Fuzzy Application for Tracking Heterogeneous Sensor Node to Prolong System Lifetime in WSN Word Template

Raju Dutta
Assistant Professor
Narula Institute of Technology
Kolkata-700109, W.B., India

Sajal Saha
Assistant Professor
Narula Institute of Technology
Kolkata-700109, W.B., India

Asish K. Mukhopadhyay
Professor
Dr.B.C.Roy Engg. College
Durgapur-713206, W.B.,India

ABSTRACT
Mobility of sensor node in Wireless Sensor Network (WSN) is one of the key advantages of wireless over fixed communication system. But to track the sensor node in the heterogeneous network is more challenging and difficulties. In heterogeneous system, generally power consumption is more then homogeneous system. Thus, tracking the location of sensor node is not only one of the challenges for location management but to prolong the system lifetime is also very much important in WSN. Fuzzy application is a new era in communication system. Using fuzzy in heterogeneous system, can easily track the sensor node and consequently prolong the system lifetime. In this paper we introduce a movement pattern learning strategy system to track the node's movement using adaptive fuzzy logic. Every node of different category identified as a cell in a location. Here fuzzy inferences system extracts pattern from the past data records as occupying cell number, date and time of sensor node of particular type. Here in this paper this strategy has been implemented and we propose a mathematical model, that model has been verified with real time data. This mechanism reduces sensor node's location tracking cost. All together overall it prolong the system lifetime Here in this paper we have discussed and proposed a mathematical model to find an optimal solution to optimize energy consumption of the sensor node and to maximize system life time. Through an extensive simulation results show that the proposed model has good performances in the aspects of energy consumption and efficiency of the system network to prolong the system life time.

Keywords
Sensor Nodes, System Life Time, Fuzzy Logic, WSN, Node Deployment, Fuzzy inference system.

1. INTRODUCTION
A wireless sensor network forms by self-organized sensors and many of the node are not perform well all the time in the network. Every node has a specific application in an area in the network. Each sensor can sense the data from the environment, performing simple computations and transmitting this data over wireless medium either directly to specific position or through some cluster head. WSNs although have some similarities with ad-hoc networks but they differ from ad-hoc networks mainly due to their more severe energy constraints, much larger density of sensor nodes, lower cost and usually static nature of nodes. In WSNs the positions of the node are not always fixed in the network and different algorithms are also applied in different situation to track the location of the node because the topology of sensor network changes very frequently. It is evident that most of the sensors follow a particular pattern distinct for set of users while moving within a wireless network. In this paper, we consider four distinct types of sensor nodes – Sensor type-1(Mobile), Sensor type-2, Sensor type-3 & so on. We can easily get a picture of the movement pattern of a particular category of sensor by studying the historic records of their movement patterns. We implement this movement pattern into a fuzzy inference engine and get the location of a sensor node in terms of cell number as the output.

1.1 Hardware Analysis of Sensor Node:

Here a simple architecture of a sensor node is given and the components are explained. Since Wireless sensor networks are comprised of a number of distributed sensor nodes which cooperate to monitor the environment. Commercial wireless sensor node are generally composed of a single microcontroller and a number of other components.

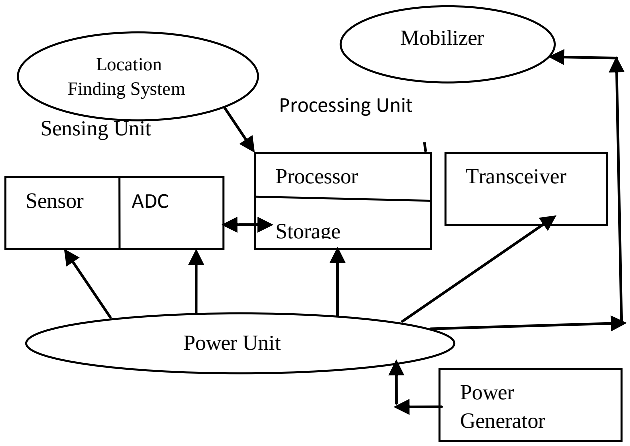

Fig : 1 The Components of a Sensor Node

The controller performs different task such as processes data and controls the functionality of other components in the sensor node. Now the microcontroller is the most common controller. A microcontroller is frequently used in several embedded systems such as sensor nodes because of it is flexibility and low cost to connect to other devices, for of programming, and minimum power consumption. Since the positions of the node are not always fixed in the network and different algorithms are also applied for the communication to the other sensor node in different situation because the topology of sensor network changes very frequently. A sensor node has major four basic components [10], as shown in Figure 1.These components are sensing unit, a processing unit, a transceiver unit, and power unit. Sensors and analog-to-digital converters (ADCs) are two main components of Sensing unit. The analog signals produced by the sensor node are converted to digital signals by ADC and then fed into the processing unit. The processing unit (microcontroller) is associated with a small storage unit which performs the procedures that make the sensor node collaborate with the other nodes. A transceiver unit connects the every node to the sensor network. The transmitter and receiver both are combined into a single device which is known as transceivers. Generally the sensor nodes are consumes power for sensing other node, communicating and data processing. Power is stored in batteries which is both rechargeable and non-rechargeable, are the main source of power supply for sensor nodes. To carry out the assigned tasks mobilizer may be needed to move sensor nodes when it is require.

2. RELATED WORK

Several works have already been done on the movement prediction of the user. Various approaches for optimizing the energy usage in wireless sensor networks have been proposed [1, 2, 3]. The potential of fuzzy logic is being fully explored in the fields of signal processing, speech recognition, aerospace, robotics, embedded controllers, networking, business and marketing [4]. Fuzzy logic has been successfully applied in various areas including communications and has shown promising results [4, 5]. However, the potentials of fuzzy logic in wireless sensor networks still need to be explored. Optimization of wireless sensor networks involve various tradeoffs, for example, lower transmission power vs. longer transmission duration, multi-hop vs. direct communication, computation vs. communication *etc.* Fuzzy logic is well suited for applications having conflicting requirements. In different way, Aircraft dry bays and engine compartment fire detection system is proposed in [6], where the authors used a fuzzy inference system to detect a fire. This research focuses on fire detection system using image analysis technique. Using statistical measures of histogram and subtraction data of successive frames, fuzzy if then rules are used to compute the probability of fire. The work in [7, 8] proposes fire detection system using neural network and fuzzy inference system. Two basic mechanisms to determine a sensor node's (MN's) current location are: location update (or) registration and paging. There are three types of location update technique – time based, location based and movement based [9], [10]. On call arrival, the current location of the terminal is predicted based on the information received during the last update or last call termination. This can be done by using Self Organizing Feature Map (SOFL) and Multilayer Perception (MLP) [11], [12]. It uses the location based update technology. In [13], a new paging strategy named sectored sequential paging (SSP) is introduced intended to divide the location area (LA) into different sectors. Each sector has some weighted probability according to the direction of entrance. When a call comes to a certain mobile node (MN), the cell with the highest probability is paged first. In another scheme, users can be classified into different categories based on their characteristics where, each category of users follows a routine. User pattern learning location prediction scheme reduces the location and paging requests [14]. It uses time-based location update scheme. In [15], topology independent Markov Chain Monte Carlo method (MCMC) has been used to predict the user location. A radial based function (RBF) network is used in [16] to estimate the location of a MN in the coverage area of CDMA cellular network.

We propose a strategy that differs from the other strategies in various aspects. We use fuzzy inference system to depict the sensor node's usual movement schedule. We classify sensor node into several categories each following a particular routine. Based on the category, the location of the sensor node is predicted. The cost of paging is reduced since the location detail is always known in advance. Therefore no registration is required to update a location enabling further reduction of cost which depends on the user behaviors.

3. FUZZY SETS

The fuzzy set theory was to deal with problems involving knowledge expressed in vague, linguistic terms. In crisp set, there is no ambiguity or vagueness as for the belonging of each element to the set concerned. A fuzzy set is a set with each element in a set having graded membership in the real interval [0, 1]. That is, elements can belong to a fuzzy set to a certain degree. Fuzzy set theory can be defined as a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague, and otherwise fuzzy. Consider a universal set *X* and whose elements in the set are *x*. A fuzzy subset in *X* is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X a real number in the interval [0, 1]. The function value $\mu_{\tilde{A}}(x)$ represents the graded membership of x in $\tilde{A}$. A fuzzy set $\tilde{A}$ in X is a set of ordered pairs and is given as $\tilde{A} = \{x_i, \mu_{\tilde{A}}(x_i)\}\ x_i \in X$ where x_i represents the element in a set $\tilde{A}$, $\mu_{\tilde{A}}(x)$ represents the membership value of element x_i in the set $\tilde{A}$ and upper stroke tilde denotes the fuzzy set [17, 18, 19]

3.1 Overview of Fuzzy Logic

A Fuzzy system basically consists of three parts: fuzzifier, inference engine, and defuzzifier. The fuzzifier maps each crisp input value to the corresponding fuzzy sets and thus assigns it a truth value or degree of membership for each fuzzy set. The lifetime of the sensor network will be maximized is objective of our fuzzy routine and to determine the value of cost for a link between two sensor nodes. The fuzzy rule base not only extend the life time of the sensor network but also to balance the routing load among sensor nodes effectively so that a maximum number of nodes have sufficient energy to continue performing their own sensing tasks.

3.2 Fuzzy Inference System for Node detection

The decision network of the new detection algorithm is realized by a fuzzy expert system. Fig. 2 illustrate the structure of a fuzzy logic system with multi-sensor Fuzziness describes event uncertainty and impreciseness of linguistic terms. WSN is typically used to monitor some parameters of an environment process. The atmospheric events are complex, ambiguous and vagueness embedded in their nature. Consequently, a fuzzy based approach is a viable option. The model of fuzzy logic system as shown in Fig. 3 consists of fuzzification, fuzzy rules, FIS and defuzzification process.

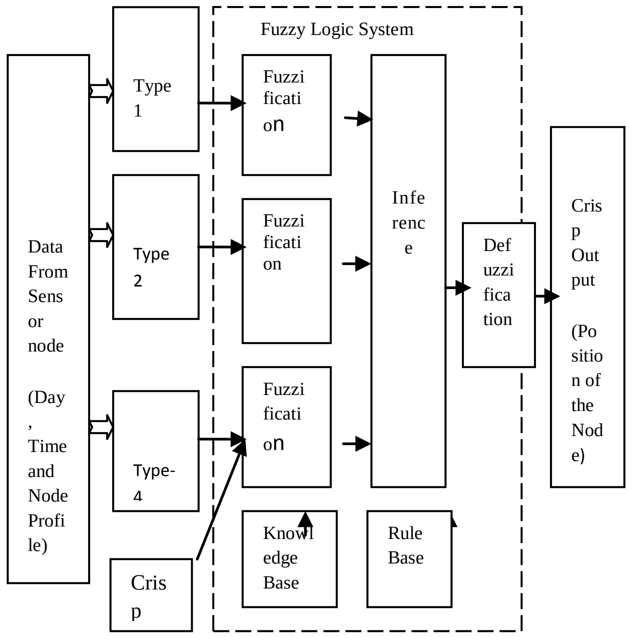

Fig- 2. Structure of a fuzzy logic system with multisensors

3.3 System Mathematical Model and Problem Definition

The fuzzy logic controller (FLC) provides mechanism to convert the linguistic control strategies based on intuition, heuristic learning and export knowledge into an automatic control strategy [9]. The FLC is made of fuzzifier, inference engine, fuzzy rule base and defuzzifier. Basic block diagram of FLC is shown in fig 2. Input to the FLC is shown in fig 3.

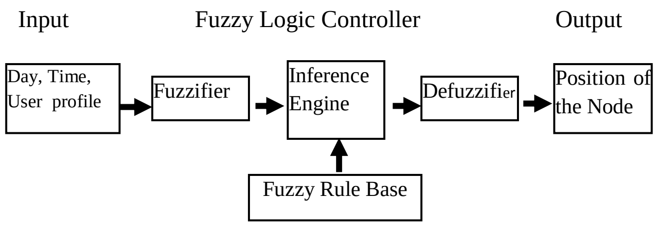

Fig 3: Input and output to the fuzzy logic controller

We choose triangular function as a membership function [10]. This function is nothing more than a collection of three points forming a triangle. The membership functions for input and output linguistic parameters are shown in the Fig.4.

We select day and time as the two inputs for our Fuzzy Inference System (FIS) for a certain user profile. Below, we define these inputs and discuss their relationship to the rule set and output signal.

Input 1: Days: include weekdays and holidays.

- Membership functions: weekdays and holidays
- Use in rule set: Set the user paging area based on some defined rule. Thus using the membership function weekdays, fall among when the user found Monday, Tuesday, Wednesday, Thursday, and Friday. For Saturday and Sunday, use the membership function holidays.

Input 2: Time: Specific moment when user to be located.

- Membership function: Divide 24 hours into 39 triangular membership functions.
- Use in rule set: Depending on the user profile LA is being selected by the FLC.

Output: Location area of user.

The output has 19 membership functions (c1 to c19) depicting the 19 hexagonal cells in our region of survey as shown in Fig. 7. The MN falls under the special cell c0 if it does not fall under any one of the 19 cells. The markings 1,2,3,4,5,6,7 denote the week days sequentially from Monday to Sunday as shown in Fig. 3. The total span of 7 days are divided into two trapezoidal membership functions d1 and d2 depicting the week days and holidays respectively. The function d1 has a peak membership function of 1 from day values 0 to 5.5 and decays to 0 value at 7; whereas, d2 rises from 0 value at 5.5 and reaches peak value 1 at 7.

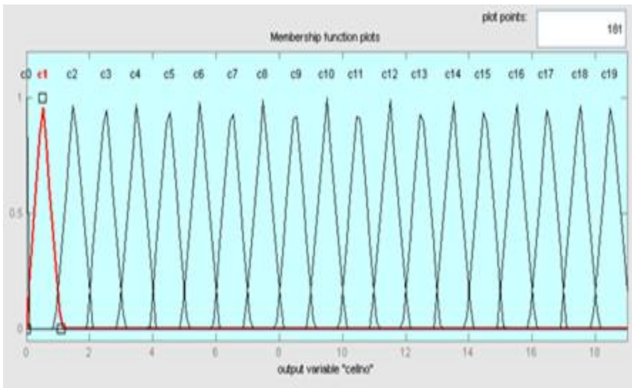

Fig 4: Output: Cell number

The system has total four layers as shown in Fig. 2.

Layer1: Every node in this layer is an adaptive node performing triangular membership function

$O_{1,i} = \mu_{A,i}(a_i)$, Where μ is the grade of membership of a_i in A, i and i=1, 2... in the input layer.

Layer2: Every node in this layer is a fixed node whose output is the product of the entire incoming signals. These nodes perform the fuzzy AND operation of the function. $O_{2i} = \mu_{A,i}(a_i).\mu_{B,i}(b_i)$

Layer3: The single node in this layer computes the summation of all incoming signals and defuzzified. $O_{3,i} = \sum f_i$.

Layer 4: This layer provide specified to output according to input. $O_{4,i} = LA_i$. This structure can update membership function and rule base parameters accordingly. We consider a two input first-order Mamdani fuzzy model with fuzzy IF THEN rules. A typical IF-THEN rules are depicted below.

If (time is t1) and (day is d1) then (cell no. is c17) for user Manager. 48 IF-THEN rules generated in the user specific rule base. We consider four category of sensor. So the rule base contains 248 rules.

Now consider the problem that an MN is deviated from it's usual LA to another location as specified in the rule base. Suppose the general routine of a mobile user is to arrive at cells C(t-2),C(t-1),C(t),C(t+1),C(t+2) at times t-2,t-1,t,t+1,t+2 respectively. We assume that this deviation can occur over a range of four hours. The user may reach C(t), up to two hours late or early than expected time t. Therefore, if the user at time t is not located at C(t), then its next probable location would be at C(t-1) or C(t+1). If he is still not located, then the MSC must search the next probable location at C(t-2) or C(t+2). So at time t, the user has the highest location probability at C(t) followed by C(t±1) and C(t±2). So paging signals are sent first to C(t). If the user is not located then paging signals are sent simultaneously to C(t-1) and C(t+1). If the user is still not located, then paging signals are sent to C(t-2) and C(t+2).

3.4 SIMULATION AND Results Analysis

We consider our survey area over the region of Saltlake city, Kolkata, India. We have randomly taken mobile user's as Sensor node (Type-1) and collected data about their movement in 7 x 24 hour basis. The mobility pattern of the different sensors is divided in the area of interest into 19 hexagonal cells. The wikimapia snap-shot of the region along with the logical alignment of the cells is shown in the fig 5.

Fig 5: Aerial view of Salt Lake City, Kolkata

Simulation has been carried out using MATLAB R2008b.We created five different rule bases corresponding to five different categories of users in MATLAB fuzzy logic toolbox. MATLAB generates surface view graphs corresponding to a particular rule base. In the surface view as shown in Fig 6, 7, 8 and time is set along X- axis, day is set along Y- axis and LA with respect to cell number is the output along the Z- axis. We collected four consecutive LA values of MN with specific day and time as input and take the ceiling value. On the basis of survey results the experimental results of LA of several sensor are shown in Table 1. Considering the stability of the movement of the users we use R^2 test [20], [21] of the polynomial of the user movement curve.

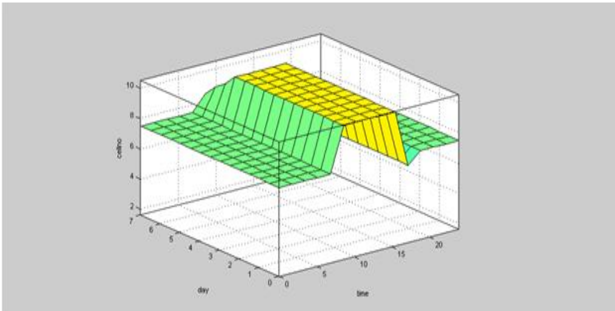

Fig 6 : surface view of Sensor node Type-1

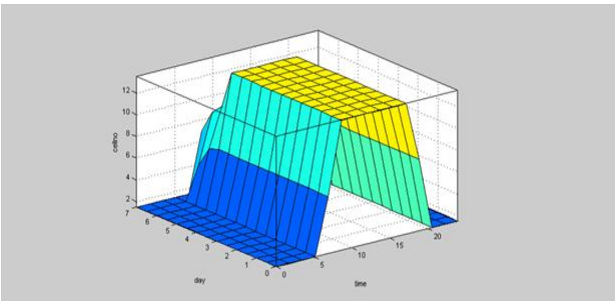

Fig 7 : surface view of Sensor node Type-2

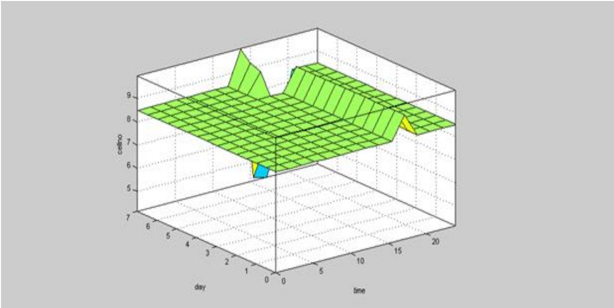

Fig 8: surface view of Sensor node Type-3

Table 1: Experimental LA of users at specific day and time

Sensor profile	Day	Time	Sequence	Cell number
Sensor node Type -1	w	12:00	10.5043,10.5043,10.5043,10.5043,10.5043	c11,c11,c11,c11,c11
	w	18:30	7.5005,1.4960,7.5005,10.5043,7.5005	c8,c2,c8,c11,c8
	h	12:00	7.5005,7.5005,7.5005,7.5005,1.4960	c8,c8,c8,c8,c2
	h	18:30	3.5014,13.5019,7.5005,4.4967,7.5005	c4,c14,c8,c5,c8
Sensor node Type -2	w	12:00	13.5019, 13.5019, 13.5019, 13.5019, 13.5019	c14,c14,c14,c14,c14
	w	18:30	3.5014,13.5019,9.5000,13.5019, 1.4960	c4,c14,c10,c14, c2
	h	12:00	1.4960, 1.4960, 1.4960, 1.4960, 1.4960	c2,c2,c2,c2,c2
	h	18:30	9.5000,10.5043,9.5000,10.5043, 9.5000	c10,c11,c10,c11,c10
Sensor node Type -3	w	12:00	8.4957, 8.4957, 8.4957, 8.4957, 8.4957	c9,c9,c9,c9,c9
	w	18:30	8.4957,9.5000,8.4957,9.5000,8.4957	c9,c10,c9,c10,c9
	h	12:00	8.4957, 8.4957, 8.4957, 8.4957,10.5043	c9,c9,c9,c9,c11
	h	18:30	9.5000,3.5014,8.4957,4.4967,8.4957	c10,c4,c9,c5,c9
	h	18:30	6.5052,8.4957,16.4938,8.4957,16.4938	c7,c9,c17,c9,c17

3.5 MATHEMATICAL MODEL AND ANALYSIS

Here we have find a model of a 7th degree polynomial of the movement pattern curve for the Sensor node(Type-1)is

$$(-10^{-4})x^7+2(10^{-3})x^6-2.6(10^{-2})x^5+1.5(10^{-1})x^4-3.3(10^{-1})x^3+8.4$$

R^2 value of the polynomial is 0.459 as shown in Fig.9. As the R^2 value is less than 1, that indicates the polynomial is stable. R^2 value of the polynomial of Sensor Node (Type-2) is 0.392, sensor Node (Type-3) is 0.276 and so on. Hence we come to the conclusion is that the movement of all category of sensor node is predefined and stable. Movement pattern of a Sensor Node (Type-1) randomly chosen from survey data collected by us is compared with our simulation result and plotted in the logical cellular region is shown in fig. 10. From real time data and simulation data the least square regression parabola to the bivariate samples are similar to each other.

$$y=2.69+0.22x-0.09x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$y=2.98+0.08x-0.13x^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

respectively.

Now we observe that goodness of fit of the best fitting parabola

$$y=a_0+a_1x+a_2x^2+\ldots\ldots\ldots\ldots\ldots\ldots+a_kx^k$$

to the bivariate sample become high if the value S^* become low and vice-versa. So S^* gives an inverse measure of goodness of fit, where

$$S^*=\frac{1}{n}\Big[\sum_{i=1}^{n}y_i^2-a_0\sum_{i=1}^{n}y_i-a_1\sum_{i=1}^{n}x_iy_i-a_2\sum_{i=1}^{n}x_i^2y_i-\ldots\ldots\ldots\ldots\ldots\ldots\ldots-a_k\sum_{i=1}^{n}x_i^ky_i \quad \ldots\ldots\ldots\ldots(iii)$$

Again if $R_Y=\rho(\hat{U},\hat{Y})$ be the co-relation co-efficient between $\hat{U}$ and $\hat{Y}$ then we get $S^*=s_y^2(1-R_y^2)$, where, $0\le R_y\le 1$ and s_y is the standard deviation of $\hat{Y}$.Therefore, $R_y^2=1-\frac{S^*}{s_y^2}$, where $s_y\rangle 0$ …………………(iv)

3.6 Test for goodness of fit

The simplest hypothesis concerning the cell probabilities specifies a numeric value for each cell. Here we test if the sample comes from a specified population. We want to know how good the fit is, i.e., how will expected cell count measures with the observed values.

The test statistic

follows a χ^2 distribution with (k-1) degrees of freedom. We reject our model at level α if,

$$\chi^2_{(\text{Observed})}\ \rangle\ \chi^2_{\alpha\,;\,k-1\ (\text{Tabulated})}$$

where $\chi^2_{\alpha\,;\,k-1}$ is tabulated value for the upper $100\,\alpha\%$ point of χ^2 -distribution with (k-1) degrees of freedom.

Here calculating we get,

$$\chi^2_{(Observed)}=14.7909<\chi^2_{0.05\,;\,47\ (Tabulated)}=16.919$$

Which implies that our Mathematical model is well accepted for tracking the Sensor Node in Wireless Sensor Network and simultaneously it increases the system lifetime.

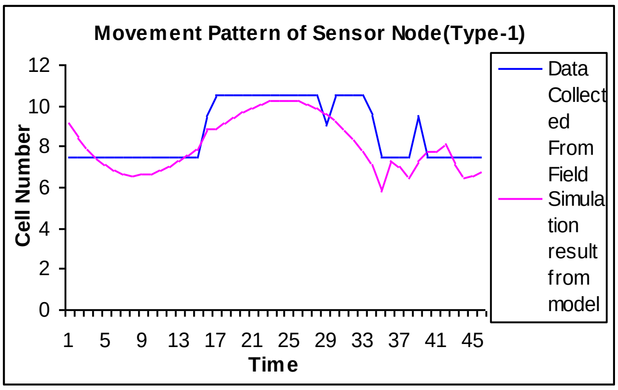

Fig 9 : 7th degree polynomial equation curve of the movement pattern of the Sensor node (Type-1)

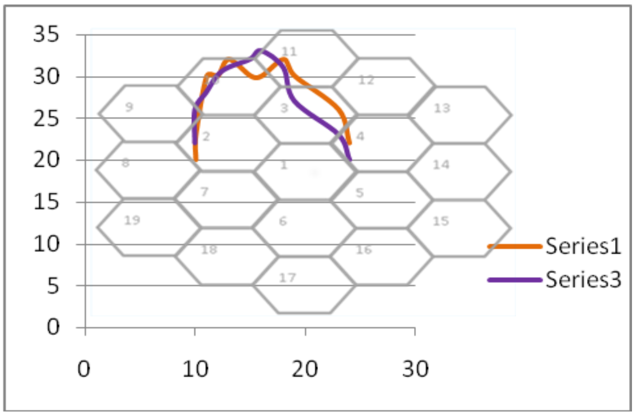

Fig 10: real time movement (series 1) vs. simulation result (series 2) of the movement of the user curve with respect to cell no. (Y-axis) and time (X-axis).

Now we see that S^* decreases as R_Y $(0 \le R_Y \le 1)$ increases. So, R_Y can be taken as a direct measure of goodness of fit of the least square regression parabola to the bivariate sample.

4. CONCLUSION

In this paper a new fuzzy logic based intelligent paging scheme has been developed. This scheme reduces the paging cost of the local MSC for a new call and save the radio bandwidth resource. Prediction accuracy is also been enhanced by minimizing the standard deviation we have verified our model by χ^2. test which implies that our mathematical model is well accepted and very much perfect for instantly tracking the sensor node in wireless sensor network and simultaneously it increases the system lifetime. fig-9 shows that the movement pattern on the sensor node (type-1) is very much similar with the real field collected data and Movement pattern of a Sensor Node (Type-1) randomly chosen from survey data collected by us is compared with our simulation result and plotted in the logical cellular region is shown in fig. 10. From real time data and simulation data the least square regression parabola to the bivariate samples are similar to each other. However $R_Y^{\ 2}$ for the equation (i) and (ii) are 0.5 and 0.35 respectively, indicates the linear relationship between the equations are also preserved. The principal assumption considered here is that the user movement posses a mobility pattern. Hence, historical data of user movement is fed into the rule base to produce the LA with respect to the cell number. In this scheme, our rule base is fed offline by the movement of large number of users. However efficiency of the scheme is highly dependent on the size of the rule base.

5. REFERENCES

[1] Akyildiz,I.F., Weilian, S., Sankarasubramaniam, Y.,and Cayirci, E. 2011 "A survey on sensor networks", Communications Magazine, IEEE, 40(**8**), 102 - 114.

[2] Heinzelman, R. 2000 "Energy Scalable Algorithms and Protocols for Wireless Sensor Networks", in Proceedings of the International Conference on Acoustics, Speech, and Signal Processing (ICASSP'00), Turkey, 773-776.

[3] Wang, A. 2001 "Energy-Scalable Protocols for Battery: Operated Micro Sensor Networks", Computer Journal of VLSI Signal Processing, 29(**3**), 223-237.

[4] Ghosh, S., Razouqi, Q., Schumacher, H., and Celmins, A.1998 "A Survey of Recent Advances in Fuzzy Logic in Telecommunications Networks and New Challenges", Computer Journal of IEEE Transactions on Fuzzy Systems, 6(**3**), 443-447.

[5] Nikaein, N., Bonnet, C. 2002 "ALM Adaptive Location Management Model Incorporating Fuzzy Logic for Mobile Ad Hoc Networks", in Proceeding of Med Hoc Net, Italy, 1489-1499.

[6] Simon, Y. 2001 "A fuzzy logic approach to fire detection in aircraft dry bays and engine compartments", IEEE Transaction on Industrial Electronics, 47(**5**), 1161–1171.

[7] Li, Z. B., Zhou, H., 2004 "Research on the application of fuzzy data fusion to cable fire detecting system", in Proceedings of the Third International Conference on Machine Learning and Cybernetics, 4, 2083– 2085.

[8] Bao, H., Li, J., Zeng, X., and Zhang, J. 2003 "A fire detection system based on intelligent data fusion technology", in Proceedings of the Third International Conference on Machine Learning and Cybernetics, 2, 1096– 1101

[9] Mukhopadhyay, A. K. et al , 2009 "An Efficient Call Admission Control Scheme on Overlay Networks Using Fuzzy Logic", 3rd International Symposium on Advance Networks and Telecommunication System (IEEE ANTS), 1-3.

[10] Sivanandam, S. N., Sumathi, S., and Deepa, S. N. 2007 "Introduction to Fuzzy Logic using MATLAB", Springer-Verlag Berlin Heidelberg.

[11] Majumdar, K., Das, N. 2005 "Mobile User Tracking Using A Hybrid Neural Network ", Kluwer Academic publisher Hingham, MA, USA, 275-284.

[12] Majumdar, K., Das, N. 2003 "Neural Networks for location management in mobile cellular communication system", TENCON 2003, conference on convergent technologies for Asia-Pacific,2, 647-651.

[13] Misra, I. S. et. al. 2002 ”Intelligent paging strategy in 3G personal communication system”, EurAsia-ICT 2002,LNCS, Springer Berlin / Heidelberg, 899-906.

[14] Amar, J., Singh, P., and Karnan, M. 2010 “Intelligent location management for UMTS networks using fuzzy neural networks”, Journal of Engineering and Technology Research, 2 (**1**), 001-012.

[15] Akoush, S., Sameh, A. 2007 “Movement Prediction Using Bayesian Learning for Neural Networks”, proceedings of Second International Conference on Systems and Networks Communications (ICSNC 2007), 6-10.

[16] Fakhreddine, O., Karray, D., and Silva, C., “Mobile position estimation using RBF network in CDMA cellular systems”, from book Soft Computing and Intelligent Systems Design Theory, Tools and Applications, 522-532.

[17] Ross, T. J. 1997 Fuzzy logic with engineering applications, New York, McGraw-Hill Inc.

[18] Bowles, J., Pelaez, C. E. 1995 “Application of fuzzy logic to reliability engineering”, in Proceeding of the IEEE, 83(**3**), 435–449.

[19] Kenarangui, R. 1991 “Event-tree analysis by fuzzy probability”, IEEE Transactions on Reliability, 40(**1**), 120–124.

[20] Bouzaouache, H., Braiek N.B. 2008 “On the stability analysis of nonlinear systems using polynomial Lyapunov functions”, Journal of Mathematics and Computers in Simulation, Elsevier Science Publishers,76(**5-6**), 316-329.

[21] Kolokolov, A. A., Sykov, A. I. 1975 “Instability in the higher modes of a nonlinear equation”, Journal of Applied Mechanics and Technical Physics, Springer New York, 16(**4**), 519-522.