Modeling of Social Network using Graph Theoretical Approach

Sweata Mishra
School of computing sciene
Kziranga University
Assam,India

Rupam Borboruah
School of computing science
Kaziranga University
Assam,India

Bharadwaj choudhury
School of computing science
Kaziranga University
Assam,India

Sandip Rakshit
School of computing science
Kaziranga University
Assam,India

ABSTRACT
This paper describes the model of formation and properties of social network via a graph theoretical approach. The paper considers the interaction between different sets of people in a social network. It also describes the additional information about each individual in a network. We have also described the matrix representation of social network.

The purpose of this paper is to review the graphical model of social networking. This paper is about how two persons are related to each other and how they can access each other's resources. A social network is a social structure made up of individuals (or organizations) called "nodes", which are tied(connected) by one or more specific types of interdependency, such as friendship, kinship, common interest, financial exchange, dislike, sexual relationships, or relationships of beliefs, knowledge or prestige.

Keywords
Cut vertex and cut edge, graph metric, adjacency matrix, Social graph topology

1. INTRODUCTION
Social network is a" group of internet-based application, that is build on the ideological and technological foundation of web, and allow the creation and exchange of user generated content"[10]. Understanding such a network provides us useful insights of ways regarding how social communities are formed and interactions thereafter. Therefore it is required to generate a graphical view in order to understand the network, so that refinement or analysis in future is easier.

Social network is the structure, which shows the relations between individuals and organization. It indicates the ways in which they are connected through various social familiarities ranging from casual acquaintance to close familiar bonds [1]

In this paper, we have shown the mapping of relationship between individuals and organizations.

We have represented it using graph G(V,E) which is a collection of vertices and edges. Vertices are the people and groups and edges are relationships between them. Most important part of modeling a social network is their sub models, through which we can get more information about connections between two different vertices i.e. individuals or communities.

2. SOCIAL GRAPH TOPOLOGY
Social network can be represented as undirected graph G(V,E) where V is the set of vertices representing users and E is the set of edges representing social ties such as friendship, kinship and the likes. Graphs are used to represent communication networks or social networks and the analysis of some of their basic properties can help on evaluating and improving the performance of networking solution.[2]

3. NODE DEGREE
In a graph, the degree of a vertex is denoted by deg(v) is the number of edges incident in that vertex.

Since in a graph G(V,E) , vertex represents the user and edge represents the relationship or friendship , a user's degree represents the number of friend a user can have or the pages he likes. A page may belong to any organization, communities etc. Understanding the degree of a node gives the overall idea of how many social acquaintances an individual is having. A 'like' in a social network typically involves information sharing.

Different studies have shown that increase in the number of participants results in a weak link on privacy of users [3], [4], [5].

4. SOCIAL MODEL
The reason behind using graphical model in social network is to describe the network more elaborately and systematically. Using graphical model allows us to describe the properties of network elaborately and distinctly. The relation between individuals can be mapped using graphical method and how one can access the resource of other user(s) can be described. There is a big issue of privacy while using social network, as it contains huge amount of sensitive data.

4.1 Modeling:
Let us consider two friends A and B. If we want to model the relation between them we can simply draw the graph below. We have used undirected graph to show the relation as A and B both can share each other resources. The reason for using undirected graph is that an edge from one friend to another always means that the access of resource is on both the direction.

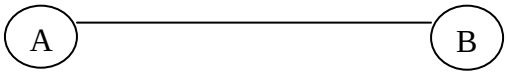

Fig1: Graph with two vertexes

Now if there are more friends suppose A, B, C, D then the graph can be:

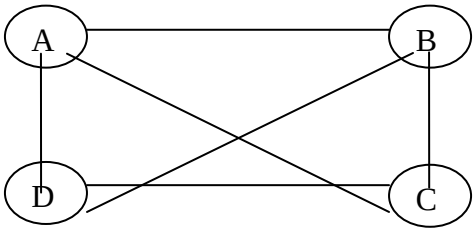

Fig2 : Maximum connectivity among four vertex

Here A, B, C, D four friends are included. Each o them can share each others resource.

a) Representation:

If A and B are friends then we can say they can share each other's resource, i.e.

AUB implies A and B are friends.

If AUB= True, then maximum connectivity between them will be 1. If there are more than one node or number of friends are N, then maximum connectivity between them can be expressed as:

$$N=N(N-1)/2$$

This approach is only for direct connection. For indirect connection .i.e. connection between individuals via some group or community or some other pages, this approach cannot be used.

5. DISTANCE BETWEEN INDIVIDUALS AS METRIC

Let 'f ' be a function that maps the distance between individual, then we can map the distance between node or individuals in terms of function called metric.

Let A, B and C are two friends. then the connection between them can be measured in terms of function. Here f(x,y) is the distance between x and y. Therefore, we can conclude three conditions regarding the distance between individuals:

a) $f(A,B) \geq 0$

b) $f(A,B) = f(B,A)$

c) $f(A,B) \leq f(A,C) + f(C,B)$

Here, distance between two nodes cannot be 0. Hence, $f(A,B) \geq 0$.

Again, distance from node A to node B is same as from node B to A.

Third condition says that distance from a node A to B is always less than the sum of distance between A and C and C and B.

6. CLUSTERING OF SOCIAL NETWORK GRAPH

An important aspect of social network is that they contain communities that are connected by many edges [6]. These communities may be a group of friends, group of researchers interested in same topic and so on. Sociologist have studied many of the properties of social networks. Milgram [7] shows the average path length between two American is 6 hops, and Pool and Kochen[8] prvide an analysis of small-world effect. The paper by Granovetter [9] says that the social network can be partitioned into strong and weak ties, and the strong ties are strongly clustered.

Representation:

We can represent this type of interaction graphically:

Let A and B are two friends, they are connected through an edge which means that there is direct connection between A and B. Again, they belongs to some community, which can be any group or some common pages, then the relation between them can be modeled as shown:

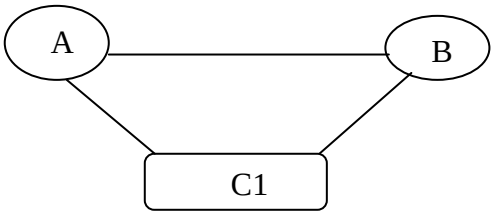

Fig3 : Common community between two vertex

Here, A, B are two connected friends. C1 be a community to them. Hence, A and B are connected via two different paths, one is the direct connection, another is a connection via community C1.

So, we can conclude that the connection between A and B are strong, as they belongs to same community too. Again if there is another friend D and F, and the graph represented as:

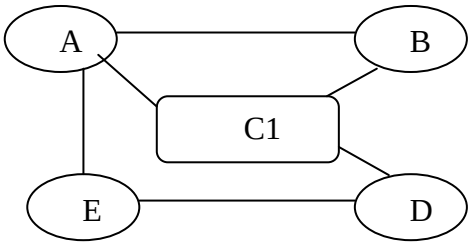

Fig4 : Strong and weak ties among four vertex

Here A and B are strongly connected, as we have discussed. Similarly, the connection between B and D is weak, as there is no direct connection between B and D, they are connected via community C1 only.

Again, connections between D & E and A & E are also weak, as there are direct connections only. Also, we can see that A and D are also connected via community C1, here also connection is weak.

Also, B and E are connected via A .i.e. A is a common friend between B and E.

7. USE OF MATRIX TO REPRESENT SOCIAL NETWORK

Matrix representation of social network is very simple, it will consist of numbers of rows and columns. It can be represented using binary data .i.e. '0' and '1'[11]. We can construct a table containing many cells. If there will be a tie or connection between two friends then a '1' will be entered and if there is no such connection between friends, then a '0' will be entered. This type of representation in matrix is called 'adjacency matrix'. In the undirected graph, the sender will be row and the receiver will be column. [12] [13]. To illustrate this, let us look a simple graph which consist of five individual, namely A, B, C, D, E. Their connection can be represented graphically as follows:

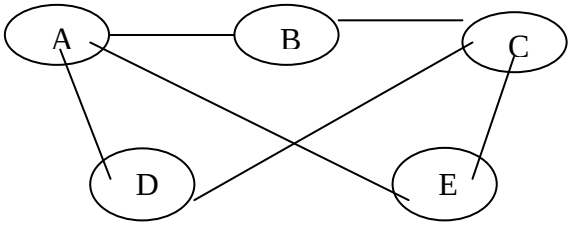

Fig5: graph to represent social relation

This graph can be modelled using adjacency matrix as given below[1]:

	A	*B*	*C*	*D*	*E*
A	-	*1*	*0*	*1*	*1*
B	*1*	-	*1*	*0*	*0*
C	*0*	*1*	-	*1*	*1*
D	*1*	*0*	*1*	-	*0*
E	*1*	*0*	*1*	*0*	-

Fig6: Adjacency matrix representation of graph

Here, connection between a node to itself is not possible in social network, hence the respective cell which shows the connection between a node to itself is blank.

8. CUT EDGE AND CUT VERTEX IN SOCIAL NETWORK

As the social network model is a vast graph, we can partition it into smaller sub graphs to study the properties of graph more elaborately. One of the approaches of graph simplification is cut edge and cut vertex, whose removal partitions a graph into smaller sub graph [14].

Removing an edge or a vertex from a social graph means that the connection between two nodes fails, this can be viewed as an individual has been removed from friend list of another friend. cutting an edge or a vertex also means that each individual in a disconnected graph will be no longer be able to access the resources of the individuals of another disconnected sub graph.

Let us consider an example with individuals A, B, C, D, E, F. They are connected as shown in Fig5. We can describe how cut edge and cut vertex can be applied to simplify the graph. Let us refer to Fig 6 for an illustration:

The graph below shows the cut edge for the social relationship between A, B,C,D,E,F

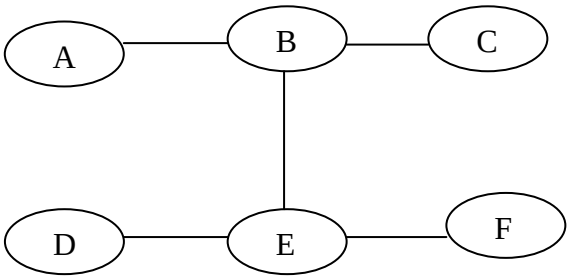

Fig7: Graph with six vertices

If an edge has been removed between B and E. The graph will be divided into two components or sub graph. This edge is called the cut edge of the graph. This implies that B will not able to interact directly with E.

The graph in Fig8 shows the cut edge of graph 7.

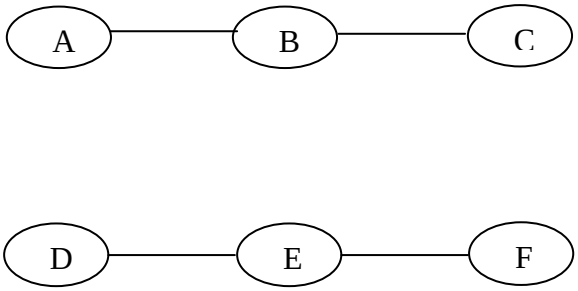

Fig8: Graph with cut edge

By removing the edge between B and D, we have get two sub graph, first containing vertices A, B, C and the second containing vertices D, E, F. This can be seen in social network graph, when the connection between individuals has been removed.

The graph in Fig9 shows the cut vertex of graph7.

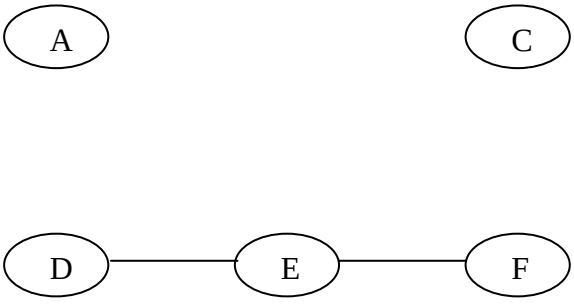

Fig9: Graph with cut vertex

Removing a vertex B from the graph in Fig6, left us with the graph Fig8. By removing the vertex we have got three components, as the edges connected with vertex B has also been removed.

Hence, in a social graph if an individual is removed, it left the graph disconnected with and the individuals will no longer be able to view posts from friends of friends in a public setting.

9. CONCLUSION

In this paper, we have reviewed social networks, studied different properties of social networks and formal methods to depict them. This paper investigates the strong relationship between the topological properties of the social network graph.

We have described the metrics and the clustering phenomenon visible in social networks. Paper also describes how cut edge or cut vertex affects the entire network.

As a future work, we intend to study more about the social networking model using graph theory approach, such as using bipartite graph to describe social network. Efforts may be exercised in future to present a graph theoretical approach of how chat functionality operates within a social network. A more elaborate graph theoretical model of a social network can help strengthen the ease of use of social networks and make user's information remain secured within the social network community and beyond.

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