Modal Analysis of Optical Waveguide using Finite Element Method

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ABSTRACT
The optimization of the performance of optical waveguides requires the knowledge of the propagation characteristics, field distribution and their dependence on the fabrication parameters. As the range of guiding structures and the depending parameters becomes more intricate, the need for computer analysis becomes greater and more demanding. Therefore, there is a great deal of interest in theoretical methods of waveguide analysis. The Finite Element Method (FEM) gives the elaborate and in depth analysis of the waveguide problems in all dimensions. This project presents a method for computing the propagation modes of an optical fiber. Finite element Method Analysis reduces Maxwell's equation to standard eigen value equation involving symmetric tri-diagonal matrices. Routines compute their eigen values and eigenvectors, and from these the waveforms, propagation constants, and delays (per unit length) of the modes are obtained. The method is reliable, economical, and comprehensive, applying to both single and multimode fibers with different refractive index profiles.

Keywords
Eigenvalue, Eigenvector, FEM, Field distribution, Open boundary problem.

1. INTRODUCTION
Recently, a method employing finite element analysis to investigate the propagation characteristics of circular waveguide with arbitrary refractive index profile has attracted the attention of many researchers. Routines compute their eigen values and eigenvectors, and from these the waveforms, propagation constants, and delays (per unit length) of the modes are obtained. The method becomes a powerful tool throughout engineering [9]. Generally, Optical fiber is dielectric waveguide that operates at optical frequencies. It confines electromagnetic energy in the form of light. Optical fiber Guides the light in a direction parallel to its axis.

2. TYPES OF ANALYSIS
Analysis of optical waveguide is divided into following two types.

2.1: Analytical method

2.2: Numerical method

2.1 Analytical method
The analytical method of analysis gives the exact solution of the analysis. In this method, the scalar wave equation is obtained from Maxwell's equations. The mode field distribution in optical fiber can be solved using Maxwell's equation. Even though the method gives exact solution, the method is somewhat complicated as compare to numerical method.

2.2 Numerical method
The numerical method is another method used for the same purpose. The Finite Element Method is one type of this method. Recently, a method employing finite element analysis to investigate the propagation characteristics of circular waveguide with arbitrary refractive index profile has attracted the attention of many researchers.

Finite element method (FEM) is a numerical method for solving a differential or integral equation. It has been applied to a number of physical problems, where the governing differential equations are available. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution. In this article, a brief introduction to finite element method is provided. The method is illustrated with the help of the plane stress and plane strain formulation.

2.2.1 Brief History of FEM
The term finite element was first coined by Clough in 1960. In the early 1960s, engineers used the method for approximate solutions of problems in stress analysis, fluid flow, heat transfer, and other areas. The first book on the FEM by Zienkiewicz and Chung was published in 1967. In the late 1960s and early 1970s, the FEM was applied to a wide variety of engineering problems.

2.2.2 Description of FEM
FEM cuts a structure into several elements (pieces of the structure) then reconnects elements at "nodes" as if nodes were pins or drops of glue that hold elements together. This process results in a set of simultaneous algebraic equations.

Finite element method analysis reduces Maxwell's equation to standard Eigen value equation involving symmetric tridiagonal matrices. The method is reliable & economical. It can be applied to both single & multimode fibers.

2.2.3 Finite Element Method

Finite element method (FEM) is a numerical method for solving a differential or integral equation. It has been applied to a number of physical problems, where the governing differential equations are available. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution. In this article, a brief introduction to finite element method is provided. The method is illustrated with the help of the plane stress and plane strain formulation [8].

FEM formulation for a linear differential equation A linear differential equation can be of the following form:

$$\mathbf{Lu + q = 0} \qquad \textbf{(1)}$$

Where u is the vector of primary variables of the problem, which are functions of the coordinates, L is the differential operator and q is the vector of known functions. This differential equation will be subjected to boundary conditions, which are usually of two types.

(i) The essential boundary conditions: - The essential boundary conditions are the set of boundary conditions that are sufficient for solving the differential equations completely.

(ii) the natural boundary conditions:- The natural boundary conditions are the boundary conditions involving higher order derivative terms and are not sufficient for solving the differential equation completely, requiring at least one essential boundary condition.

For example, consider the differential equation:

$$\frac{d}{dx}\left(EA\frac{du}{dx}\right)+q=0 \qquad (2)$$

This problem can be solved completely under one of the following two conditions:

(i) u is prescribed at both ends.

(ii) u is prescribed at one end and du/dx is prescribed at the same or other end.

However, the problem cannot be solved if only du/dx is prescribed at both ends. Thus, we surely require one boundary condition prescribing u Therefore, for this problem u= u* is an essential boundary condition and du/dx= (du/dx)* is a natural boundary condition, where * indicates the prescribed value.

Now consider the differential equation:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right)-q=0 \qquad (3)$$

This differential equation can be solved completely by specifying w and dw/dx at both ends. One can also specify d2w/dx2 and/or d3w/dx3 as boundary conditions, however out of total four boundary conditions; two must be of one of the following forms:

(i) w prescribed at both ends.

(ii) w prescribed at one end and dw/dx prescribed at the other end.

Thus, the prescribed values of w and dw/dx form the part of essential boundary conditions and prescribed values of d2w/dx2 and d3w/dx3 form the part of natural boundary conditions.

Two popular FEM formulations are Galerkin formulation and Ritz formulation. In Galerkin formulation, the primary variable is approximated by a continuous function inside the element. When the approximate primary variable u^e is substituted in Eq. (1), we shall get residue depending on the approximating function

$$L\boldsymbol{u}^e + \boldsymbol{q} = \boldsymbol{R} \qquad (4)$$

Ideally, the residue should be zero everywhere. In that case, approximation becomes equal to true value. As it is very difficult to make the residue 0 at all points, we make the weighted residual equal to zero, i.e.,

$$\int_D w\boldsymbol{R}\,dA = 0 \qquad (5)$$

Where w is the weight function. In order to weaken the requirement on the differentiability of the approximating function, we integrate Eq. (5) by parts to redistribute the order of derivative in w and R. In Galerkin method, the weight function is chosen of the same form as the approximating function. The approximating function is some algebraic function. It is common to replace the unknown coefficients of the function by unknown nodal degrees of freedom. Thus, typically,

$$\boldsymbol{u}^e = [N]\left\{\boldsymbol{u}^{ne}\right\} \qquad (6)$$

Where [N] is the matrix of shape functions and $\{u^{ne}\}$ is the nodal degrees of freedom.

In Ritz formulation, the differential equation Eq. (1) is converted into an integral form using calculus of variation. (Sometimes the integral form itself may be easily derivable from the physics of the problem.) The approximation (Eq. (6)) is substituted in the integral form and the form is extremized by partially differentiating with respect to $\{u^{ne}\}$.

After obtaining the elemental equations, the assembly is performed. A simple way of assembly is to write equations for each element in global form and then add each similar equations of all the elements, i.e., we add the equation number 1 from each element to obtain the first global equation, all equation number 2 are added together to give second equation, and so on. The boundary conditions are applied to assembled equation and then are solved by a suitable solver. Then, post-processing is carried out to obtain the derivatives.

3. METHODOLGY USED FOR THE ANALYSIS

The present work concentrates on Finite Element Method for evaluation of mode field distribution in case of single and multimode fiber. The Finite Element Method gives elaborate and in depth analysis of the waveguide problems. FEM using Linear and Quadratic elements is used to reduce scalar wave equations to standard eigen values and eigen vectors. These are further used to calculate the mode field distribution and propagation constant for LP01, LP11, LP21 and LP31 modes. These results are compared with the results of Analytical method. Dispersion of single mode and multimode fibers are also obtained using FEM.

Analytical method used for the analysis of mode field distribution gives the exact solution. Analytical method reduces the Maxwell's equation into scalar wave equation. By changing the values of core radius (a), we obtain different field and contour distribution for modes such as LP 01 mode, LP 11 mode, LP 21 mode & LP 31 mode.

4. TYPES OF FEM USED FOR THE ANALYSIS

4.1. Linear Finite Element Method.

In this section, we develop the Finite-Element Method (FEM) using linear elements for solving the scalar wave equation for an optical fiber with arbitrary refractive index profile. The method is developed for various refractive index profiles. The convergence studies have been carried out. The key in the procedure is to select the number and the location of the nodes in the element so that the geometry of the latter is uniquely defined. The number of nodes must be sufficient to allow the assumed degree of interpolation of the solution in terms of primary variables. For a linear polynomial approximation, two nodes with one primary unknown per node are sufficient to define geometry of the element. In this method the core radius is divided into 'L' elements. Each element has two nodes which are the ends of the element.(See fig.4.1)

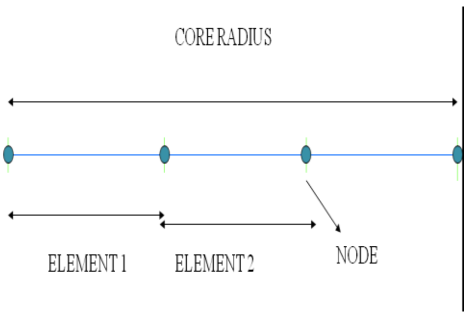

Fig. 4.1

4.2. Quadric Finite Element Method.

This is another type of finite element method used for the analysis of optical fiber. In this method, let us consider an axially symmetric optical fiber of radius a with refractive index n(r) in the core and n_2 in the cladding. To solve the scalar equation each element is divided into three nodes (unlike two nodes in Linear FEM). To define the geometry, two nodes must be end points of the element. The third element is the midpoint. This process is nothing but Discretization.(See fig. 4.2)

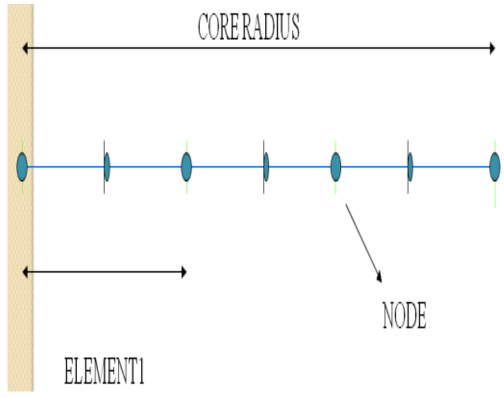

Fig. 4.2

6. ADVANTAGES OF FINITE ELEMENT METHOD

The following are some advantages of finite element method.

i. It can readily handle very complex geometry.

ii. The heart and power of the FEM can handle a wide variety of engineering problems such as solid mechanics, electrostatic problems

iii. This method can handle complex restraints.

iv. This method can convert complicated mathematical calculations into simple calculations.

7. RESULTS

Three different modes are implemented namely LP01, LP11, LP21 and LP31.The analysis of fiber modes is carried out for different mode numbers. The results of the four different modes are compared with analytical method. The mode field approximation is increased from Linear to Quadratic FEM.

8. CONCLUSION

Analytical method used for the analysis of mode field distribution gives the exact solution. Analytical method reduces the Maxwell's equation into scalar wave equation. By changing the values of core radius (a), we obtain different field and contour distribution for modes such as LP 01 mode, LP 11 mode, LP 21 mode & LP 31 mode.

Finite element method (FEM) is used to analyze the modal quantities of an optical fiber. FEM analysis reduces the Maxwell's equation into standard Eigen value equation. The FEM provides approximate solution to the scalar wave equation. Accuracy increases from Linear to Quadratic FEM and increasing the no. of elements. The Finite Element Method (FEM) gives the elaborate and in depth analysis of the waveguide problems in all dimensions.

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