Scattering Matrix Calculation of Waveguide Corner Distorted by Discontinuities using FEM Solution of 2D Helmholtz Equations

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ABSTRACT
A simple method is presented to obtain S-parameters of two dimensional dielectric waveguide corner distorted by discontinuities. We can analyze by using three different cases are considered: Length of the waveguide remains same on both axes (x and y axis), changing the length in x-axis and y-axis directions. S-parameters can compute by using integral expressions of S_{11} and S_{12} can be derived from FEM solution of two dimensional Helmholtz equation and numerical results are tabulated and compared.

Keywords
Waveguide discontinuities, finite element method (FEM), scattering matrix, two dimensional Helmholtz equations.

1. INTRODUCTION
Discontinuities in waveguides can be defined as defects in waveguide structure produced by two or more factious boundaries. These defects occur inevitably in many waveguide systems due to construction tolerance and misalignment in the component intersection [1]-[3]. Physical mathematical model is explaining the exciting and classification of electromagnetic waves in an anisotropic waveguide in the three dimensional case. Finite difference techniques are used to find s-matrix parameter calculation of waveguide discontinuity problem [4].This paper explain the application of the generalized scattering matrix resemble to analyses of waveguide structure, in which the finite difference time domain (FDTD) method is use to find generalized s-matrix [5].Waveguide transitions containing ports where mode are in cutoff are analyzed by using cross section method and also coupling co-efficient for modes are solved [6].

In this paper, FEM solution of 2D Helmholtz equations is applied to the analysis of waveguide corner distorted by discontinuities of using three different cases and numerical results are tabulated and compared.

2. THEORY
Simulation of electromagnetic wave propagation is a very simple method in the two dimensional Helmholtz problem is also called in-plane wave propagation. TM wave propagation will be analyzed in a parallel plane waveguide structure with a 90 deg bend. Bend is an example of waveguide discontinuity and two dimensional simulations are useful for the design of three dimensional real life structures at next stage

Important theory background of a TM in plane wave propagates in the x and y plane and there is no propagation in the z-direction. Therefore we have components E and H that is $E_z(x,y)$, $H_x(x,y)$ and $H_y(x,y)$ are not equal to zero. Using two dimensional Helmholtz equation TM mode solutions can be obtained as follows [8].

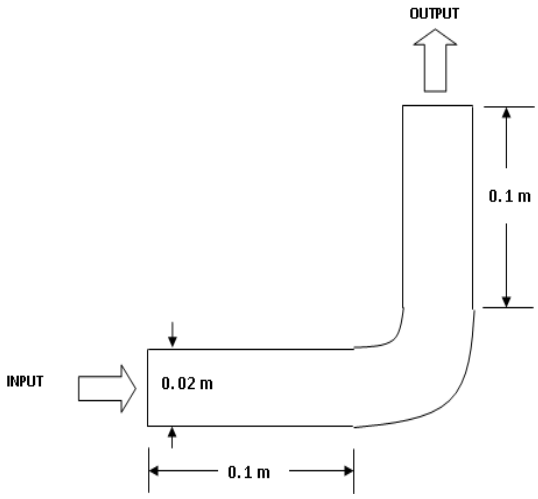


Figure.1

Consider two dimensional Helmholtz equations

$$\nabla_t E_z + \gamma^2 E_z = 0 \quad \ldots (1)$$

Where $\gamma^2=\omega^2 \mu\varepsilon$ is the propagation constant $\mu = \mu_0,\ \varepsilon = \varepsilon_0$ material properties of the waveguides and the frequency $f=9\times10^9$ Hz. Waveguide walls are made up of conducting materials that is perfect electric conductor (PEC), we experience following boundary conditions

$$E_{z(x,y)}=0 \ (x,y)\epsilon\delta_{1\Omega} \quad \ldots\ldots\ldots (2)$$

For FEM calculations, consider bend type waveguide (90deg bend) and with PEC walls as shown in figure. Usually connected waveguides to the bend are infinitely long. In order to obtain standard solution consider waveguide with finite at some distance from the bend we have to terminate our waveguides. The termination ports are called input and output ports and to be defined in such way that no reflection of electromagnetic wave back in to the structure. Waveguide terminations is called absorb boundary conditions (ABC).

First consider the x-axis port that is input port of our structure shown in figure. We assume that our structure will be set in motion with fundamental even mode of a two dimensional parallel waveguide the following analytically solution is obtained.

$$E_{01}\ (x,y) = C(y)e^{-J\gamma\ x} \quad \ldots\ldots\ldots. (3)$$

Where c is a constant. Electromagnetic wave propagates along x-axis via middle of the waveguide and y-axis with parameters γ_x and γ_y are the x and y-axis components of the wave numbers

$$\gamma = \omega\sqrt{\mu\varepsilon} = \sqrt{\gamma_x{}^2 + \gamma_y^2} \text{ .... (4)}$$

The PEC is an electromagnetic to be sense the material with $\sigma = \infty$.At any point of waveguide on x-axis, the electric field Ez will be equal super position that is incoming wave and the reflected wave.

This can written in mathematical form as follows

$$E_z \ (x, y) = C(y)e^{-j\gamma x} + \mathrm{RC}(y)e^{j\gamma x} \text{ .... (5)}$$

$$\text{Where R=} \frac{Incoming \ \ wave}{reflected \ \ wave} = -e^{-2\gamma x} \text{ .... (6)}$$

From the above equation.5 the reflected wave is positive as opposed to the negative components at the incoming wave and equation (5) cannot be used as the ABC at input port without knowing the reflection co-efficient in advance, the equation (5) can be modified and used as ABC mode with a particular mode.

Differentiate equation (5) with respect to

$$\frac{\delta E_z}{\delta x}(x,y) = -J\gamma_x C(y)e^{-j\gamma x} + J\gamma_x \mathrm{RC}(y)e^{j\gamma x} \text{ ... (7)}$$

From equation (5)

$$\mathrm{RC}(y) \ e^{j\gamma x} = E_z \ \ (x, y) - C(y)e^{-j\gamma x} \text{ ....(8)}$$

From above equation (5) we can obtain the formula

$$\frac{\delta E_z}{\delta x}(x,y) = -2J\gamma_x C(y)e^{-j\gamma x} + J\gamma_x E_z(x, y) \text{....... (9)}$$

The above equation shows a special case of a much more general radiation boundary condition used in scattering method theory [7,8] same boundary condition can be derived at the output of waveguide that is propagation of wave in y-direction.

$$E_{02} \ \ (x, y) = \mathrm{T} \ \mathrm{C}(x)e^{-J\gamma \ y} \text{ .... (10)}$$

$$\frac{\delta E_z}{\delta y}(x, y) = -J\gamma_y E_z(x, y) \text{........... (10.a)}$$

$$\text{Where R=} \frac{Incoming \ \ wave}{Transmitted \ \ awave} = -e^{-2\gamma y} \text{. (11)}$$

Using the same procedure we can obtain the following ABC condition at the output port of bend type of waveguide in the y-direction.

$$\frac{\delta^2 E_z}{\delta x^2} + \frac{\delta^2 E_z}{\delta y^2} + \gamma^2 \ E_z = 0, \ (x, y)\epsilon\Omega \text{ ... (12)}$$

$$E_{z(x,y)} = 0 \ (x, y)\epsilon\delta_1\Omega - \text{Waveguide wall (PEC) ... (13)}$$

$$\frac{\delta E_z}{\delta x}(x,y) = -2J\gamma_x C(y)e^{-j\gamma x} + J\gamma_x E_z(x, y), (x, y)\epsilon\delta_2\Omega$$

Input port (ABC)....... (14)

$E_{02} \ (x, y) \ = -J\gamma_y E_z(x, y), \ \ (x, y)\epsilon\delta_1\Omega$ Output port (ABC)........... (15)

Using above solution we can compute the scattering matrix or S-Parameters:

$$S_{11} = \frac{\int_{\delta 2\Omega}(E_z - E_{1z}).E_{1z} \, dl}{\int_{\delta 2\Omega} E_{1z} \, .E_{1z} \, dl} \text{............. (16)}$$

$$S_{12} = \frac{\int_{\delta 3\Omega}(E_z \, .E_{2z} \, dl}{\int_{\delta 3\Omega} E_{2z} \, .E_{2z} \, dl} \text{......... (17)}$$

3. RESULTS AND DISCUSSION

Case.1 Waveguide with 90 deg bend, length remains same on both axes (Length in mtrs)

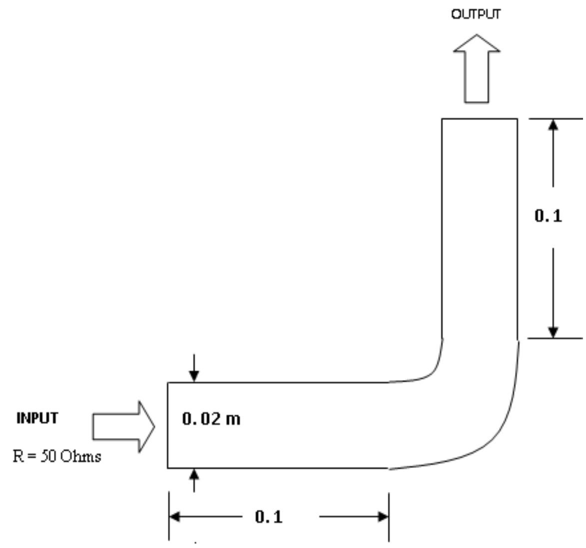
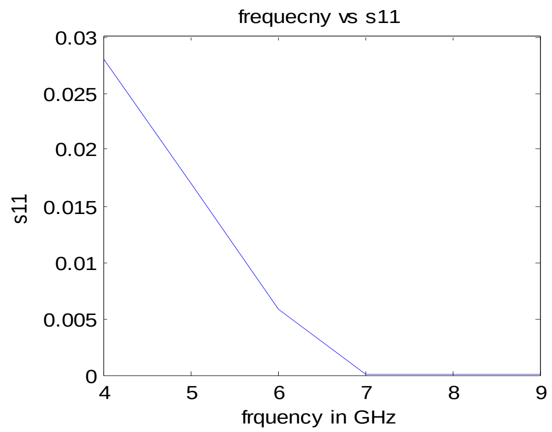
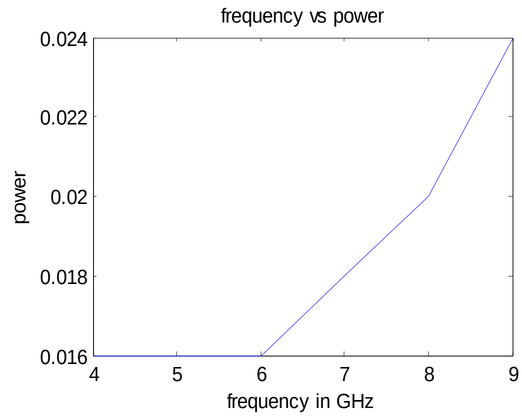


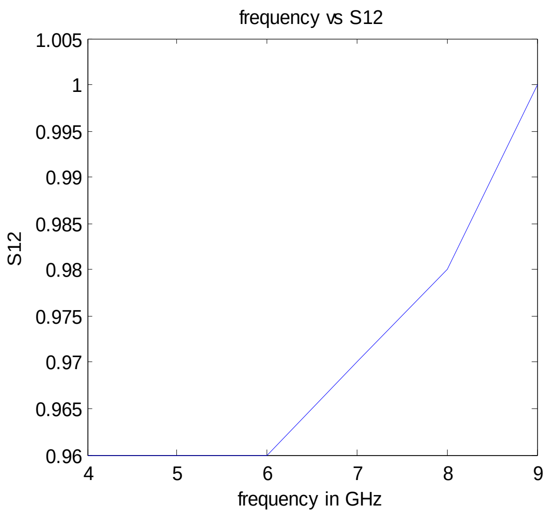
Figure.2

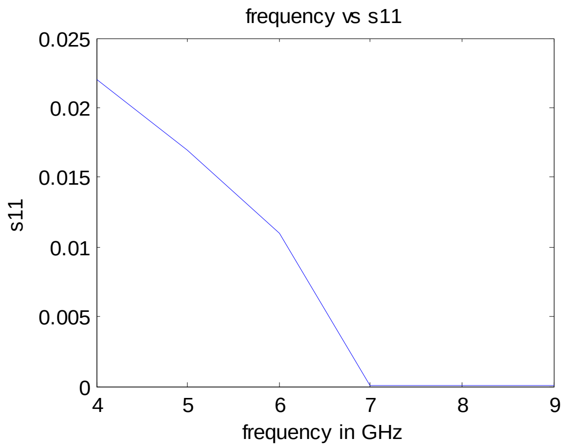
Table 1: Paramters of waveguide with 90 deg bend

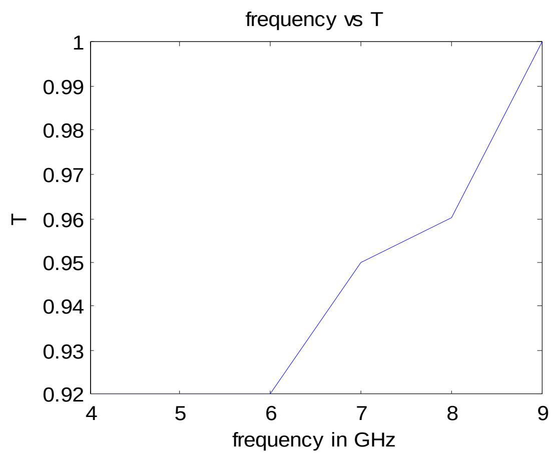
Freq in GHz	Wave number (γ)	S11	S12	T=|S11|2+|S12|2	S11/S12	P=V2/R
4GHz	83.69	0.028	0.96	0.92	0.029	0.0169 w
5GHz	104.8	0.017	0.96	0.92	0.017	0.0169 w
6GHz	125.74	0.0058	0.96	0.92	6.041x 10-3	0.0169 w
7GHz	146.71	0.07014x 10-3	0.978	0.95	7.171x 10-5	0.0180 w
8GHz	167.66	0.06772x 10-3	0.98	0.96	6.9102 x10-5	0.0184 w
9GHz	188.30	0.001727 x10-3	1.052	1.106	1.641x 10-6	0.0244 w











Case.2 Waveguide with 90 deg bend, length varies on both the axes (Length in mtrs).

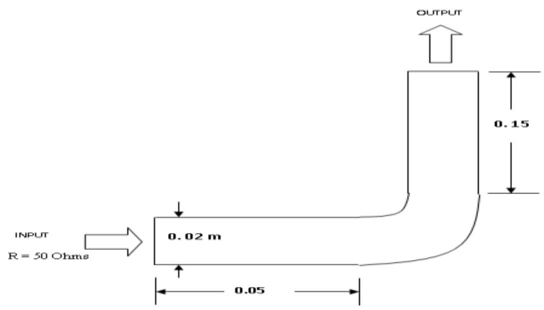
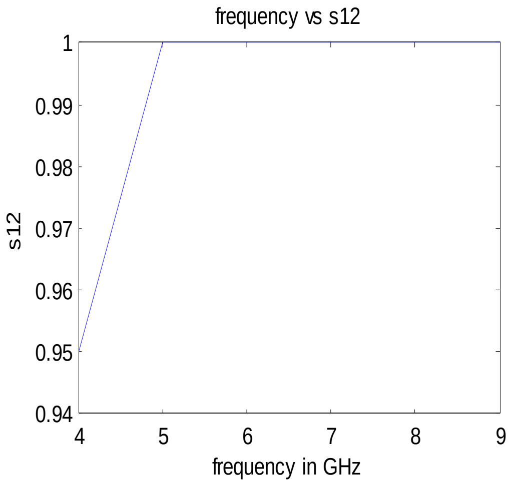
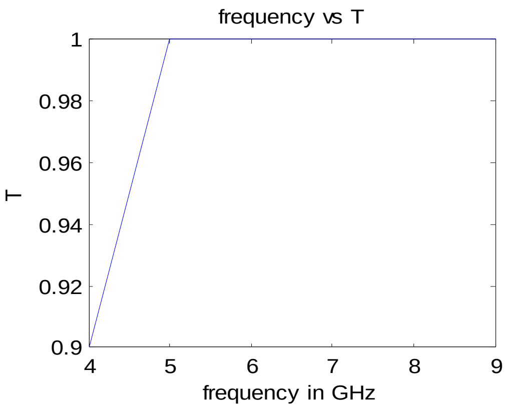


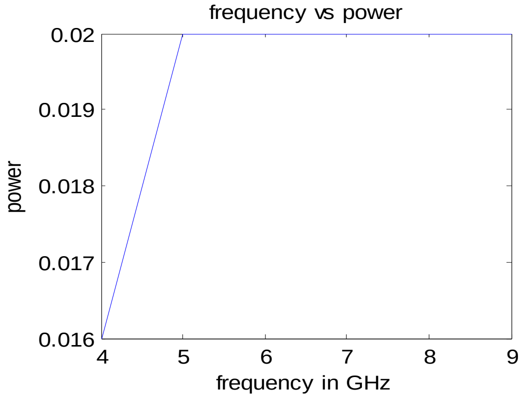
Figure.3

Table.2 Parameters of waveguide with 90 deg bend

Freq in GHz	Wave number (γ)	S11	S12	T=|S11|2+|S12|2	S11/ S12	P=V^2/R
4GHz	83.69	0.022	0.95	0.90	0.023	0.016 w
5GHz	104.8	0.017	1.00	1.00	0.016	0.020 w
6GHz	125.74	0.011	1.00	1.00	0.010	0.020 w
7GHz	146.71	0.09948x 10-3	1.00	1.00	9.919x10 -5	0.020 w
8GHz	167.66	0.0873x1 0-3	1.00	1.00	8.558x10 -5	0.020 w
9GHz	188.30	0.07163x 10-3	1.00	1.0	7.120x10 -5	0.020 w







Case.3 Waveguide with 90 deg bend, length varies on both the axes (Length in mtrs).

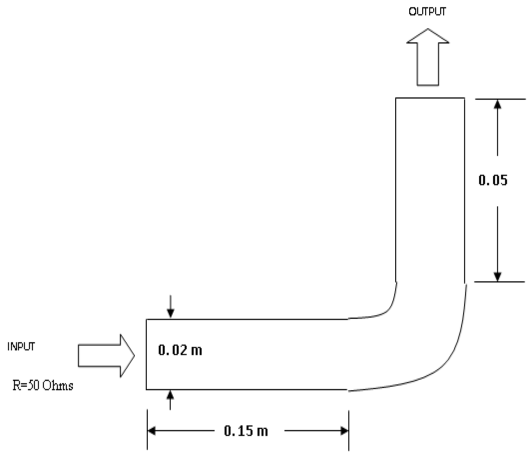
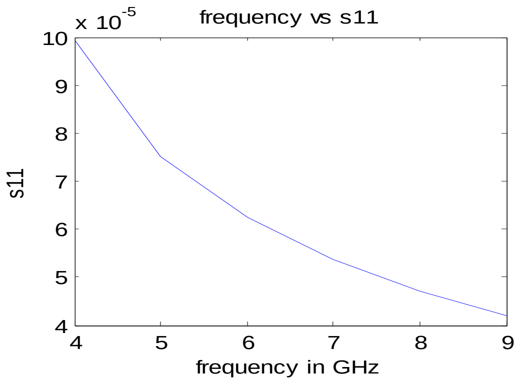
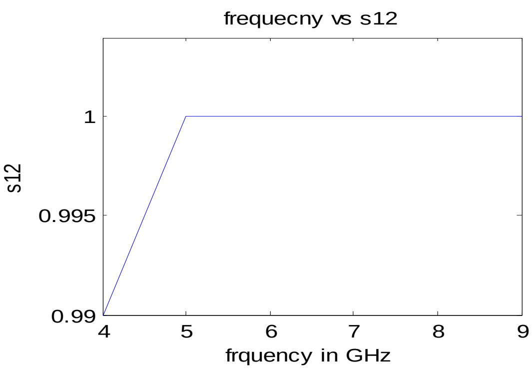


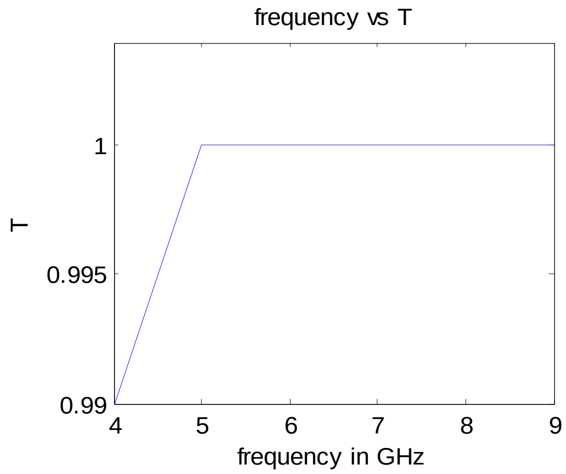
Figure.4

Table.3 Parameters of waveguide with 90 deg bend

Freq in GHz	Wave number (γ)	S11	S12	T=|S11|2+|S12|2	S11/S12	P=V^2/R
4GHz	83.69	0.0926x10-3	1.00	1.00	0.096x10-3	0.022 w
5GHz	104.8	0.07515x10-3	1.00	1.00	0.073x10-3	0.021 w
6GHz	125.74	0.06244x10-3	1.00	1.00	0.0615x10-3	0.020 w
7GHz	146.71	0.0538x10-3	1.00	1.00	0.0531x10-3	0.019 w
8GHz	167.66	0.04712x10-3	1.00	1.00	0.0466x10-3	0.018 w
9GHz	188.30	0.04186x10-3	1.00	1.00	0.0415x10-3	0.018 w







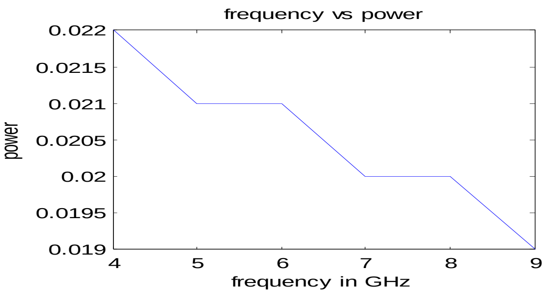


Table.1, 2 and 3 shows results for the magnitude of the different parameters like gamma, S11, S12 T, S11/S12 and Power are obtained using Integral form FEM

Case 1: Length remains same on both axes (l=0.1 meters) and diameter d=0.02mtrs.

Case 2: Length varies on both the axes (l=0.05meters x-axis & l=0.15 meters y-axis) and d=0.02mtrs.

Case 3: Length varies on both the axes (l=0.15meters x-axis & l=0.05metersy-axis) and d=0.02mtrs

Case 1: As frequency increases wave number increase, because frequency is directly propositional to wave number reflection co-efficient (S11) parameter is decrease, Transmission co-efficient (S12) and total transmitted (T) increase exponentially, ratio of reflection co-efficient and transmission co-efficient decrease (S11/ S12) and power increases, all the parameter variations are shown in the graph.

Case 2: As frequency increases wave number increase, because frequency is directly propositional to wave number reflection co-efficient (S11) parameter is decrease, Transmission co-efficient (S12) , total transmitted (T) and power increases and remains constant comparing with case.1, ratio of reflection co-efficient and transmission co-efficient decrease (S11/ S12) and power increases and remains constant, all the parameter variations are shown in the graph.

Case 3: As frequency increases wave number increase, because frequency is directly propositional to wave number reflection co-efficient (S11) parameter is decrease, Transmission co-efficient (S12) and total transmitted (T) increases and remains constant comparing with case.1and case.2, ratio of reflection co-efficient and transmission co-efficient decrease (S11/ S12) and power gets attenuated, all the parameter variations are shown in the graph.

Referring all parameters in the three cases we have made the following conclusion, all parameters are depending on frequency, discontinuity (90deg bend), length and diameter of the waveguide.

4. CONCLUSION

In conclusion, a waveguide with discontinuity like bend, twist, double bend and double twist or any shape of discontinuity can be analyzed by using FEM solution of 2D Helmholtz equations and desired parameter can be calculated by using Integral form reflection co-efficient and transmission co-efficient(S-Matrix)

5. REFERENCES

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