Design of Integer and Fractional order PID Controller using Dominant Pole Placement Method

Vineeta Ranjan
RAIT
Nerul
Navi Mumbai

Sharad Jadhav
RAIT
Nerul
Navi Mumbai

M.D. Patil
RAIT
Nerul
Navi Mumbai

ABSTRACT
Over the last few decades, controllers found in the industries are mostly PID controllers. They have found large recognition and applications in several industries. The majority of the controllers used in process control are of PID type. The control loops which are not properly tuned, give reduced output and inappropriate and undesired performance. There are continuous researches going on to develop new methods for PID tuning and designing. A number of algorithms have been developed by scientists and suitable methods depending upon the applications are adopted by the industries for PID tuning and designing. Several tuning methods are present but they have some restrictions. These methods do not give desired tuning parameterizations for the control systems which have higher order and delay systems. The method which is named as Dominant pole placement method provides better tuning parameterizations for the above mentioned type of systems. In this method a pair of desired poles is chosen such that the requirements of the control system are converted in terms of these chosen poles. These poles are termed as dominant poles. This is an easy design method which when implemented for various types of plant processes gives desired result. This type of controller can tune plant processes with long dead times, long time constants, and monotonic or oscillatory responses. In this method, desired closed loop performance which is performance specifications, are identified and then the dominant poles are converted in terms of these performance specifications. In this paper, the performance specifications are settling time and peak overshoot. Also constraints have been put on complementary sensitivity function to handle the high frequency noise rejection and to get more mathematical equations to solve further. The method is then extended to fractional order system with a fractional order controller. For fractional order model there is no direct method of expressing dominant poles in terms of performance specifications, so the method starts with the assumption of dominant poles. The procedure is simple, efficient and gives better performance for different types of control systems.

General Terms
PID controller designing, Integer order model, Fractional order model

Keywords
Dominant Poles, PID tuning, Oustaloup Recursive Approximation technique

1. INTRODUCTION
A variety of new control systems have come into existence owing to the advances in the technology and electronic industry. These new control systems play a vital role in the development of industry [1]. This fast pace expansion leads to the growth in almost all areas of controls: new theory, innovative controllers, actuators, sensors, novel industrial processes, new applications etc. It is predominantly important in space vehicle systems, robotics, missile guidance etc. There are two chief divisions in control theory, they are classical and modern. Of these two, the extent of classical theory is restricted to single-input and single-output system design while modern control theory is done in the state space which deals with multi-input and multi-output systems. In an automatic control system, the controller receives information from input devices and generates instructions for remedial action to uphold control system performance. The controller may be a hardware or software. The Proportional-Integral-Derivative (PID) controllers have been most commonly used in automatic control systems for decades [2]. Most of the industrial controllers in use today are PID controllers or modified PID controllers [3]. The controller structure is generally simple which makes it easy to operate and handled by process engineers. It provides feedback, can eliminate steady state offsets through integral action and anticipate the future through derivative action [4]. These controllers are robust and flexible across. The PID controller can eliminate steady-state offset for step inputs through integral action. The most basic and important requirement of any system design is the stability of the system but PID controllers are still struggling with the maximum possible stability as the requirement is not yet sufficiently achieved. PID controllers can be effectively used in most of the control systems, especially when the analytical design methods cannot be used because the mathematical model of the plant is not known, PID controllers establish to be the most appropriate type of controllers [5]. Numerous PID tuning methods have been projected over the years. The conventional methods of tuning PID controllers were developed by Ziegler and Nichols (1942). These methods are still extensively used and often form the basis for tuning procedures used by controller manufacturers and process industry [6]. In state space i.e. time domain, the design of a single-input, single-output system is such that the dominant closed loop poles have a desired damping ratio and a desired un-damped natural frequency [7]. Significant specifications such as the settling time, rise time and overshoot of the output step response are used. These specifications may be converted into the damping ratio and un-damped natural frequency, and then represented by a pair

of poles. Pole placement in the state space and polynomial settings is extensively used [8]. Initially a pair of poles is chosen which replaces the closed loop poles in the desired locations such that all the other poles are distant and to the left of chosen poles [9]. Considering the above condition is achieved, the closed- loop systems may have good probability of the desired performance. In order to achieve arbitrary pole placement for single-input-single output systems, the order of output feedback controller should be one less than the order of the plant. The challenge in the above method is that it leads to the complex controllers [10]. If the requirement is of low order feedback controller for a higher order plant, it is difficult to be achieved. It is almost impossible for the plant processes involving delay [11]. Dominant pole placement can overcome this challenge. Unlike the arbitrary pole placement, it only places a pair of conjugate poles which represent the requirements on the closed-loop response and tries to make all other poles have insignificant effects on the control performance. One design for dominant pole placement was first introduced by P. Persson (Persson and Astrom, 1993) [12] and further explained in Astrom and Hagglund (1995). Their method however is fairly efficient and well recognized in PID controller designing and tuning techniques [13]. The method however works fine only for the plants of first or second order with small time delay. If the system is higher order, plant's reduced models are used. The reduced models are generally of second order. In such cases, the chosen poles might not be dominant actually and the control performance might not be satisfactory. If not properly managed, the performance of the system may become sluggish or there are chances of unstable closed- loop system [14]. Dominant pole Placement method is simple, easy to implement, has superior computational efficiency, *y*ields good quality results. Fractional calculus was a matter of limited interest for few mathematicians and theoretical physicists in 1970's. In the last few decades the broad-spectrum interest in such a tool has experienced an ongoing growth. On the other hand inspiration for this growing interest has been the engineering applications, especially the control engineering discipline. This primary step towards the application of fractional calculus in control led to the adaptation of the FC concepts to frequency-based methods. A generalization of the PID controller, namely the $PI^{\lambda}D^{\mu}$ controller, involving an integrator of order λ and a differentiator of order μ [11]. FO systems are used for controller design purposes. With the fractional-order integral and derivative actions more powerful and supple design methods to satisfy the controlled system specifications can be developed. Further to the integer order system, fractional order plant and controllers are considered [15].

2. PID CONTROLLER

A PID controller aims at minimizing the error between a measured process variable of the controlled system and a reference, by calculating the error and generating a correction signal to the system from the error. Generally *r*(*t*) denotes the reference value, *Y*(*t*) is the output of the controlled system, *e*(*t*) is the error between *r*(*t*) and *Y*(*t*), whereas *u*(*t*) is the output control signal of the PID controller [15]. A conventional PID controller consists of three components: the proportional part, the integral part and the derivative part as shown in Fig (1).The proportional term produces an output value that is proportional to the current error value. The contribution from the integral term is proportional to both the magnitude of the error and the duration of the error. Derivative control is used to reduce the magnitude of the overshoot produced by the integral component and improve the combined controller-process stability. The output control signal of an integer order PID controller is described as follows,

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s$$

where, K_p, K_i, and K_d refers to the proportional gain, the integral gain and the derivative gain, respectively.
K_p, K_i, and K_d should satisfy following equations,

$$k_i = k_p T_i$$

$$k_d = k_p T_d$$

where, T_i and T_d refers to the integration time and derivative time, respectively.

PID controller structure for fractional order is described as,

$$C(s) = K_p + \frac{K_i}{s^l} + K_d s^u$$

where *l* and *u* represent real numbers.

3. DOMINANT POLE CALCULATION FOR INTEGER ORDER MODEL

For higher order system dominant poles are the poles are the poles which are closest to the imaginary axis [14]. As the poles go away from the imaginary axis, the system becomes more stable, thus the poles nearest to the imaginary axis will dominate the behavior of the system. Consider the original open-loop system is of high order, the closed-loop system behaves like a second-order system:

$$G(s) = \frac{w_n^2}{s^2 + 2w_n \mathcal{E} + w_n^2}$$

Consider a unit step response of a second order system. It is possible to calculate exactly maximum percent overshoot as a function of damping ratio,

$$M_p = e^{\frac{-\mathcal{E}\pi}{\sqrt{1-\mathcal{E}^2}}}$$

and settling time is given as,

$$t_s = \frac{4}{\mathcal{E} w_n}$$

Peak overshoot and the settling time are the performance specifications taken.

Dominant poles for the second order integer system in terms of performance specifications is given is as,

$$S_{1,2} = \frac{-4}{t_s}\left(1 \pm j\pi \frac{1}{\ln M_p}\right)$$

Since there is no direct method for calculation of dominant poles for fractional order, an initial assumption will be taken.

4. INTEGER ORDER APPROACH

Let the PID has transfer function,

$$G_c(s) = k_p + \frac{k_i}{s} + k_d s$$

Where k_p, k_i, k_d represents proportional, integral and derivative gains respectively.

Let the plant at dominant poles be represented by a complex number,

$$G_p = X1 + jX2$$

The characteristic equation of the system is,

$$1 + G_p(s)G_c(s) = 0$$

Substituting plant and controller in characteristic equation the following equation is obtained,

$$1 + \left(\frac{k_p s + k_i s + k_d s^2}{s}\right)(X1 + jX2) = 0$$

Substituting dominant poles –a + jb in characteristic equation and comparing real-real and imaginary-imaginary parts in equation, it can be written as,

$$a = k_p(\text{-aX1 –bX2}) + k_i\, X1 + k_d\,(X1(a^2 - b^2) + 2abX2)$$

and

$$-b = k_p(\text{-bX1 –aX2}) + k_i\, X2 + k_d\,(X2(a^2 - b^2) + 2abX1)$$

To ensure high frequency noise rejection a constraint on the complementary sensitivity function T can be defined as,

$$\mathrm{T} \le A$$

where *A* is the desired value of the complementary sensitivity function at dominant poles.

$$|T(-a+jb)| = \left|\frac{G_p(-a+jb)G_c(-a+jb)}{1+G_p(-a+jb)G_c(-a+jb)}\right| \; dB \le \sqrt{a^2 + b^2}$$

After simplification, equations in terms of a, b, X1, X2 are obtained where a, b, X1, X2 are known quantities and only unknown quantities are K_p, K_i, and K_d . These controller parameters are calculated using the LINPROG function in MATLAB considering one of the two equations which we get from complementary sensitivity function as objective function. LINPROG finds the optimized solution i.e. the best solution with minimum errors.

5. DESIGN METHOD AND IMPLEMENTATION FOR INTEGER ORDER MODEL

Step 1: Find the dominant poles
Step 2: Obtain response of plant at dominant pole
Step 3: Minimize the function given with equality constraints Set A = [ ], b=[ ], the initial starting point *x*0=[ ], ub=[ ] and lb=[ ] in MATLAB's linprog function.
Step 4: If the constraints given are not satisfied, repeat the step 1 to 3 by changing the *settling time*. This ensures the guaranteed solution of the problem
Step 5: Finally, obtain the controller parameters Kd, Ki, and Kp using the minimized variable.

5.1 Design Example I

Consider a second order plant with OLTF,

$$G_p(s) = \frac{5}{s^2 + 3s + 5}$$

Performance specifications,

$$t_s \le 1s$$

$$M_p \le 15\%$$

The resulting controller parameters are,

k_d = 4.6664
k_p = 46.1262
k_i = 100.0000

The resulting controller is,

$$G_p(s) = \frac{4.666s^2 + 46.13s + 100}{s}$$

Transfer function of controller and plant in series,

$$G_c(s)G_p(s) = \frac{23.33s^2 + 230.6s + 500}{s^3 + 3s^2 + 5s}$$

System transfer function with unity feedback,

$$G(s) = \frac{23.33s^2 + 230.6s + 500}{s^3 + 26.33s^2 + 235.6s + 500}$$

The step response of the open and closed loop control systems are shown in the figure.

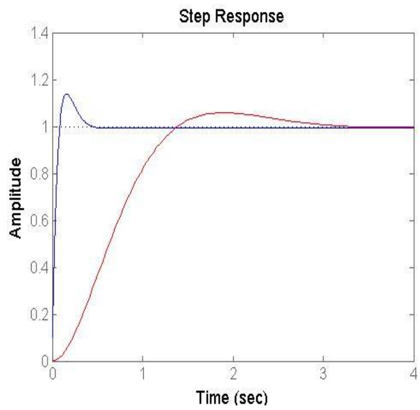


Fig 1: Step response of open and closed loop system (IO), Example I

5.2 Design Example II

Consider a second order plant with OLTF,

$$G_p(s) = \frac{30}{s^2 + 3s}$$

Performance specifications,

$$t_s \le 1s$$

$$M_p \le 20\%$$

The resulting controller parameters are,

k_d = 1.3733
k_p = 23.4659
k_i = 100.0000

The resulting controller is,

$$G_p(s) = \frac{1.3733s^2 + 23.47s + 100}{s}$$

Transfer function of controller and plant in series,

$$G_c(s)G_p(s) = \frac{41.2s^2 + 704s + 3000}{s^3 + s^2}$$

System transfer function with unity feedback,

$$G(s) = \frac{41.2 + 704s + 3000}{s^3 + 44.2s^2 + 704s + 3000}$$

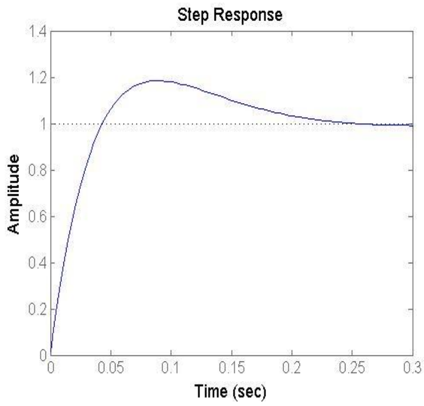


Fig 2 : Step response of closed loop system (IO), Example II

The performance of open loop and closed loop systems are compared as tabulated below, for Example I.

Table 1. Comparison of rise time, settling time and peak overshoot for open and closed loop system (IO), Example I

Parameters	Open loop	Closed loop
Rise time	1.15s	0.0643s
Settling time	3.25s	0.467s
Peak overshoot	6%	13%

6. FRACTIONAL ORDER APPROACH

Let the dominant pole be given by, -a + jb. This complex quantity can be written as,

$$-a + jb = Me^{j\theta}$$

Let the plant at dominant poles be given by,

$$G_p = X1 + jX2$$

Let the fractional order controller be FOPI, so transfer function will be,

$$G_c(s) = k_p + \frac{k_i}{s^\alpha}$$

The characteristic equation of the system will be,

$$1 + G_p(s)G_c(s) = 0$$

Substituting plant and controller in characteristic equation the following equation is obtained,

$$1 + \left(k_p + \frac{k_i}{s^\alpha}\right)(X1 + jX2) = 0$$

Substituting dominant poles –a + jb in characteristic equation we get,

$$M^\alpha \cos(\alpha\theta) + jM^\alpha \sin(\alpha\theta) + [k_p M^\alpha \cos(\alpha\theta) + jk_p M^\alpha \sin(\alpha\theta) + k_i](X1 + jX2) = 0$$

and comparing real-real and imaginary-imaginary parts in equation, it can be written as,

$$R = M^\alpha \cos(\alpha\theta) + (k_p M^\alpha \cos(\alpha\theta) + k_i)X1 - k_p X2 M^\alpha \sin(\alpha\theta)$$

And,

$$I = M^\alpha \sin(\alpha\theta) + k_p X2 M^\alpha \cos(\alpha\theta) + k_i X2 + k_p X2 M^\alpha \sin(\alpha\theta)$$

Here M and θ are known quantities. k_p, k_i and α are computed using fminsearch function in MATLAB.

7. DESIGN METHOD AND IMPLEMENTATION FOR FRACTIONAL ORDER MODEL

Step 1: Assume the dominant poles
Step 2: Obtain response of plant at dominant pole
Step 3: Select fractional order controller structure.
Step 4: Obtain frequency response of fractional order controller.
Step 5: Use characteristic equation to design fractional-order controller at dominant poles.
Step 6: Obtain parameters of fractional order controller by evaluating the expression obtained in step 5.
Step 7: Obtain integer order approximation of fractional-order plant and controller.
Step 8: Simulate the result.

7.1 Design Example

Consider a fractional order plant with OLTF,

$$P(s) = \frac{1}{0.8s^{1.2} + 1}$$

The assumed dominant pole is -2 + i.

Using fminsearch function values of k_p, k_i and α are found to be,

$$\alpha = 1.1854$$
$$k_p = 5.7090$$
$$k_i = 11.9812$$

Thus the transfer function of PI controller can be written as,

$$G(s) = 5.7090 + \frac{11.9812}{s^{1.1854}}$$

The overall integer order approximation for fractional order controller using Oustaloup recursive approximation is,

$$G(s) = \frac{1353s + 2819}{248s + 1}$$

The overall integer order approximation of the plant is,

$$P(s) = \frac{s+251.2}{202s+252}$$

The overall integer order approximation for the closed loop transfer function is,

$$C(s) = \frac{1353s^2 + 3.462x10^5 s + 7.081x10^5}{4.877x10^4 s^2 + 4.09x10^5 s + 7.084x10^5}$$

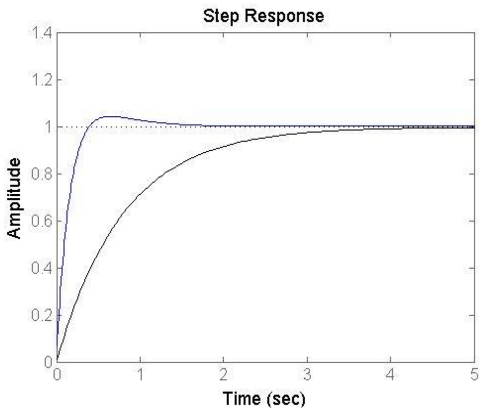


Fig 3: Step response of open and closed loop system (FO)

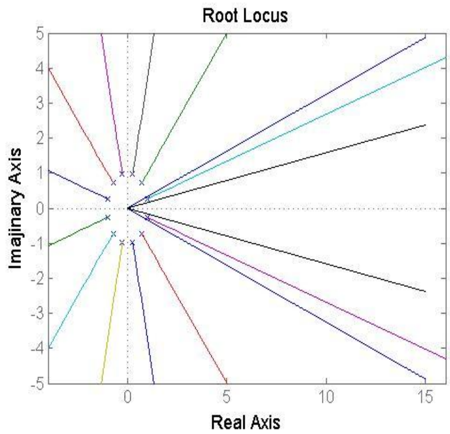


Fig 4: Root locus for fractional order model

Root locus gives the idea about the stability of fractional order model. There are several Riemann sheets but poles are looked only in principal Riemann sheet. Beyond that if poles fall on any other Riemann sheet the system is stable. Here in this case the assumed dominant poles hold good as the system is stable.

The performance of open loop and closed loop systems are compared as tabulated below,

Table 2. Comparison of rise time, settling time and peak overshoot for open and closed loop system (FO)

Parameters	Open loop	Closed loop
Rise time	2.26s	0.293s
Settling time	4.09s	1.72s
Peak overshoot	0%	4%

8. CONCLUSION

Integer order and fractional order PID controller design using dominant pole placement method is implemented for integer-order as well as fractional order plants. The simulation result shows that the rise time and settling time for closed loop system in both the cases is less than that of open loop system. However there is a small overshoot in closed loop system but is under the limit set for it. This design gives satisfactory desired response. The table below gives the percentage reduction in rise and settling time for both the cases.

Table 3. Percentage reduction in rise and settling time(IO)

Parameter	Percentage reduction
Rise time	94%
Peak overshoot	86%

Table 4. Percentage reduction in rise and settling time(FO)

Parameter	Percentage reduction
Rise time	87%
Peak overshoot	58%

The charts below clearly show the difference between the open loop and closed loop systems for integer order and fractional order system.

Chart 1: For IO model

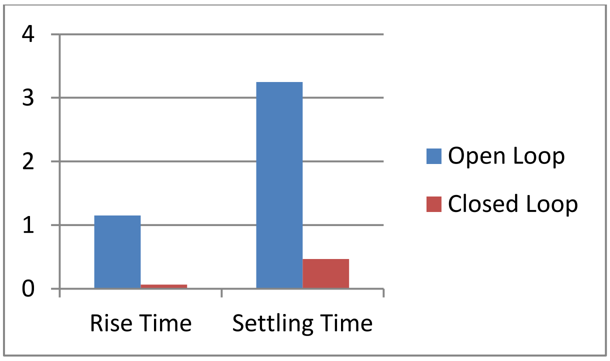
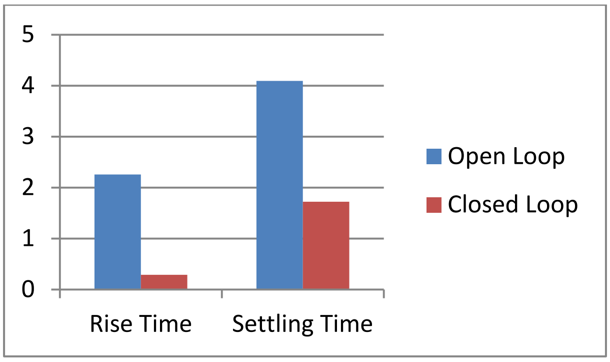


Chart 2: For FO model



9. FUTURE SCOPE

The method can be extended to designing of PID i.e. including derivative component in fractional order systems. Also fractional order controller can be designed for plants having delay or long dead times. Considering it gives the desired response, the assumption of dominant poles would be correct.

10. REFERENCES

[1] I. J. Nagrath and M. Gopal, *Control Systems Engineering*. New Age International (P) limited Publishers.,

[2] K. Ogata., *Modern Control Engineering*, Upper Saddle River, NJ.: Prentice Hall Inc., 4th edition., 2002Aidan O'Dwyer. 2006 PI and PID controller tuning rules: an overview and personal perspective. Proceedings of the {IET} Irish Signals and Systems Conference.

[3] L. M. G. M. Malwatkar., Design of controllers for higher-order-plus-delay-time processes: A practical solution.," *International Journal of Computer Applications (0975-8887) 1(21) 34:39*, 2010.

[4] K. H. G. Chong Ang and Y. Li.,\PID control system analysis, design and technology.," *IEEE Transaction on Control Systems Technology 13*, 2005.Arbor, MI: University of Michigan Press 1975, Second edition, Cambridge, MA: The MIT Press, 1992.

[5] K. J. Astrom and T. Hagglund., PID controller: Theory, design and tuning, second edition.," *Instrument Society of America. Research Triangle Park,* 1995.KiamHeongAng, Gregory Chong and Yun Li, "PID Control System Analysis, Design and Technology", IEEE Transactions On Control Systems Technology, vol 13, no 4, pp. 555-576, 2005.

[6] T.-H. L. Ho-Wang Fung Qiang Bi Qing-GuoWang and Y. Zhang., Pid tuning for improved performance," *IEEE Transactions on Control System Technology,7(4):457-65*, 1999.

[7] M. Q. G.Wang B. Huang Liu and C. C. Hang., \Improved identification of continuous time delay processes from piecewise step tests.," *Journal of Process Control, (17):51-57,*, 2007.

[8] Trans. P. Persson and K. J. Astrom, Dominant pole design - a unified view of PID controllertuning.," Adaptive systems in control and signal processing 1992:selected papers from the Fourth IFAC Symposium, Grenoble, France, pp. 377{382, 1993

[9] P. Persson and K. J. Astrom, Dominant pole design - a unified view of PID controller tuning.," *Adaptive systems in control and signal processing 1992:selected papers from* the Fourth IFAC Symposium, Grenoble, France, pp. 377{382, 1993.

[10] Y. G. P. Rui Ping Wang, Fractional order proportional and derivative controller design for second-order systems with pure time-delay," ," *Mechatronic Science, Electric Engineering and Computer (MEC), International Conference*, pp. 1321 { 1325, 2011.

[11] Y. C. e. a. C A Monje, *Fractional-order Systems and Controls*. Glasgow, Scotland, UK: Springer, 2010.

[12] G. Z. Ioan D. Landau, *Digital Control System* Saint Martin d'Heres, France: Springer, 1938.

[13] Y. Luo, Synthesis of robust pid controllers design with complete information on pre specifications for the foptd systems ," *American Control Conference(ACC)*, pp. 5013, 5018, 2011.

[14] J. Rommes, Effcient computation of transfer function dominant poles using sub space acceleration," *Power System, IEEE Transactions*, vol. 21, pp. 1218 { 1226, 2006.

[15] A. Gambier, Digital pid controller design based on parametric optimization," *Control Applications, CCA, IEEE International Conference*, pp. 792 { 797, 2008.