Comparative Study of Adaptive Algorithms for Identification of Filter Bank Coefficients of Wavelets

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ABSTRACT
In this paper, a technique to identify the filter bank coefficients of Wavelets db4 and coif5 using adaptive filter NLMS algorithm is presented. Filter bank coefficients of the wavelet are treated as the weight vector of adaptive filter, changes with each iteration and approach to the desired value after few iterations. When we compare the two adaptive algorithms viz. Least Mean Square (LMS) and Normalized Least Mean Square (NLMS), NLMS performs better due to its insensitivity to step size, faster convergence and better accuracy. New Scaling and wavelet functions of the Wavelets db4 and coif5 are generated with the filter bank coefficients obtained by NLMS algorithm iteratively.

General Terms
Wavelet, Multi-resolution Analysis, Discrete Wavelet Transform.

Keywords
Approximation Coefficients, Detail Coefficients, Filter Bank, Quadrature Mirror Filters.

1. INTRODUCTION
Wavelets are playing important role in digital signal processing today [1-3]. In widely adopted Fourier representation in signal processing, one can get the features of the signal either in Time domain or in Frequency domain, one at a time, but both are extensively used in analysis, design and various applications. However there are many instances in which the localization in time as well as localization in frequency, both is required simultaneously. Short duration signals need to be localized in time and small bandwidth signals localized in frequency. In musical signals, small duration signals or small bandwidth musical pieces are placed at an effective temporal position to give special effects. They need to be captured in time as well as in frequency simultaneously [4]. Fourier representation is not suited for such requirements. Wavelet transform replaces Fourier transform's sinusoidal waves by a family generated by translations and dilations of a window called wavelet [4a]. The idea of multi-resolution analysis is studying signals at different scales of resolution [5].

The paper is organized as follows: Section 2 describes wavelet and Filter bank theory. Theory related to least mean square (LMS) and normalized least mean square (NLMS) algorithms is discussed in Section 3.This section compares the differences between the two algorithms for adaptive filter design. The experimental results using the two algorithms for standard wavelet filter bank design are dictated in Section 4. Some directions for future research are given in Section 5.

2. WAVELET AND FILTER BANK
Wavelets are manipulated in two ways. The first one is translation where the position of the wavelet is changed along the time axis. The second one is scaling. Scaling means changing the frequency of the signal. To understand wavelet analysis, two special functions: the (1) wavelet function and (2) the scaling function are used, they have unique expressions:

$$\psi(t) = \sum_{-\infty}^{+\infty} g(n)\, \phi(2t-n) \qquad (1)$$

$$\phi(t) = \sum_{-\infty}^{+\infty} h(n)\, \phi(2t-n) \qquad (2)$$

$$g(n) = (-1)^{1-n} h(1-n) \qquad (3)$$

Wavelet can be generated from a set of transfer function H(z) and G(z). A wavelet can be reconstructed from its approximation and detail coefficients [6]. The wavelet acts as a band pass filter [7]. Most of the wavelet applications are dealt with the coefficients h(n) and g(n) from (1) and (2). These are represented as quadrature mirror filters, having mirror image spectra. The filter formed by the mother wavelet act as constant-Q filters, whose Q-factor is given by:

Q-factor = Centre frequency/ bandwidth (4)

This breaks down into the filter bank implementation of discrete wavelet transform. In figure 1, the filters g(-n) form the high pass filters segment of the QMF filter [8] that gives the detail coefficients of the wavelet and h(-n) forms the low pass filter. The approximation coefficients obtained at the output of h(-n) is down sampled and passed through another pair of QMF filters and the process of decomposition continues. We can reconstruct the wavelet by using the same QMF filters by giving the approximation and detail coefficients as filter inputs. In this paper, we propose two algorithms LMS and NLMS to find out the h(-n) and g(-n) in an iterative manner and the comparison of the results of two algorithms is discussed. If h(-n) is obtained, then g(-n) can be obtained using (3) and the comparison is discussed. The h(-n) of the standard wavelets are fed into the LMS and NLMS algorithms. The role played by adaptive filter is to find out the filter coefficients with minimum error. This algorithm suggests that the estimated signal is up sampled, obtained at the output end of the adaptive filter and gives this new signal as input, a new set of filter weights can be obtained with minimum error. The above steps are repeated iteratively for many times to get the final result. This set of filter weights is used to reconstruct the h(-n) with minimum error. This process is carried out at some of the standard wavelets available in literature. With this set of filter weights, on the standard wavelets, their respective h(-n) are successfully reconstructed.

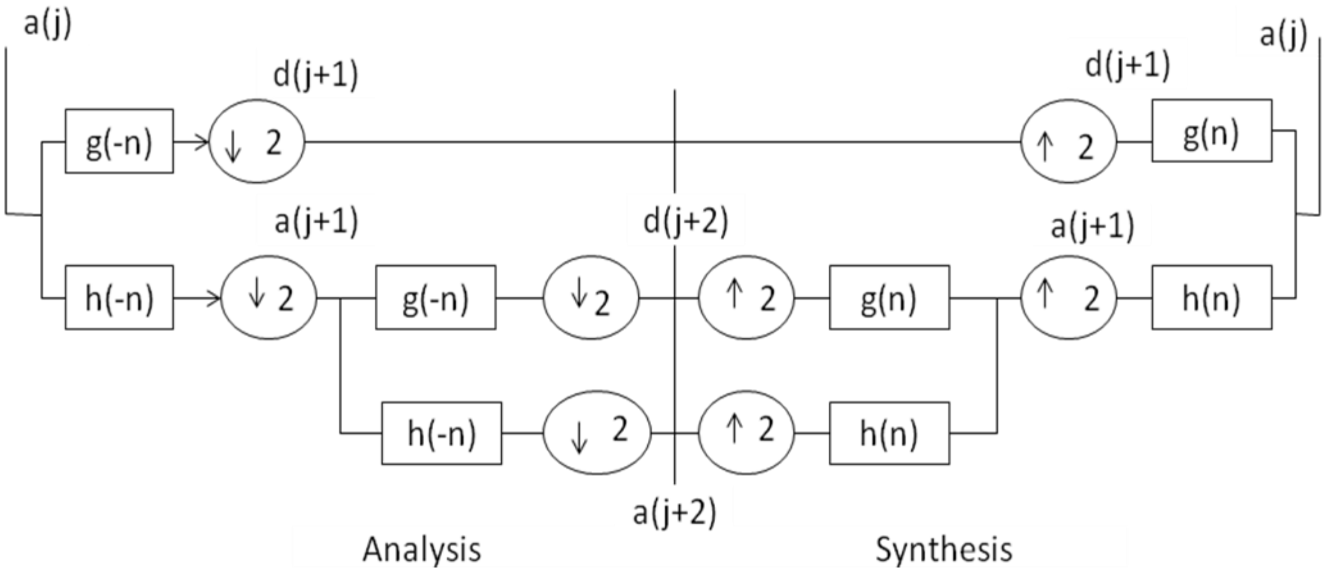

Figure1: Concept of DWT as Filter bank

3. LEAST MEAN SQUARE ALGORITHM

A uniform linear array with N isotropic elements, which forms the integral part of the FIR adaptive filter design [9] is shown in the figure 2.

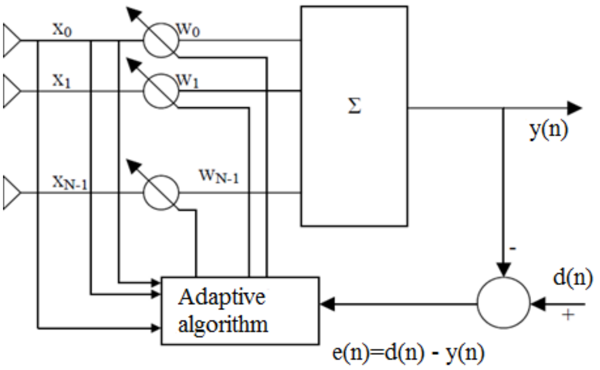

Figure2. LMS algorithm Flowchart

The outputs of the individual sensors are linearly combined after being scaled using corresponding weights to have maximum possible gain in the direction of the desired signal and nulls in the direction of the interferes. The weights are computed using LMS algorithm based on minimum squared error (MSE), therefore the spatial filtering problem involves estimation of the signal s(n) from the received signal x(n) by minimizing the error between the reference signal d(n), which closely matches or has some extent of correlation with the desired signal estimate and the output y(n).

From the method of steepest decent, the weight vector equation is given by [10]

$$w(n+1) = w(n) + \frac{1}{2}\mu[-\nabla(E\{e^2(n)\})] \qquad (5)$$

Where μ is the step size parameter and controls the convergence characteristics of the LMS algorithm; $e^2(n)$ is the mean square error between the output y(n) and the reference signal which is given by,

$$e^2(n) = [d^*(n) - w^h x(n)]^2 \qquad (6)$$

The gradient vector in the above weight update equation can be computed as

$$\nabla_w(E\{e^2(n)\}) = -2r + 2Rw(n) \qquad (7)$$

The LMS algorithm takes the instantaneous values of covariance matrices r and R rather than their actual values i.e.

$$R(n) = x(n)x^h(n) \quad \text{And} \quad r(n) = d^*(n)x(n) \qquad (8)$$

Therefore the weight update can be given by the following equation

$$w(n+1) = w(n) + \mu x(n)[d^*(n) - x^h(n)w(n)]$$
$$= w(n) + \mu x(n)e^*(n) \qquad (9)$$

The LMS algorithm is initiated with an arbitrary value w(0) for the weight vector eventually leads to the minimum value of the mean squared error. Hence the LMS algorithm can be summarized in the following equations;

Output $\quad y(n) = w^h x(n) \qquad (10)$

Error $\quad e(n) = d^*(n) - y(n) \qquad (11)$

Weights $\quad w(n+1) = w(n) + \mu x(n)e^*(n) \qquad (12)$

An LMS adaptive filter which has p+1 coefficients will require p+1 additions and p+1 multiplications for updating the filter coefficients, and it is necessary to compute the error;

$$e(n) = d(n) - y(n) \qquad (13)$$

Since w(n) is a vector of random variables, the convergence of the LMS algorithm [11] should be considered within the statistical framework. For the convergence of the algorithm the step size should satisfy the following condition;

$$0 < \mu < \frac{2}{\lambda max} \qquad (14)$$

Where λmax is the largest eigenvalue of the correlation matrix. The drawback of the LMS algorithm is that, it is very sensitive to the change in the input signal x(n), which result in having difficulty to decide the optimum size of convergence parameter μ for convergence of the algorithm and minimum time of convergence also. Normalized LMS algorithm may be a suitable alternative [12] which normalizes the LMS step size with the power of the input. When the input signal is too small, the NLMS algorithm can be modified by adding a small positive value ε to the power of the input signal.

$$w(n+1) = w(n) + \beta \frac{x^*(n)}{\varepsilon + |x(n)|^2} e(n) \qquad (15)$$

In the algorithm developed, h(-n) is reconstructed using LMS algorithm. In this algorithm the desired signal is first down-sampled and up-sampled before giving to the filter, but in this process value of h(-n) is changing with the step size as shown in the table 1. This problem of sensitivity of step size can be resolved by taking NLMS algorithm in consideration, which is not sensitive to step size as shown in table2.The stream of impulses is given as input to the adaptive filter, up-sampling and then passing the estimated output iteratively to the filter helps in calculating the scaling function h(n). The maximum five iterations are required. The iteration method is shown in figure3 is implemented on coif5 and db4 wavelets, which are the standard wavelets.

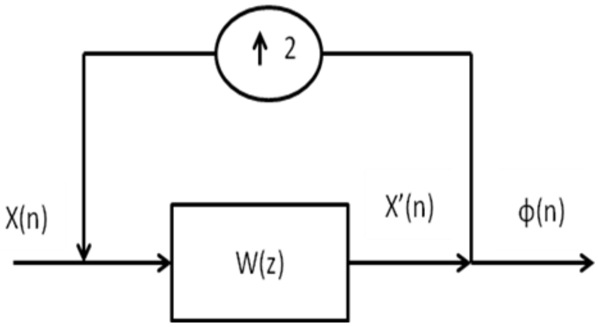

Figure3: Flow chart for scaling function

4. EXPERIMENTAL RESULTS

4.1 Using LMS Algorithm

We implemented LMS algorithm to reconstruct the standard wavelet db4 and coif5. Since LMS algorithm is sensitive to step size hence reconstruction is not successful as shown in the table 1.

Table1: LMS algorithm on wavelet db4

Actual Filter Coefficients h(n)	Step Size (α)	LMS Filter Coefficients
[0.3258, 1.0109, 0.8922, - 0.0396, -0.2645, 0.0436, 0.0465, -0.015]	0.2	[0.2376, 2.0112, 0.2633, 0.1232, -0.4325, -1.3456, -0.8723, 0.2432]
	0.6	[0.3299, 0.1324, 0.1244, -0.4567, 0.4576, -0.2345, -1.0365, 0.4536]
	1.3	[-.5647, 0.1137, 0.4578, -0.4568, 1.2234, -0.1235, 0.5634, 0.3568]

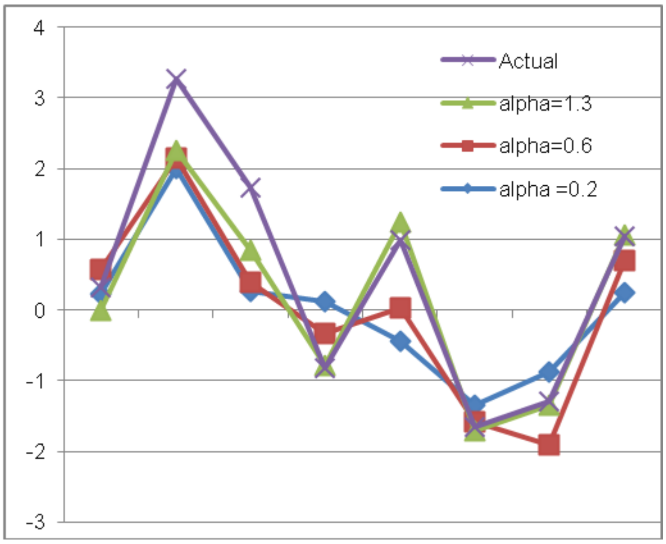

Figure4: Graph of h(n) using LMS algorithm

4.2 Using NLMS Algorithm

We repeated the same experiment for constructing the above wavelets with NLMS algorithm and the reconstruction is successful and also not sensitive to step size as shown in the table 2. The new scaling and wavelet functions of the wavelet coif5 and db4 are shown in the figures 6, 7and 8.

Table2: NLMS algorithm on wavelet db4

Actual Filter Coefficients h(n)	Step Size (α)	LMS Filter Coefficients
[0.3258, 1.0109, 0.8922, -0.0396, -0.2645, 0.0436, 0.0465, -0.015]	0.2	[0.3233,1.0109,0.8767, -0.0399,-0.2578,.0438, 0.0478,-0.027]
	0.6	[0.3260,1.0109,0.8932, -0.0398,-0.2650,0.0437, 0.0469,-0.015]
	1.3	[0.3245,1.0136,0.8678, -0.0369,-0.2278,0.0466, 0.3356,-0.039]

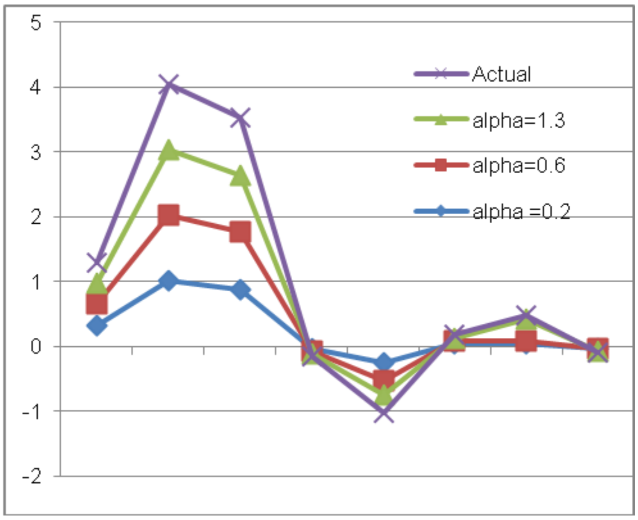

Figure5: Graph of h(n) using NLMS algorithm

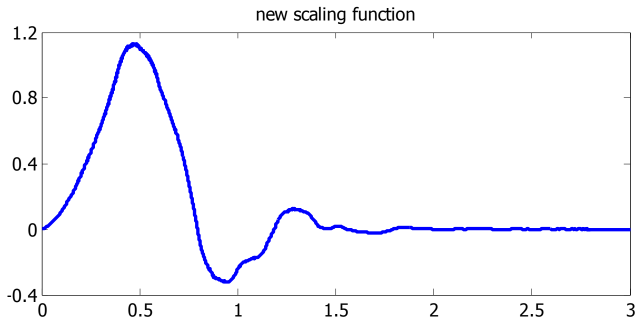

Figure6: New scaling function of db4

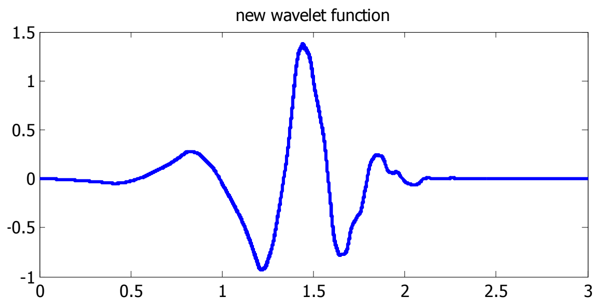

Figure7: New wavelet function of db4

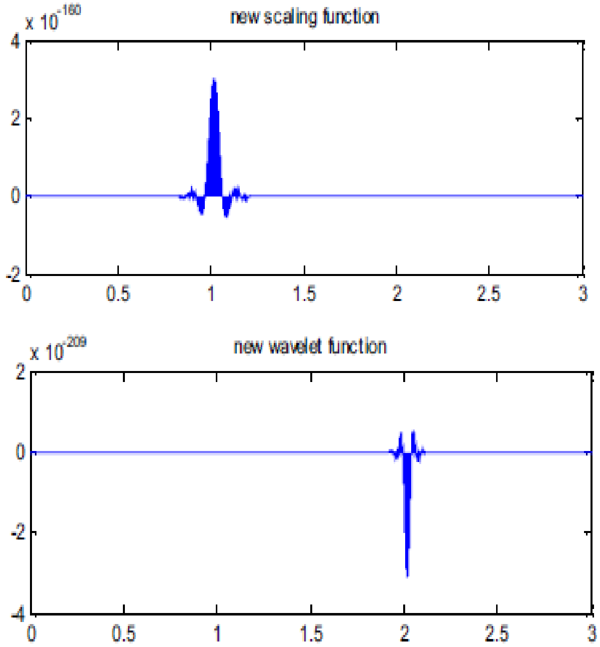

Figure8: New scaling and wavelet functions of coif5

5. CONCLUSSIONS

In this paper we conclude that LMS and NLMS algorithms are two important algorithms for adaptive filter design. NLMS algorithm is more useful than LMS due to its quality of insensitivity to step size. We can implement this algorithm for the identification of filter bank coefficients of standard wavelets db4 and coif5 and reconstruct the wavelets also. The number of iterations used for the convergence of the algorithm is five only; hence convergence time is the least. Wavelets are very useful for characterization of music signals. So our future work is related to the identification of signature wavelet of the sound signals of some Indian musical instruments.

6. REFERENCES

[1] Bultheel, A., 2003, Wavelets with Applications in Signal and Image Processing, Chapter- 5, pp 1-181.

[2] Pruysers, C., Schnapp, J., and Kaminskyj, I., 2005. Wavelet Analysis in Musical Instrument Sound Classification, IEEE Transaction on Acoustic, Speech and Signal Processing, Vol. 1, pp. 1 – 4.

[3] Mallat S.G, 2009. A Wavelet Tour of Signal Processing, The Sparse Way, 3rd Edition, Academic Press.

[4] Serrano E .P Fabio, 2002. Application of the Wavelet Transform to Acoustic Emission Signals Processing, IEEE Transaction on Signal Processing, Issue 5, pp. 1270-1275.

[4a] Soman, K.P, Ramchandran, K.I, and Resmi, N.G, 2010, Insight into Wavelets from Theory to Practice, PHI Learning Private Limited New Delhi, 3rd Edition.

[5] Mallat S.G, 1989, A Theory for Multiresolution Signal Decomposition: The Wavelet Representation, IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol. 11, pp. 674 – 693.

[6] Joseph.O.Chapa and Raghuveer M. Rao, 2000, Algorithms for Designing Wavelets to match a Specified

Signal, IEEE Transactions on Signal Processing, vol.48, No.12, pp. 3395-3406.

[7] Vetterli. M and Harley. C, 1992, Wavelets and Filter Banks: Theory and Design, IEEE Transactions on Signal Processing, Vol.40.

[8] Gianpolo Evangelista, 1993, Pitch- Synchronous Wavelet Representation of Speech and Music Signals, IEEE Transactions on Signal Processing, Vol.41, No.12, pp. 3313-3330.

[9] Markus R, 1993, The Behaviour of LMS and NLMS Algorithms in the Presence of Spherically Invarient Process, IEEE Transactions on Signal Processing, Vol.41, No.3, pp. 1149-1160.

[10] Douglas S. C and Markus R, 2000, Convergence issue in the LMS adaptive Filter", CRC press LLC.

[11] Dirk T.M Slock, 1993, On the Convergence Behaviour of LMS and Normalized LMS Algorithm, IEEE Transactions on Signal Processing, Vol.41, No.9, pp. 2811-2825.

[12]Tarrab. M and Feuer A, 1988, Convergence and Performance Analysis of Normalized LMS Algorithm with Uncorrelated Gaussian Data, IEEE Transaction on Information Theory, vol. 34, pp. 680-691.