Satellite Image Compression using Kekres Wavelet Transform

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ABSTRACT

The evolution of satellite image technology is enabling the manipulation of a greater range of data contained in increasing types of satellite images. Efficient and effective utilization of transmission bandwidth and storage capacity have been a core area of research for remote sensing images. Hence image compression is required for multi-band satellite imagery. In addition, image quality is also an important factor after compression and reconstruction.

The wavelet transform is anticipated to provide economical and informative mathematical representation of many objects of interest.

In the proposed system, Kekres wavelet transform is used for compression of multispectral satellite image based on compressive sampling method.

The compressed image performance is analyzed using Compression Ratio (CR), Peak Signal to Noise Ratio (PSNR),Mean Square Error.

General Terms
Image compression, multispectral image,, kekres wavelete transform(KWT)

Keywords
Compressive sensing, Incoherence,measurement matrix. Compression Ratio (CR),Mean Square Error (MSE) ,Peak Signal to Noise Ratio (PSNR)

1. INTRODUCTION
Remote sensing multispectral images are of interest for a large number of applications, like meteorology, geology, earth resource management, pollution monitoring ,and military surveillance. Often they are composed by only a few spectral bands, but some sensors provide up to some hundreds of bands, in which case the images are also called hyperspectral, so as to cover in great detail a wide spectral window and provide more valuable information about the land covers of the region under investigation. The “hyper” in hyperspectral means “over” as in “too many” and refers to the large number of measured wavelength bands. Hyperspectral images are spectrally
overdetermined, which means that they provide ample spectral information to identify
and distinguish spectrally unique materials. Hyperspectral imagery provides the potential for more accurate and detailed information extraction than possible with any other type of remotely sensed data. Whatever the case, such images occupy hundreds or thousands of Mbytes each, giving rise to serious problems for their archival, and especially for their transmission from the satellite to the earth station. As an example, a single multispectral image acquired by the Thematic Mapper (TM) sensor carried on board of the

2. COMPRESSED SENSING
Compressed sensing or compressive sampling (CS) is a simple and efficient signal acquisition technique that collects a few measurements about the signal of interest and later uses optimization techniques for reconstructing the original signal from what appears to be an incomplete set of measurements [1]. Accordingly, CS can be seen as a technique for sensing and compressing data simultaneously (thus the name). The CS technique relies on two fundamental principles :

a) sparse representation of the signal of interest in some basis, which is called the representation basis. Sparsity expresses the idea that the “information rate” of a continuous time signal may be much smaller than suggested by its bandwidth, or that a discrete-time signal depends on a number of degrees of freedom which is comparably much smaller than its (finite) length. More precisely, CS exploits the fact that many natural signals are sparse or compressible in the sense that they have concise representations when expressed in the proper basis Ψ.

b) Incoherence between the sensing matrix and the representation basis. Incoherence extends the duality between time and frequency and expresses the idea that objects having a sparse representation in Ψ must be spread out in the domain in which they are acquired, just as a Dirac or a spike in the time domain is spread out in the frequency domain. Put differently, incoherence says that unlike the signal of interest, the sampling/sensing waveforms have an extremely dense representation in Ψ.

2.1 Limits of the Shannon-Nyquist sampling

The Shannon/Nyquist sampling theorem specifies that to avoid losing information when capturing a signal, one must sample at least two times faster than the signal bandwidth. In many applications, including digital image and video cameras, the Nyquist rate is so high that too many samples result, making compression a necessity prior to storage or transmission. In other applications, including imaging systems (medical scanners and radars) and high-speed analog- to-digital converters, increasing the sampling rate is very expensive.

2.2 Compressible signals

Consider a real-valued, finite-length, one-dimensional, discrete-time signal **x**, which can be viewed as an $N \times 1$ column vector in with elements x[n], n = 1, 2, . . . , N. (We treat an image or higher-dimensional data by vectorizing it into a long one-dimensional vector.) Any signal in can be represented in terms of a basis of N × 1 vectors $\{\Psi_i\}_{i=1}^{N}$ .For simplicity, assume that the basis is orthonormal. Using the N×N basis matrix $\Psi = [\Psi_1 | \Psi_2 | \Psi_3 | \ldots . | \Psi_N]$ with the vectors I $\{\Psi_i\}$ as columns, a signal x can be expressed as

$$x = \sum_{i=1}^{N} s_i \Psi_i$$ or

$$x = \Psi S \qquad (1)$$

where s is the N×1 column vector of weighting coefficients $s_i = \langle x, \Psi_i \rangle = \Psi_i^T$ x and $.^T$ denotes transposition. Clearly, x and s are equivalent representations of the signal, with x in the time or space domain and s in the Ψ domain.

The signal x is K-sparse if it is a linear combination of only K basis vectors; that is, only K of the s_i coefficients in (1) are nonzero and (N − K) are zero. The case of interest is when K N. The signal x is compressible if the representation (1) has just a few large coefficients and many small coefficients.

2.3 Transform coding and its inefficiencies

The fact that compressible signals are well approximated by K-sparse representations forms the foundation of transform coding [2]. In data acquisition systems (for example, digital cameras) transform coding plays a central role: the full N-sample signal x is acquired; the complete set of transform coefficients {si} is computed via s = Ψ_i^T x; the K largest coefficients are located and the (N − K) smallest coefficients are discarded; and the K values and locations of the largest coefficients are encoded.

Unfortunately, this sample-then-compress framework suffers from three inherent inefficiencies. First, the initial number of samples N may be large even if the desired K is small. Second, the set of all N transform coefficients $\{s_i\}$ must be computed even though all but K of them will be discarded. Third, the locations of the large coefficients must be encoded, thus introducing an overhead

2.3 A basic model for compressive sampling

A basic model [3] for compressive sampling is shown in Figure 1. The N-dimensional signal x is assumed to be K-sparse with respect to some orthogonal matrix V. The “sampling” of x is represented as a linear transformation by a matrix Ψ yielding a sample vector y = Ψ x. Let the size of Ψ be M-by-N, so y has M elements; we call each element of y a measurement of x. A decoder must recover the signal x from y knowing V and Ψ, but not necessarily the sparsity pattern of the unknown signal u.

Since u is K-sparse, x must belong to one of $\binom{N}{K}$ subspaces in $\mathbb{R}^N$ Similarly, y must belong to one of $\binom{N}{K}$subspaces in $\mathbb{R}^M$For almost all ΦS with M ≥ K+1 an exhaustive search through the subspaces can determine which subspace x belongs to and thereby recover the signal’s sparsity pattern and values. Therefore, in principle, a K sparse signal can be recovered from as few as M = K + 1 random samples.

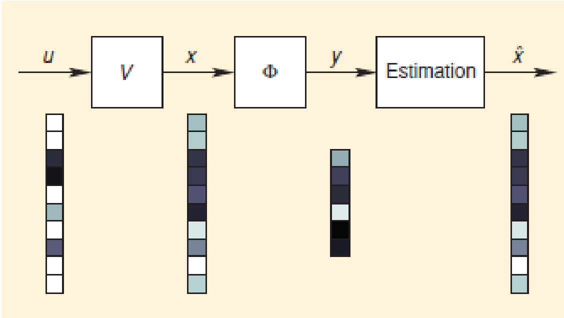

Figure.1 Basic compressive sampling model

3 KEKRE’S TRANSFORM

Kekre’s transform matrix [4] [5] can be of any size NxN, which need not to be an integer power of 2. All upper diagonal and diagonal elements of Kekre’s transform matrix are 1, while the lower diagonal part except the elements just below diagonal is zero. Generalized NxN Kekre’s transform matrix can be given as,

$$K_{N\times N} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 & 1 \\ -N+1 & 1 & 1 & \ldots & 1 & 1 \\ 0 & -N+2 & 1 & \ldots & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \ldots & 1 & 1 \\ 0 & 0 & 0 & \ldots & -N+(N-1) & 1 \end{bmatrix}$$

The formula for generating element Kxy of Kekre’s tansform matrix

$$K_{xy} = \begin{cases} 1 & ; x \le y \\ -N+(x-1) & ; x = y+1 \\ 0 & ; x > y+1 \end{cases}$$

Kekre’s Wavelet transform is derived from Kekre’s transform. From NxN Kekre’s transform matrix, we can generate Kekre’s Wavelet transform matrices of size

(2N)x(2N), (3N)x(3N),……, (N2)x(N2). For example, from 5x5 Kekre’s transform matrix, we can generate Kekre’s Wavelet transform matrices of size 10x10, 15x15, 20x20 and 25x25. In general MxM Kekre’s Wavelet transform matrix can be generated from NxN Kekre’s transform matrix, such that M = N * P where P is any integer between 2 and N that is, 2 ≤ P ≤ N. Consider the Kekre’s transform matrix of size NxN shown in Figure

3.1 Properties of kekre’s transform

A. Orthogonal

The transform matrix K is said to be orthogonal if the following condition is satisfied.

$$[K][K]^T=[D]$$

where, D is the diagonal matrix.

Kekre's Transform matrix satisfies this property and hence is orthogonal. The diagonal matrix values of Kekre's transform matrix of size NxN can be computed as

D(x,y) =

$$\begin{cases} 2 & , if x = y = N \\ N & , if x = y = N \\ 0 & , if \ x \neq y \\ D(x+1, y+1) + 2(N - x + 1) & , if x = y = p \& p \neq 1 or N \end{cases}$$

B. Asymmetric

In linear algebra symmetric matrix is square matrix, which is equal to it's transpose. As the Kekre's transform matrix is upper triangular matrix, it is asymmetric.

C. Non Involutional

An involutionary function is a function that is it's own inverse. So involutionar transform is a transform which is inverse transform of itself. Kekre's transform is non involutional transform.

D. Transform on Vector

The transform of a vector **q** is given by

Q = [K] q

And inverse is given by

q = [K]T [D]-1 Q

4 PROPOSED SYSTEM

In this proposed system,Compressive sampling method is used for compression.We first build a dictionary of coherence of measurement of Kekres wavelet; haar and Db4;Db8 wavelet.Secondly we compare the coherence of three wavelet and finally compression of image

4.1 Dictionary of Coherence of measurement

In the proposed system , the compressive sampling method two parameters are of the most important ,sparsity and the coherence between between two basis.

The measurement matrix Phi (Φ) and the wavelet basis psi(Ψ) from which theta(θ)is extracted must be sufficiently incoherence.

The sparse matrix is obtain as S= [KWT* KWT^{-1}] [x] where the x can be viewed as an N×N multispectral image.

In our system, we compute the incoherence of sparsity basis with the kekres wavelet transform (KWT) ,Haar wavelet ,Daubechies wavelet.

The phi (Φ) is the random matrix of N=145, Psi is basis of the kekres wavelet transform(KWT) ,Haar wavelet ,Daubechies wavelet(Db4 ,Db8) .Each iteration is consist of 1000 run and in each run it count how many time the mutual coherence of the different wavelet with the sparsity basis.the Table 5.1 summerised the comparison of count of different wavelet with sparsity basis.

From which it is found that KWT is having high mutual incoherence than that of the haar ,Db4 and Db8

So from this we come to know that the KWT is highly incoherence with the randam matrix.The graph in figure 5.1 is showing the comparision for the eight iteration.

Table 1. Comparison of Incoherence of different wavelet

Iteration	DB4	DB8	Haar	KWT
1	301	300	289	1000
2	333	271	281	1000
3	297	288	318	1000
4	315	301	285	1000
5	334	286	284	1000
6	295	293	301	1000
7	307	300	292	1000
8	329	300	279	1000

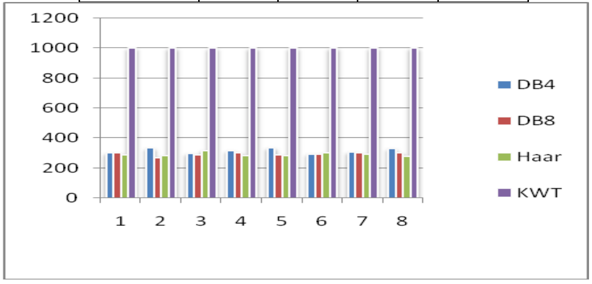

Figure2. Comparison of Incoherence of different wavelet

The mutual coherence(μ) of measurement matrix Phi(Φ) and the wavelet basis psi(Ψ) is calculated

as $\mu(\Phi, \Psi) = \max_{(i,j)} \langle \Phi_i, \Psi_j \rangle$

it is found that ,the mutual coherence of KWT in 1000 runs is less than haar,Db4,Db8.that is KWT is highly incoherence than these wavelet.the value of mutual coherence is given in the table below. Also it is representade graphically in figure 5.2.

Because of the high incoherence of KWT it can be used as a bases matrix for compressive sampling for the multispectral satellite image compression.

Table 2. Incoherence (μ) of different wavelet transform

Iteration	DB4	DB8	Haar	KWT
1	4.2671	4.1508	4.4699	1.139
2	4.4591	4.4591	4.4599	1.201
3	4.2836	3.9583	4.0685	0.8558
4	4.2613	4.2613	4.2613	0.9943
5	4.4323	3.9907	3.7312	0.9583
6	4.1783	4.2474	4.6904	0.9511
7	4.4386	3.8896	4.5411	0.9804
8	3.9507	4.226	4.4963	0.8607

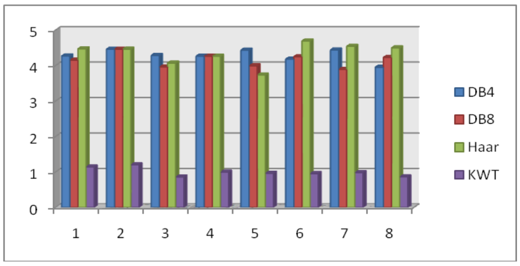

Figure 3 Incoherence (μ) of different wavelet transform Compresion of multispectral satellite image

This scene was gathered by AVIRIS sensor over the Indian Pines test site in North Western Indiana and consists of 145 X 145 pixels and 224 spectral reflectance bands in the wavelength range 0.4 -2.5 X 10^(-6) meters.[6][7] This scene is subset of a larger one.

Steps for compression

Step 1: load the multispectral satellite image (X) and read bands of image and select test band

Step 2: Generate the Kekres Wavelet transform (KWT) from Kekres transform.

Step 3: Generate the bases matrix X1 as X1=ww*sparse(X)*ww' where ww is KWT and ww's its inverse transform

Step 4: Generate random measurement matrix of different values say M

Step 5: Take the product of measurement matrix and wavelet bases matrix this is forward transform

Step 6: Apply inverse matrix and obtain reconstructed image.

Step 7: Calculate the image quality parameter such as MSE, PSNR .CR

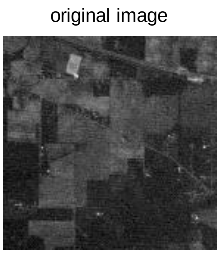

Figure 4. original satellite image ,test band

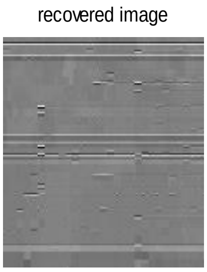

Figure 5. reconstructed satellite image ,test band

Following table shows the result of compression for different values of M

Table 5. Image quality parameter for different values of M

M	psnr	mse	error x	Cr
77	2.0962	4.0129	8.4372	0.531
88	3.5659	2.8608	6.0149	0.6069
99	4.9439	2.083	4.3795	0.6828

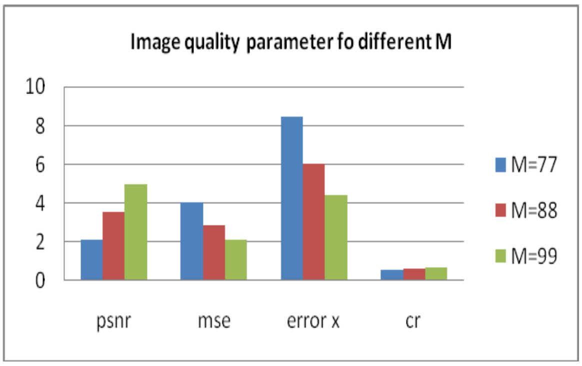

Figure 5. Image quality parameter for different values of M

CONCLUSION:

In this paper , we used the concept of compressive sampling for the compression of multispectral satellite image. The kekres wavelet matrix is having the high incoherence with the bases matrix than the haar, Db4, Db8 wavelet. The image quality parameter such as MSE PSNR are calculated Such as for M=99.PSNR=4.9439MSE=2.083.

The limitation of our method is that since the data size is very large, the required computation time is not optimal and there is a constraint on memory.

Future research can be directed at reducing time for compression by using parallel processing.

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