Channel Estimation using LS and MMSE Algorithm

Barnali Dey
Sikkim Manipal Institute of Technology

Awanish Kumar
Sikkim Manipal Institute of Technology

Prashant Kumar
Sikkim Manipal Institute of Technology

Sanjit Lal
Sikkim Manipal Institute of Technology

Naveen Kumar
Sikkim Manipal Institute of Technology

Bikash Sharma
Sikkim Manipal Institute of Technology

ABSTRACT
In this paper, we describe the channel estimation based on time domain channel statistics. Using a general model for a slowly fading channel, we present the MMSE and LS estimators. The mean square error and symbol error rate for a BPSK system is presented by means of simulation results. Depending upon estimator complexity, up to 4 dB in SNR can be gained over the LS estimator.

Keywords
Channel estimation, Minimum mean square error (MMSE), Least square (LS), Kalman filter, Orthogonal frequency division multiplexing(OFDM).

1. INTRODUCTION
In performance evaluation of wireless communication system, It is crucial to account for the effects of communication channel. Mobile communication where either the transmitter or receiver is in motion w.r.t. the transmission environment, Presents specific challenges for channel modeling, as it is necessary to account for the perceived time variation of the channel. Currently, orthogonal frequency division multiplexing (OFDM) systems are subject to significant investigation. Orthogonal frequency division multiplexing (OFDM) signaling in fading channel environments has gained a broad interest. In the design of wireless orthogonal frequency division multiplexing system , The channel is assumed to have finite length impulse response. This paper discusses channel estimation technique in wireless orthogonal frequency division multiplexing systems that use the property of channel impulse response (CIR).In section 2. We describe the system model. Section 3.discusses the minimum mean square error (MMSE) and least square (LS) channel estimators. In section 4. The estimators are evaluated by simulating a binary phase shift keying (BPSK) signaling scheme. The performance is presented both in terms of mean square error (MSE) and symbol error rate (SER).

2. OFDM CHANNEL MODEL
OFDM is generated by choosing the spectrum required, based on the input data and modulation scheme used. Each carrier produced is assigned some data to transmit. The required amplitude and phase of the carrier is then calculated based on BPSK modulation scheme. The input is transformed into a multi-level signal reducing the symbol rate to D$=\frac{R}{\log_2 N}$ symbol per second where R is the bit rate of the data stream in bits per second. If the serial data is converted to parallel, the data rate gets further reduced by N, where N is the number of parallel channels. These parallel channels are essentially low data rate channels and since they are narrow band they experience flat fading. This is the greatest advantage of the OFDM technique. This parallel data stream is then subjected to an IFFT and summed. This constitutes the OFDM modulation. Mathematically it can be expressed as

$$x(t) = \sum_{n=1}^{N} (a_n + jb_n)(\cos\omega_n t + \sin\omega_n t)$$

After modulation, a guard interval is inserted to suppress ISI caused by multipath distortions. This guard interval is also called cyclic prefix. The guard interval also allows time for multipath signals from the previous symbol to die away before the info from the current symbol is gathered. The symbols are then decoded using the FFT. At the receiver end the signal may be expressed as

$$y_k = h_k x_k + n_k ,\ k = 0,1\ldots,N\text{-}1$$

where, h_k is the complex channel attenuation. In matrix notation we may write

$$y = XFg + n$$

where X is a matrix with the elements of x on its diagonal and

$$F = \begin{bmatrix} W_N^{00} & \cdots & W_N^{0(N-1)} \\ \vdots & \ddots & \vdots \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

Is a DFT matrix with

$$W_N^{nk} = \frac{1}{\sqrt{N}} e^{-j2\pi\frac{nk}{N}}$$

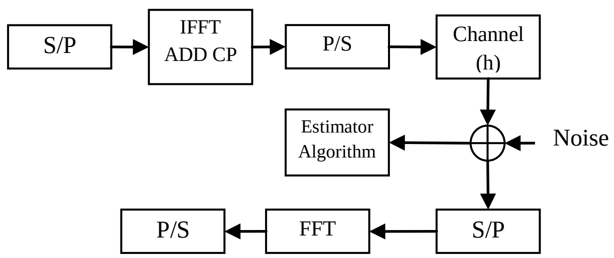

Figure 1: General OFDM System

To send a transmitter signal to the receiver with the feature to combat the channel problems, some characterstics of the channel must be estimated in terms of delay or channel impulse response(CIR). Channel estimation is nothing but estimating CIR at the receiver.

3. CHANNEL ESTIMATION

Two estimators are derived based on the system model in the previous section. These estimation techniques have the general structure presented in fig.2. The transmitted symbol x_k, appearing in the estimator expression are either training symbols or quantized decision variables in a decision directed estimator.

3.1 MMSE estimators

If the channel vector g is Gaussian and uncorrelated with the channel noise n, the MMSE estimate of g becomes

$$\hat{g}_{MMSE} = R_{gy} R_{yy}^{-1} y$$

where

$$R_{gy} = E\{gy^H\} = R_{gg}F^H X^H$$

$$R_{yy} = E\{yy^H\} = XFR_{gg}F^H X^H + \sigma_n^2 I_N$$

are the cross covariance matrix between g and y and the auto covariance matrix of y which are assumed to be known. $\hat{g}_{MMSE}$ generates the frequency domain MMSE estimate $\hat{h}_{MMSE}$ by

$$\hat{h}_{MMSE} = F\hat{g}_{MMSE} = FQ_{MMSE}F^H X^H y$$

Where Q_{MMSE} can be shown to be

$$Q_{MMSE} = R_{gg}[(F^H X^H XF)^{-1}\sigma_n^2 + R_{gg}]^{-1}(F^H X^H XF)^{-1}.$$

This MMSE channel estimator has the form shown in fig. 2

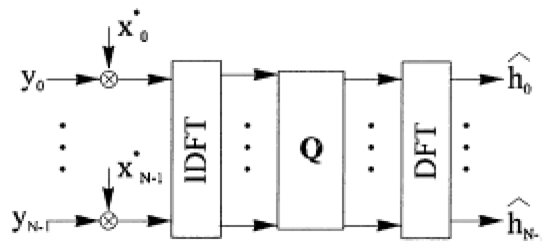

Figure 2: General Estimation Structure

3.2 LS estimators

The Least square (LS) estimator used in the channel to estimate the impulse response g minimizes the factor $(y-XFg)^H(y-XFg)$ and generates

$$\hat{h}_{LS} = FQ_{LS}F^H X^H y$$

Where,

$$Q_{LS} = (F^H X^H XF)^{-1}$$

$\hat{h}_{LS}$ reduces to the following corresponding estimator structure

$$\hat{h}_{LS} = X^{-1}y\ ,$$

LS estimator is equivalent to the zero-forcing estimator.

4. SIMULATIONS

4.1 Simulated channel

In the simulations we consider an OFDM system with BPSK modulation scheme where number of subcarriers are 64. We have used monte-carlo simulations to generate R_{gg} for this channel model. This co-variance matrix together with true noise variance σ_n^2 is used in the MMSE estimation. We have considered the channel response expression to be

g(t)=delta(t-0.5 Ts)+delta(t-3.5 Ts)

Table 1. Estimation algorithm used:

Estimator	Notation	Tap used	Size of Q'
MMSE	MMSE	0….63	64*64
LS	LS	0….63	N.A

4.2 Mean square error

Fig. 3 shows the mean square error versus SNR for the MMSE and LS estimators. For low SNRs, this approximation effect is small compared to the channel noise, while it becomes dominant for large SNRs. The curves level out to a value determined in the energy of the taps. MMSE estimator reduces the mean square error for a range of SNRs compared to LS estimator.

4.3 Symbol error rate

The symbol error rate (SER) curves shown in the fig. 4 are based on the mean square error of the channel estimation shown in previous section. For the calculation of SER we have used the formulae presented in [8].These formulae find the symbol error rate of a system given the noisy estimate of the channel.

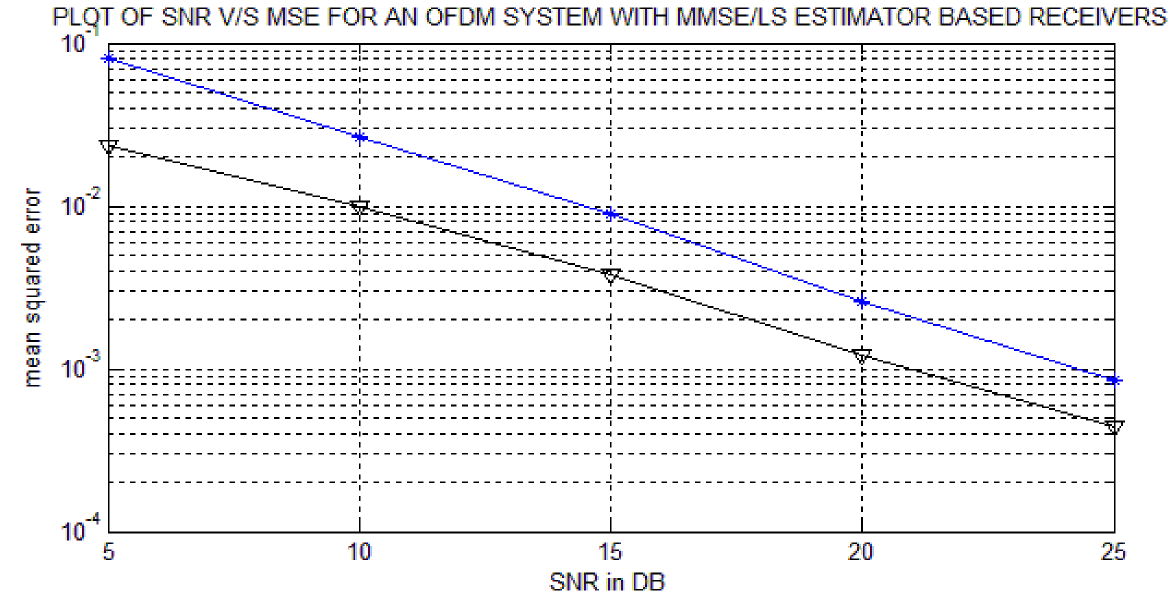

Figure 3: Plot Of SNR vs MSE

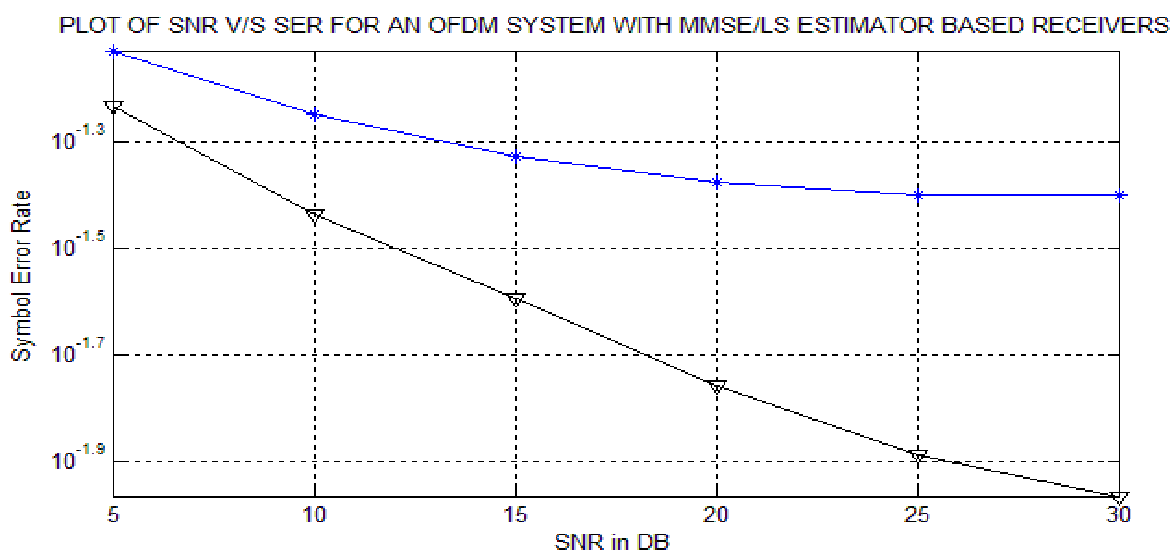

Figure 4: Plot of SNR vs SER

5. CONCLUSIONS

The algorithm of LS estimator is very simple, As LS algorithm does not require correlation function calculation nor does it require matrix inversion. MMSE estimator is complex; As MMSE algorithm requires both correlation function calculation and matrix inversion. From the results it is clear that MMSE estimator provides better performance than LS estimator in terms of mean square error (MSE) and symbol error rate(SER) whereas implementation of LS algorithm is much easier than MMSE algorithm. In future Kalman algorithm can be used for channel estimation which uses the correlation of the channel in time domain for enhancing the performance of the estimator.

6. REFERENCES

[1] John Proakis, digital communication, McGraw-Hill, 1989.

[2] Louis L. Scharf, Statistical Signal Processing, Addison-Wesley, 1991.

[3] S.Haykin and B.Widrow, Least-mean-square adaptive filters.

[4] Vineet Srivastava, Chin Keong Ho, Patrick Ho Wang Fung and Sunei Sun, Robust MMSE channel estimation in OFDM systems with practical timing synchronization.

[5] Asad Mehmood, Obaid Ullah and Macharla Kranthi Kumar, Using LS algorithm to estimate CIR for OFDM system over multipath fading channel.

[6] Shigenori Kinjo, A new MMSE channel estimation algorithm for OFDM systems.

[7] T.S.Rappaport, Principle of wireless communication, 2nd edition.

[8] Sarah Kate Wilson, Digital audio broadcasting in a fading and dispersive channel, PhD-thesis, Stanford University, August 1994.