A Novel Adaptive Stationary Wavelet-based Technique for SAR Image Despeckling

Amlan Jyoti Das
Dept. of Electronics & Communication Engineering, Gauhati University, Guwahati-14, Assam

Anjan Kumar Talukdar
Dept. of Electronics & Communication Engineering, Gauhati University, Guwahati-14, Assam

Kandarpa Kumar Sarma
Dept. of Electronics & Communication Technology, Gauhati University, Guwahati-14, Assam

ABSTRACT
In this paper, we present a Stationary Wavelet Transform (SWT) based method for the purpose of despeckling the Synthetic Aperture radar (SAR) images by applying a maximum a posteriori probability (MAP) condition to estimate the noise free wavelet coefficients. A MAP Estimator is designed for this purpose which uses Rayleigh distribution for modeling the speckle noise and Laplacian distribution for modeling the statistics of the noise free wavelet coefficients. The parameters required for MAP estimator is determined by technique used for parameter estimation after SWT. The experimental results show that the proposed despeckling algorithm efficiently removes speckle noise from the SAR images.

General Terms
SAR image despeckling, Stationary wavelet based MAP estimation technique.

Keywords
Synthetic aperture radar (SAR), despeckling, Stationary Wavelet Transform (SWT), Maximum a posteriori probability (MAP) Estimator.

1. INTRODUCTION
Synthetic aperture radar (SAR) represents a very robust observation tool as it allows the acquisition of high resolution images of different places on the earth. These systems operate under all weather conditions, night and day. SAR images are corrupted by special kind of noise known as speckle noise. In a SAR image, speckle manifests itself similar to thermal noise, by a random pixel-to-pixel variation with statistical properties. Its granular appearance in an SAR image makes it very difficult to visualize the SAR data. Therefore, in many SAR image processing operations like segmentation, speckle filtering is a crucial preprocessing step [1]. Many denoising algorithms have been developed for despeckling SAR images by using the Lee filter [2], the Frost filter [3], LG-MAP filter [13], the Gamma MAP filter [4], and their variations [5], [6]. These filters usually exhibit well in despeckling the SAR images. However, they lack in restoring sharp edge features and details of the original SAR image [7].

Since SAR images are multiplicative in nature, so many wavelet-based despeckling algorithms apply the log-transform to SAR images to statistically convert the multiplicative noise to *additive* noise prior to applying further denoising technique [7], [8]. An exponential operation is applied to convert the log-transformed images back to the nonlogarithmic format after wavelet denoising [8].

Several solutions have been proposed in the recent years, based on maximum a posteriori probability (MAP) criteria and different distributions: the gamma distribution [9], the α-stable distribution [10], the Pearson system of distributions [11], and the generalized Gaussian (GG) [12], laplacian and gaussian distribution [13] etc. MAP estimator generates a posterior probability by using a prior, likelihood and evidence probability density functions (pdf). The MAP estimate finds a solution for a noise-free image. The speckle noise pdf is approximated by a likelihood pdf of a prior which determines the knowledge of the scene and the best model can be evaluated by maximizing the evidence pdf.

In [12], a MAP criterion is derived which is associated with the Generalized Gaussian distribution and is performed in the undecimated wavelet domain. One of the major drawbacks of GG-based MAP solutions is that they can be achieved only numerically, thereby it leads to a high computational cost. In [13], a MAP criterion is derived by considering Gaussian distribution for modeling speckle noise and Laplacian distribution for modeling noise free wavelet coefficients. In [14], the noise-free image was approximated by a Gauss-Markov random field prior and the speckle noise was modeled using Gamma pdf.

Although DWT plays a major role in the area of image denoising and image compression, the downsampling operation involved in DWT results in a time-variant translation and has to face difficulties in restoring original image discontinuities in the wavelet domain. Therefore, to restore the translation invariance property, lost by classical DWT, Stationary Wavelet Transform has been preferred in many techniques [11].

In [11], Foucher *et al.* used the Pearson distribution to model the probability density function (pdf) of SWT wavelet coefficients and reconstructed the despeckled image using the MAP criterion. But the high computational complexity of the Pearson distribution makes this method less appealing in practice, although this algorithm has sound performance.

In this paper we propose an efficient SWT based despeckling method by using MAP estimation. We avoid the log-transform and derive a novel MAP estimation criteria based on Rayleigh distribution for modeling the speckle noise and

Laplacian distribution for modeling the noise free wavelet coefficients. The parameter estimation is done using the results of [11]. The despeckling algorithms were tested on Ku-Band SAR image of pipeline over the Rio Grande river near Albuquerque, New Mexico (1-m resolution) with dimensions of 256×256, and were compared, which gives a satisfactory result for the proposed algorithm.

This paper is organized as follows. In Section II, we describe statistical properties of SAR images, the two-dimensional (2-D) SWT algorithm, and signal modeling. In Section III, we have given a brief review of statistical characterization of wavelet coefficients and speckle noise. In Section IV we have shown the related work on MAP Estimator design, derive a suitable MAP criterion, and provide a mechanism to estimate noise-free wavelet coefficients. In Section V, the parameter estimation is described. Section VI provides a description of our algorithm and Section VII shows experimental results. Finally, a short conclusion is given.

2. STATISTICAL MODELS FOR SAR IMAGES

2.1 Statistical models for SAR images

Let G be the observed signal (intensity or amplitude), and F be the noise-free signal. Since speckle noise is multiplicative in nature, the observed signal can be expressed as $G = FU$, where, F is the noise free image and U is the normalized fading speckle-noise random variable, following a Gamma distribution with unit mean and variance $1/L$. Its pdf is given by

$$p(U) = \frac{L^L U^{L-1} e^{-LU}}{\Gamma(L)} \qquad , U \geq 0 \qquad (1)$$

where $\Gamma(\cdot)$ denotes the gamma function.

The observed intensity *I* of an *L*-look image has the conditional pdf given by [5]

$$p_{(I|F)}\left(\frac{g}{f}\right) = \frac{1}{\Gamma(L)}\left(\frac{L}{f}\right)^L g^{L-1} e^{-Lg/f} \qquad (2)$$

Where, g represents an observed intensity value and f is the corresponding actual intensity value. Amplitude A which is the square root of intensity is distributed with the following pdf [5]:

$$p_{(A|F)}\left(\frac{g}{f}\right) = \frac{2}{\Gamma(L)}\left(\frac{L}{f}\right)^L g^{2L-1} e^{-Lg^2/f} \qquad (3)$$

Note that, if $L = 1$ in eqn. (2) and (3) gives the distribution of monolook intensity and amplitude, which are exponential and Rayleigh distributions, respectively.

2.2 Stationary Wavelet Transform

The SWT algorithm is simple and is close to DWT. Figure1 shows the 2-D SWT decomposition algorithm, where H_j and L_j are the highpass and lowpass filters at level j, respectively. Also, it can be noted in the figure that the original image is LL_0 and that the output LL_j of each decomposition level j is applied to the input of the next level $j+1$. As shown in Figure 1, the two filters H_j and L_j are upsampled by two from filters of the previous decomposition level H_{j-1} and L_{j-1}. It is a redundant transform as SWT does not include downsampling operations. More precisely, as DWT, for level 1, all the decimated DWT for a given signal can be derived by convolving the signal with the appropriate filters without downsampling it.

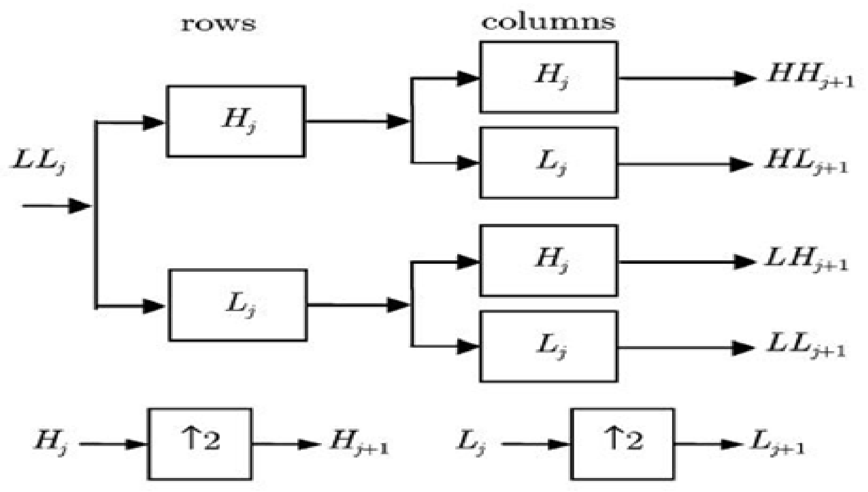

Figure 1: Block diagram of SWT Decomposition

2.3 Signal model

The observed signal is assumed to follow the following model:

$$g[n] = f[n].u[n] = f[n] + f[n].(u[n]-1) \quad = f[n] + v[n] \qquad (4)$$

where $g[n]$ is the observed signal, $f[n]$ is the speckle-free signal that we would like to estimate, $u[n]$ is the speckle noise, and $v[n]$ denotes the signal dependent speckle component in the equivalent additive model.

Let $W_x^{[j]}$ be the stationary wavelet operator applied to the signal *x*. It performs a multiresolution decomposition, where *j* is the decomposition level. We have,

$$W_g^{[j]}[n] = W_f^{[j]}[n] + W_v^{[j]}[n] \qquad (5)$$

Despeckling an image in the multiresolution domain means estimation of the speckle noise free wavelet coefficients $\widehat{W}_f^{[j]}$ and applying the inverse stationary wavelet transform (ISWT) to obtain the noise free image.

3. STATISTICAL CHARACTERIZATION OF WAVELET COEFFICIENTS AND SPECKLE NOISE

The parametric MAP estimator presumes proper modeling of prior probability distribution of signal and speckle noise wavelet coefficients. In this paper, we model the signal component of the wavelet coefficients using the Laplacian distribution, and the Rayleigh model is used for modeling the noise component.

3.1 Rayleigh model for Speckle component

The probability density function (pdf) for a Rayleigh distributed random variable v, is defined as [15]:

$$p_v(v) = \frac{v}{\alpha^2}\exp\left(\frac{-v^2}{2\alpha^2}\right) \qquad , v \geq 0 \qquad (6)$$

where, v is the amplitude of the noise and α is the fading parameter. Assuming equally probable negative and positive values of wavelet coefficients, the pdf of ε is given by:

$$p_\varepsilon(\varepsilon) = \frac{\varepsilon}{\alpha^2}\exp\left(\frac{-\varepsilon^2}{2\alpha^2}\right)Z(\varepsilon) - \frac{\varepsilon}{\alpha^2}\exp\left(\frac{-\varepsilon^2}{2\alpha^2}\right)Z(-\varepsilon) \qquad (7)$$

where $Z(\varepsilon)$ is equal to 1 for $\varepsilon \geq 0$ and zero otherwise.

3.2 Laplacian model for Signal component

The pdf of a Laplacian distributed random variable, f, defined as follows [13]

$$p_f(f) = \frac{1}{\sqrt{2}\sigma_f} e^{-\frac{\sqrt{2}|f-\mu_f|}{\sigma_f}} \qquad (8)$$

where σ_f is the standard deviation of signal, f determines the spread of the density function and μ_f is the mean of the signal f.

4. DESIGN OF THE PROPOSED MAP ESTIMATOR

The MAP estimator of the speckle-free wavelet coefficients is given by

$$\ddot{W}_f = \arg\max_{W_f} p(W_f|W_g) \qquad (9)$$

After applying the Bayes rule and the log function, we have

$$\ddot{W}_f = \arg\max_{W_f}[\log p(W_f|W_g) + \log p(W_f)] \qquad (10)$$

The proposed method is based on (10) that, by using a simplified notation and the model in (5), can be rewritten as

$$\ddot{\theta} = \arg\max_{W_f}[\log p_v(x-\theta) + \log p_\theta(\theta)] \qquad (11)$$

where $\theta = W_f[n], x = W_g[n], and\ v = W_v[n]$

From eqn. (6) and (8), we have

$$p_v(x-\theta) = \frac{x-\theta}{\alpha^2}\exp\left(\frac{-(x-\theta)^2}{2\alpha^2}\right) \qquad (12)$$

And

$$p_\theta(\theta) = \frac{1}{\sqrt{2}\sigma_\theta} e^{-\frac{\sqrt{2}|\theta-\mu_\theta|}{\sigma_\theta}} \qquad (13)$$

The MAP equation can be written as

$$\hat{\theta} = \arg\max_{W_f}\left[\log\left(\frac{x-\theta}{\alpha^2}\exp\left(\frac{-(x-\theta)^2}{2\alpha^2}\right)\right) + \log\frac{1}{\sqrt{2}\sigma_\theta}e^{-\frac{\sqrt{2}|\theta-\mu_\theta|}{\sigma_\theta}}\right]$$

(14)

Taking $\frac{d\theta}{d\theta}$ and equating it to zero, we have the solution

$$\hat{\theta} = x - \left|\frac{\sqrt{2}\alpha^2 \pm \sqrt{2\alpha^4 + 4\alpha^2\sigma_\theta^2}}{2\sigma_\theta}\right| \qquad (15)$$

The solution to this problem is given by:

$$\hat{\theta} = \begin{cases} x - \left|\frac{\sqrt{2}\alpha^2 \pm \sqrt{2\alpha^4 + 4\alpha^2\sigma_\theta^2}}{2\sigma_\theta}\right| & , if\ x > \mu_\theta + \left|\frac{\sqrt{2}\alpha^2 \pm \sqrt{2\alpha^4 + 4\alpha^2\sigma_\theta^2}}{2\sigma_\theta}\right| \\ x + \left|\frac{\sqrt{2}\alpha^2 \pm \sqrt{2\alpha^4 + 4\alpha^2\sigma_\theta^2}}{2\sigma_\theta}\right| & , if\ x > \mu_\theta - \left|\frac{\sqrt{2}\alpha^2 \pm \sqrt{2\alpha^4 + 4\alpha^2\sigma_\theta^2}}{2\sigma_\theta}\right| \\ \mu_\theta, & , elsewhere \end{cases} \qquad (16)$$

This solution is applied to the high frequency sub-bands (LH, HL, and HH) to estimate the noise free wavelet coefficients.

5. PARAMETER ESTIMATION FOR THE PROPOSED ESTIMATOR

In the solution of the MAP estimator there are three unknown parameters.

(i) μ_θ (Mean of the noise free wavelet coefficients):

$$\mu_\theta = E[\theta] = E[x] \quad (17)$$

(ii) α (Fading parameter): The value of fading parameter α ; is estimated from the noise varianceσ_v^2: Using (7), values of first and second central moments, i.e. mean and variance of the noise are found to be zero and $2\alpha^2$ respectively, which implies

$$\alpha = {}^{\sigma_v}/_{\sqrt{2}} \quad (18)$$

Based on the statistical properties of SWT transform and SAR images, Foucher *et al.* [8] estimate as

$$\sigma_v^2 = \psi_l \mu_x C_F \left(1 + C_\theta^2\right) \quad (19)$$

where, $\mu_x = E[x] \; and \; C_\theta^2$ is given by

$$C_\theta^{\,2} = \frac{C_x^{\,2} - \psi_l C_F^{\,2}}{\psi_l (1 + C_F^{\,2})} \quad (20)$$

where C_x is the normalized standard deviation of noisy wavelet coefficient and is given by

$$C_x = \frac{\sigma_x}{\mu_x} \quad (21)$$

Inserting (20) and (21) into (19), we obtain

$$\sigma_v^{\,2} = \frac{C_F^{\,2}(\psi_l \;\; \mu_x^{\,2} + \sigma_x^{\,2})}{1 + C_F^{\,2}} \quad (22)$$

where C_F is the normalized standard deviation of noise and equals $\sqrt{1/_L}$ for intensity images and $\sqrt{(4/_\pi - 1)/L}$ for amplitude images$(L \geq 1)$, where L is the look of the SAR image, and parameter ψ_l is defined as

$$\psi_l = (\sum_k h_k^{\,2})^2 (\sum_k g_k^{\,2})^{2(l-1)} \quad \text{(For diagonal subband)}$$
$$= (\sum_k h_k^{\,2})^2 (\sum_k g_k^{\,2})^{2l-1} \quad \text{(For horizontal and vertical subband)}$$

(23)

where h and g are the highpass and lowpass filters at decomposition level l, respectively.

(iii) σ_θ^2 (Variance of the noise free wavelet coefficients): It can be calculated by using

$$\sigma_\theta^{\,2} = \psi_l \mu_x^{\,2} C_\theta^{\,2} \quad (24)$$

Using eqn. (17) - (24) the parameters are estimated for each high frequency subbands at each decomposition level.

6. EXPERIMENTAL DETAILS

The block diagram of the proposed method is shown in Figure 2. The original noisy image is first transformed using SWT. From the SWT decomposed image the parameters mentioned in Section V are estimated using eqn. (17) - (24) for the high frequency subbands i.e. LH, HL, and HH subband at each decomposition level. These parameters are then used to calculate the solution of the MAP estimator using eqn. (16) for each high frequency subbands.

The solution of the MAP estimator is derived by using a prior knowledge for modeling the speckle noise and noise free wavelet coefficients. As described in Section III the speckle noise is modeled using Rayleigh distribution and the signal component is modeled using Laplacian distribution. These conditions are then applied to each wavelet coefficients of LH, HL, and HH subband for the estimation of the noise free wavelet coefficients of the high frequency subbands using the conditions of the MAP estimator. These results the formation of high frequency subbands with the noise free estimated wavelet coefficients. Finally inverse SWT is taken for the low frequency subbands and the estimated high frequency subbands for each level. The results obtained by the proposed technique are shown in Figure 3.

7. RESULTS AND DISCUSSION

For the purpose of comparison of the proposed technique we have considered the results that we have got by applying existing techniques like VisuShrink thresholding, BayesShrink thresholding, Wiener filtering and LG-MAP [13]. Among the most popular thresholding methods include VisuShrink and BayesShrink. These thresholds take the advantage of asymptotic minimax optimalities over function spaces such as Besov spaces [16]. However, for image denoising, VisuShrink yield overly smoothed images. This is because its threshold choice,$\sigma\sqrt{2\log M}$ (called the *universal threshold* and σ is the noise variance), can be unwarrantedly large due to its dependence on the number of samples, which removes too many coefficients. VisuShrink provides a single value of threshold, which is globally applied to all the wavelet coefficients. The result of this method is shown in Figure 3(b) which shows that the quality of the image is worse than other methods like BayesShrink. The BayesShrink threshold is given by

$$T_{Bayes} = {}^{\sigma^2}/_{\sigma_B}$$

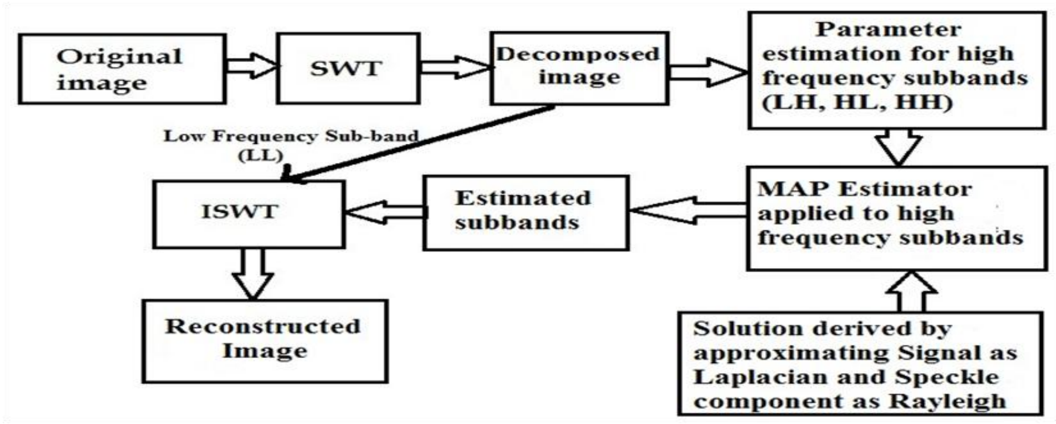

Figure 2: Block diagram of the proposed method

where σ^2 is the noise variance and σ_B is the standard deviation of noiseless coefficients in a subband. This threshold often provides a better denoising result than the SURE threshold and VisuShrink threshold. The result is shown in Figure 3(c) which shows that, this method gives better result than VisuShrink but worse than other methods.

We have also made comparisons with the Wiener filter outputs, which was claimed to be the best linear filtering possible. The results are derived using the default settings (3×3 local window size), and the unknown noise power is estimated. The PSNR results are shown in Table I. They are considerably lower than the LG-MAP and the proposed method. The image quality (shown in Figure 3(d)) is also not as good as the techniques mentioned above.

The proposed method is also compared with the LG-MAP technique in which MAP estimator is designed by modeling the speckle noise as Gaussian distribution and signal component as Laplacian distribution. This solution of this MAP estimator as given in [13]

$$\hat{\theta} = \begin{cases} x - \dfrac{\sqrt{2}\sigma_v^2}{\sigma_g} & , if\ x > \mu_g + \dfrac{\sqrt{2}\sigma_v^2}{\sigma_g} \\ x + \dfrac{\sqrt{2}\sigma_v^2}{\sigma_g} & , if\ x > \mu_g - \dfrac{\sqrt{2}\sigma_v^2}{\sigma_g} \\ \mu_g & , elsewhere \end{cases} \qquad (17)$$

where the corresponding parameters have the same meaning as described in Section III. This method results in significant reduction of speckle noise. The result is shown in Figure 3(e), which shows that result of the proposed method (shown in Figure 3(f)) have almost the same quality.

The proposed method is compared with the existing methods like LG MAP [13], Bayes Shrink thresholding etc by comparing the corresponding PSNR values. The PSNR is defined as

$$PSNR = 10 \log_{10} \frac{255^2(M.N)}{\sum_{i,j}(B(i,j) - A(i,j))^2}$$

where, B is the denoised image, A is the noise-free image and ($M \times N$) is the size of the denoised image. The comparison of the PSNR values is shown in Table I.

The PSNR is calculated for E-SAR images of Obepffafenhofen with different values of noise variance and applied to all the techniques which give a result as mentioned in Table I. It is seen that the proposed method has better PSNR for all the input images with different values of noise variance. Thus the proposed approach is suitable for SAR applications.

Table 1. Comparison of PSNR of different denoising techniques

INPUT PSNR	VISU SHRINK	BAYES SHRINK	WIENER FILTER	LG-MAP	PROPOSED METHOD
27.6135	24.3189	24.4568	24.4756	28.3828	**28.5269**
28.2838	24.1760	24.7860	24.8049	30.4351	**30.6046**
29.7210	25.9526	26.3420	26.8564	30.9375	**31.5654**
29.9965	26.0345	26.7834	26.9586	31.0219	**31.8585**

8. CONCLUSION

In this paper, a new and efficient technique for despeckling SAR images has been proposed. The speckle noise in the wavelet domain was modeled as an additive signal-dependent noise. We have designed and tested the subband adaptive

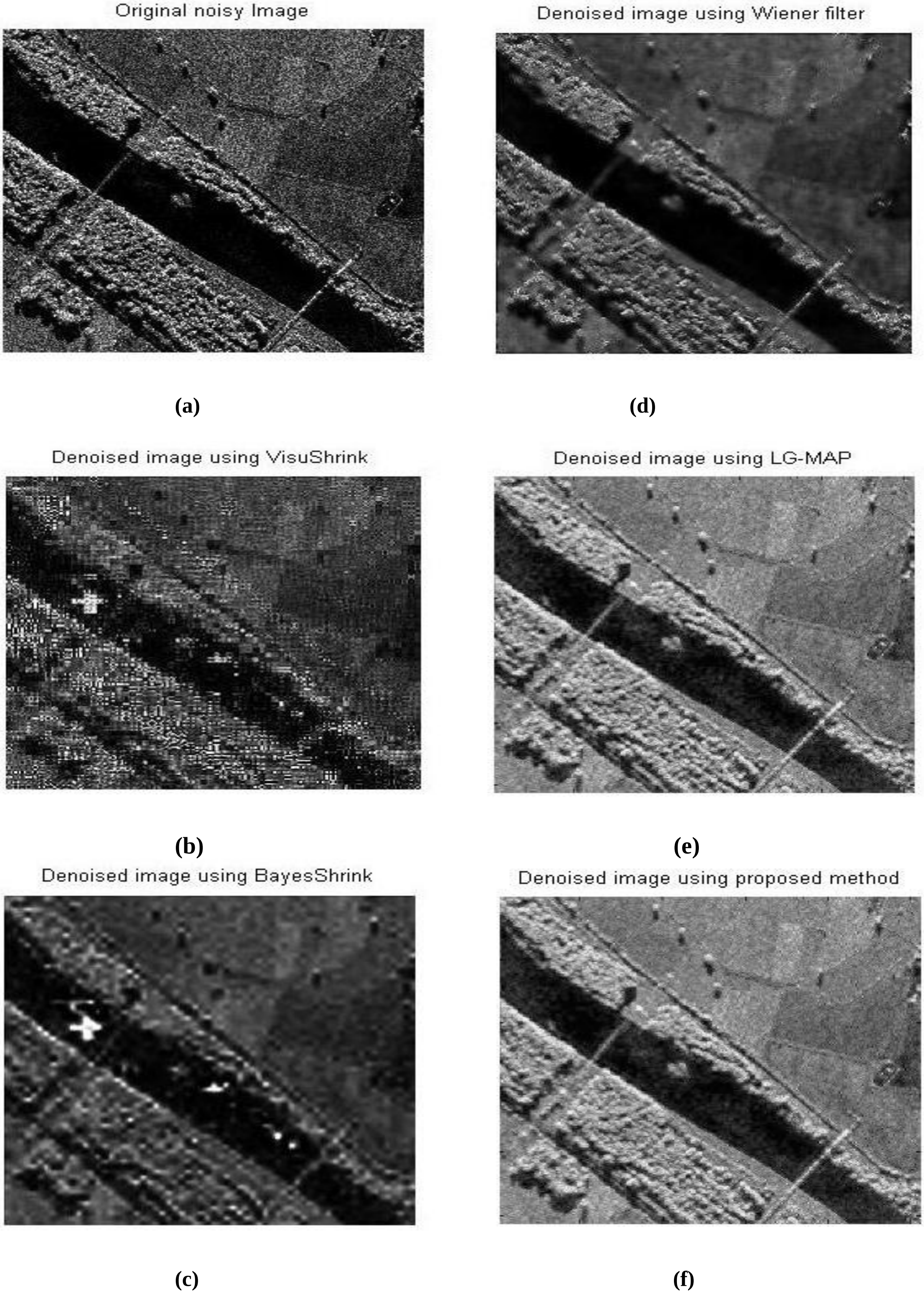

Figure 3: Original and denoised Ku-Band SAR image of pipeline over the Rio Grande river near Albuquerque, New Mexico: 1-m resolution. (a) Original noisy image (b) Denoised image using VisuShrink (c) Denoised image using BayesShrink (d) Denoised image using Wiener filter (e) Denoised image using LG-MAP (f) Denoised image using proposed method

MAP processor, which relies on the Rayleigh distribution of speckle noise and Laplacian prior for modeling the wavelet coefficients. Further, experimental results using the simulated speckled images show that, for five-level wavelet decompositions, the PSNR values of the proposed technique are higher than those algorithms mentioned above at less computational complexity. Although the speckle noise is removed by a significant amount, the denoised SAR image is smoothed in each successive decomposition levels. The subsequent stage of the proposed method will be related to sharpening the edges so that more edge information can be restored enhancing the resolution of the denoised image.

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