial and Fourier domain is of the same size. The Fourier Transform is used when to access the geometric characteristics of a spatial domain image. The image in the Fourier domain is decomposed into its sinusoidal components, it is easy to observe or process certain frequencies of the image, thus influencing the geometric structure in the spatial domain. A set of DCT domain properties for shifting and scaling by real amounts, and taking linear operations such as differentiation is also described in this paper. The discrete cosine transform (DCT) is a technique for converting an image into elementary frequency components. The DCT coefficients of a sampled signal are subjected to a linear transform, which returns the DCT coefficients of the shifted, scaled and/or differentiated image. The techniques may prove useful in compressed domain processing applications, and are interesting because they allow operations from the continuous domain such as differentiation to be implemented in the discrete domain.

Keywords

DCT, linear transform, DFT, Image compression

1. INTRODUCTION
The Discrete Fourier Transform (DFT) is a specific kind of discrete transform, used in Fourier analysis. It transforms time domain function into another function, which is called the frequency domain representation, or simply the DFT, of the original function. The DFT requires an input function that is discrete. Such inputs are often created by sampling a continuous function. The discrete input function must also have a limited (finite) duration, such as one period of a periodic sequence or a windowed segment of a longer sequence. Unlike the discrete-time Fourier transform (DTFT), the DFT only evaluates that frequency component which are sufficient to reconstruct the same finite segment which was analyzed. A discrete cosine transform (DCT) expresses a sequence of finitely many data points in terms of a sum of cosine functions which are oscillating at different frequencies. In particular, a DCT is a Fourier-related transform similar to the discrete Fourier transform (DFT), but using only real numbers. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with even symmetry (since the Fourier transform of a real and even function is real and even). Like any Fourier-related transform, discrete cosine transforms express a function or an image of a sum of sinusoids with different frequencies and amplitudes. Like the discrete Fourier transform, a DCT operates on an image at a finite number of discrete data points. The obvious distinction between a DCT and a DFT is that the former uses only cosine functions, while the latter uses both cosines and sine in the form of complex exponentials. However, the major distinction is that a DCT implies different boundary conditions than the DFT or other related transforms.

2. MATHEMATICAL DESCRIPTION OF DFT AND DCT

A) Disccrete Fourier Transform:

The Fourier Transform is used in a wide range of applications, such as image analysis, image filtering, image reconstruction and image compression. The DFT is the sampled Fourier Transform and therefore does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image. The image in the spatial and fourier domain are of the same size i.e. the number of frequencies corresponds to the number of pixels in the spatial domain region.

For a square image of size N×N, the two-dimensional DFT is given by:

$$F(k,l) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} f(i,j)\; e^{-\iota 2\pi\left(\frac{ki}{N} + \frac{lj}{N}\right)}$$

where f(a,b) is the image in the spatial domain and the exponential term is the basis function corresponding to each point F(k,l) in the Fourier space. The equation can be evaluated as: the value of each point F(k,l) is obtained by multiplying the spatial image with the corresponding base function and summing the result.

The basis functions are sine and cosine waves with increasing frequencies, i.e. F(0,0) represents the dc component of the image which corresponds to the average brightness and F(N-1,N-1) represents the highest frequency. In a similar way, the Fourier image can be re-transformed into the spatial domain. The inverse Fourier transform is given by:

$$f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l)\; e^{\iota 2\pi\left(\frac{ka}{N} + \frac{lb}{N}\right)}$$

To obtain the result for the above equations, a double sum has to be calculated for each image point. However, because the Fourier Transform is separable, it can be written as

$$F(k,l) = \frac{1}{N} \sum_{b=0}^{N-1} P(k,b)\, e^{-\iota 2\pi \frac{lb}{N}}$$

$$P(k,b) = \frac{1}{N} \sum_{a=0}^{N-1} f(a,b)\, e^{-\iota 2\pi \frac{ka}{N}}$$

Where

By using these two formulas, the spatial domain image is first transformed into an intermediate image using N one-dimensional Fourier Transforms. This intermediate image is then transformed into the final image, again using N one-dimensional Fourier Transforms. Hence, the two-dimensional Fourier Transform can be expressed in terms of a series of 2N one-dimensional transforms which decreases the number of required computations

B) Discrete Cosine Transform

Formally, the discrete cosine transform is a linear, invertible function $F : \mathbf{R}^N \to \mathbf{R}^N$ (where **R** denotes the set of real numbers), or equivalently an invertible $N \times N$ square matrix. There are several variants of the DCT with slightly modified definitions. The N real numbers $x_0, \ldots, x_{N-1}$ are transformed into the N real numbers $X_0, \ldots, X_{N-1}$ according to one of the formulas:

$$X_k = \frac{1}{2}\left(x_0 + (-1)^k x_{N-1}\right) + \sum_{n=1}^{N-2} x_n \cos\left[\frac{\pi}{N-1} nk\right] \qquad k = 0, \ldots, N-1.$$

Some authors further multiply the x_0 and x_{N-1} terms by $\sqrt{2}$, and correspondingly multiply the X_0 and X_{N-1} terms by $1/\sqrt{2}$. This makes the DCT-I matrix orthogonal, if one further multiplies by an overall scale factor of $\sqrt{2/(N-1)}$, but breaks the direct correspondence with a real-even DFT.

The DCT-I is exactly equivalent (up to an overall scale factor of 2), to a DFT of $2N-2$ real numbers with even symmetry. For example, a DCT-I of N=5 real numbers *abcde* is exactly equivalent to a DFT of eight real numbers *abcdedcb* (even symmetry), divided by two. (In contrast, DCT types II-IV involve a half-sample shift in the equivalent DFT.). However, that the DCT-I is not defined for N less than 2. (All other DCT types are defined for any positive N.)

Thus, the DCT-I corresponds to the boundary conditions: x_n is even around n=0 and even around n=N-1; similarly for X_k. The 2-D DCT is a direct extension of the 1-D case and is given by:

$$C(u,v) = \alpha(u)\alpha(v) \sum_{x=0}^{N-1}\sum_{y=0}^{N-1} f(x,y) \cos\left[\frac{\pi(2x+1)u}{2N}\right] \cos\left[\frac{\pi(2y+1)v}{2N}\right],$$

for u,v = 0,1,2,…,N −1 and α(u) and α(v) are defined as:

$$\alpha(u) = \begin{cases} \sqrt{\frac{1}{N}} & for \quad u = 0 \\ \sqrt{\frac{2}{N}} & for \quad u \neq 0. \end{cases}$$

The inverse transform is defined as:

$$f(x,y) = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \alpha(u)\alpha(v) C(u,v) \cos\left[\frac{\pi(2x+1)u}{2N}\right] \cos\left[\frac{\pi(2y+1)v}{2N}\right]$$

for x,y=0,1,2,…………N-1.

Algorithm of DCT Process:

General overview of DCT process are:

1) The image is broken into 8x8 blocks of pixels.
2) DCT is applied to each block working from left to right and top to bottom.
3) And in the next step each block is compressed through quantization.
4) The array of compressed blocks that constitute the image is stored in drastically reduced amount of space.
5) The image can be again reconstructed using the process, Inverse Discrete Cosine Transform (IDCT).

In the JPEG image compression algorithm, the input image is divided into 8-by-8 or 16-by-16 blocks, and the two-dimensional DCT is computed for each block. The DCT coefficients are then quantized, coded, and transmitted. It is computationally easier to implement and more efficient to regard the DCT as a set of basic functions which given a known input array size (8 x 8) can be pre computed and stored. This involves simply computing values for a convolution mask (8 x8 window) that get applied (sum values x pixel the window overlap with image apply window across all rows/columns of image). The values as simply calculated from the DCT formula. The 64 (8 x 8) DCT basis functions are illustrated in Fig1. The output array of DCT coefficients contains integers.

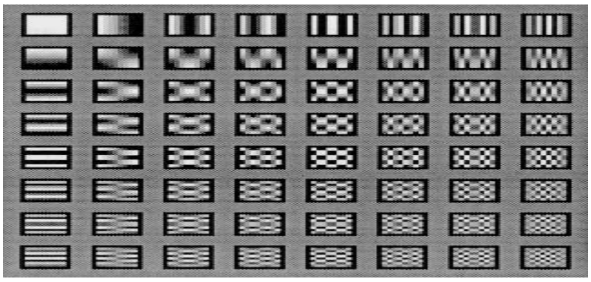

Figure 1: Two dimensional DCT basis functions (N = 8). Neutral gray represents zero, white represents positive amplitudes, and black represents negative amplitude

3. RESULTS AND DISCUSSIONS

Comparative analysis of DCT with respect to DFT is that the DCT is conceptually similar to the DFT, except, The DCT does a better job of concentrating energy into lower order coefficients than does the DFT for image data. The DCT is purely real, the DFT is complex (magnitude and phase). A DCT operation on a block of pixels produces coefficients that are similar to the frequency domain coefficients produced by a DFT operation. An N-point DCT has the same frequency resolution as and is closely related to a 2N-point DFT. The N frequencies of a 2N point DFT correspond to N points on the upper half of the unit circle in the complex frequency plane. Assuming a periodic input, the magnitude of the DFT coefficients is spatially invariant (phase of the input does not matter). This is not true for the DCT.

Figure2: Original image

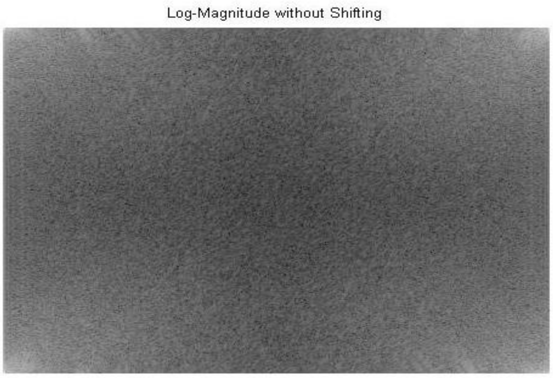

Figure 3a:Logarithmic magnitude of DFT coefficients of image without shifting of pixel values.

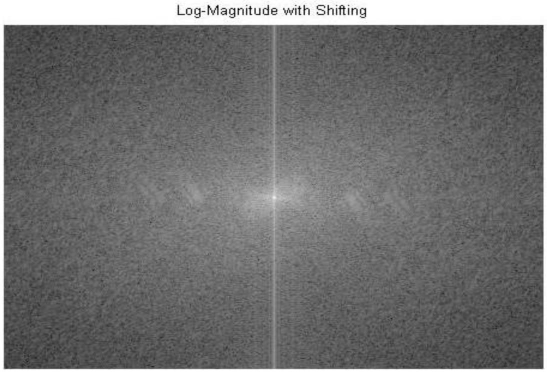

Figure 3b: Logarithmic magnitude of DFT coefficients of image with shifting of pixel values.

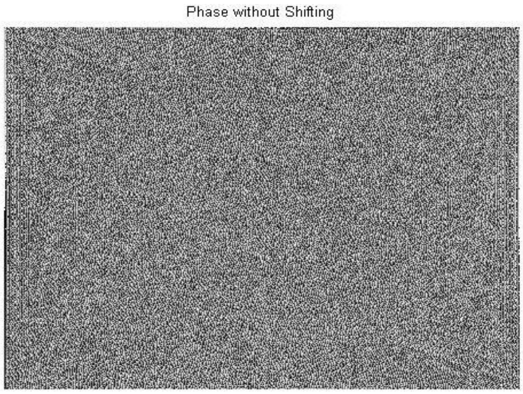

Figure 3c: Phase of DFT coefficients of image without centre shifting of pixel values.

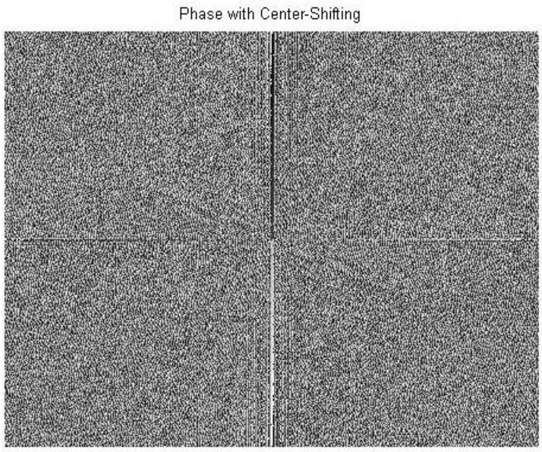

Figure 3d: Phase of DFT coefficients of image with centre shifting of pixel values.

The result of logarithmic magnitude of 2D DFT coefficients shows that the image contains components of all frequencies, but that their magnitude gets smaller for higher frequencies. Hence, low frequencies contain more image information than the higher ones. The transformed image in Fourier domain also tells us that there are two dominating directions in the Fourier image, one passing vertically and one horizontally through the center. The phase image does not yield much new information about the structure of the spatial domain image.

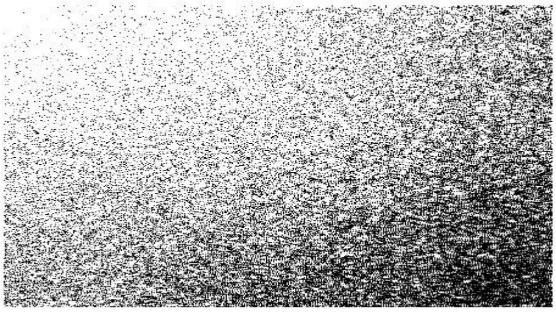

Figure 4a: 2D Discrete Cosine Transform Of Original image

Figure 4b: Compressed 2D DCT of Original image

Figure 4c: Reconstructed original image from compressed 2D DCT of original image.

The dct2 function in matlab tool computes the two-dimensional discrete cosine transform (DCT) of an image. The DCT has the property that, for a typical image, most of the visually significant information about the image is concentrated in just a few coefficients of the DCT. This is also one of the major reason why the DCT is used in compression techniques.

4. CONCLUSION

The Fourier Transform is used when to access the geometric characteristics of a spatial domain image. The image in the Fourier domain is decomposed into its sinusoidal components, it is easy to observe or process certain frequencies of the image, thus influencing the geometric structure in the spatial domain. In image processing, only the magnitude of the Fourier Transform is displayed, as it contains most of the information of the geometric structure of the spatial domain image.

The 2D DCT techniques may prove useful in compressed domain processing applications, and are interesting because they allow operations from the continuous domain such as differentiation to be implemented in the discrete domain. Although there is some loss of quality in the reconstructed image, it is clearly recognizable, even though almost 85% of the DCT coefficients were discarded.

5. REFRENCES

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