Design of SallenKey Low Pass Filter for High Bandwidth

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ABSTRACT
In this paper, an active second order filter used is a Sallen key low pass filter which is designed and simulated using AIM-Spice for different values of gain to see the effect on the bandwidth and cutoff frequency. An unequal component design is used to overcome the limitation of equal component filter i.e. overshoot and reduced bandwidth at gain more than 1.5.

Keywords
Low pass filter, AIM-Spice, Sallen Key, Equal and unequal component design, Cut off frequency (f_c), Quality factor (Q), Bandwidth (B.W).

1. INTRODUCTION
A filter is basically a "frequency selective" circuit which is designed to pass a specific band of frequency and block or attenuate input signals of frequencies outside this band. Passive and active filter are the two types of analog filter.

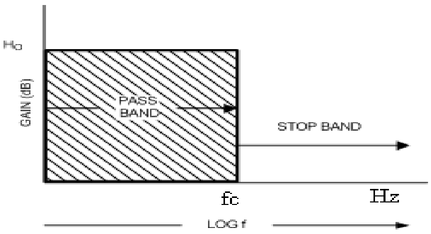

Fig.1.1 Ideal response of Low pass filter

Fig1.1 shows that a low pass filer has a constant gain from 0Hz to a high cut-off frequency f_c. The frequencies between 0 and f_c are known as "pass band frequencies" whereas the frequencies beyond f_c are known as "stop band frequencies".At f=f_c, the filter gain makes a sudden transition to zero. Therefore all the frequencies beyond f_c are completely attenuated.
Fig.1.2 shows the frequency response of a practical low pass filter. It shows that the filter gain does not change suddenly at f=f_c. Instead as f increases, the gain reduces gradually. At f=f_c, the gain is down by 3 dB and after f_c it reduces at a higher rate as shown in the Fig. 1.2.

A two-stage RC network that forms a second order low-pass passive filter is shown below

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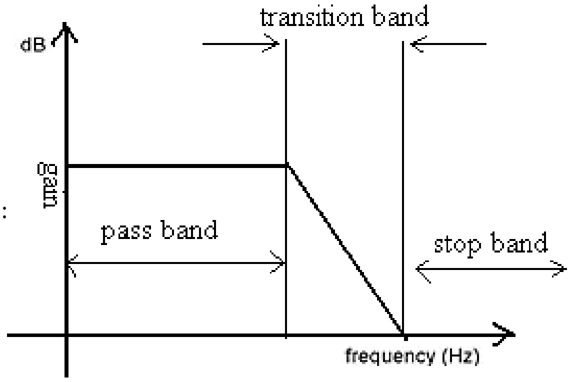


Fig 1.2 Practical response of Low pass filter

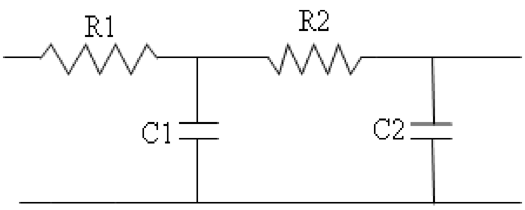


Fig. 1.3 RC passive low pass filter

The limitation of this filter is the value of Q which is always less than 1/2.for the equal component design i.e. when R1=R2 and C1=C2, Q approaches a value of 1/3. Q will approach the maximum value of 1/2 when the impedance of the second RC stage is much larger than the first. For the filters to pass a signal Q should be greater than 1/2. Larger value of Q is attainable by using a positive feedback amplifier which is nothing but an active filter. Hence a need rose to implement an active low pass filter.

II .Related Theory
The architecture that has been used to implement the Low pass filter is Sallen-Key Topology. This was chosen because of its simplicity compared to other known architectures such as multiple feedback and state variable, where the latter is for precision performance. The general diagram of Sallen Key filter is given below:

The aim is to bring a generalized active low pass active filter to a sallen key Low pass filter which will be done using the following simple steps.

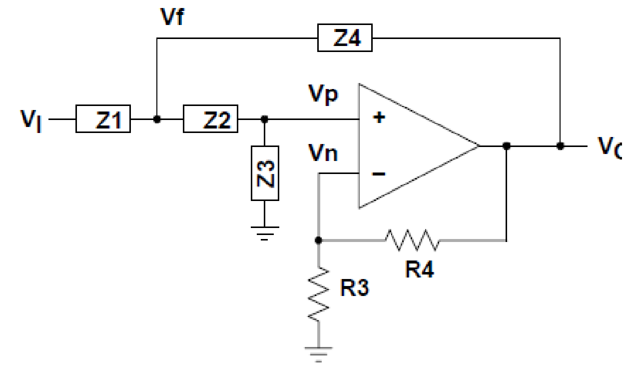


Fig. 2.1 ***Generalized Sallen key filters***

2.1) Voltage transfer function for a general single amplifier filter:

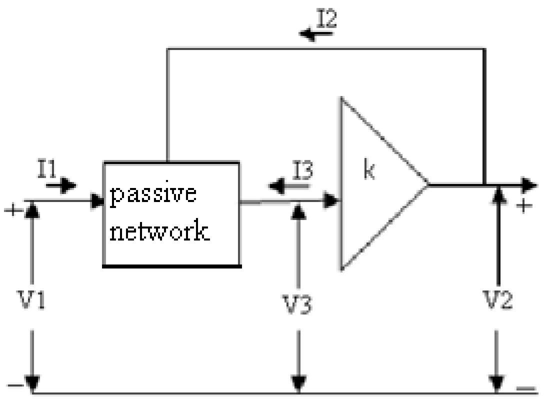


Fig. 2.2 General single low pass filter

The passive network is considered as a three port network and defined by a set of short circuit admittance parameters

1) For the passive network portion of filter we may write

$$\begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} Y_{11}(s)Y_{12}(s)Y_{13}(s) \\ Y_{21}(s)Y_{22}(s)Y_{23}(s) \\ Y_{31}(s)Y_{32}(s)Y_{33}(s) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} (1)$$

Due to passive nature of the network, $Y_{ij} = Y_{ji}$ i.e Y matrix is symmetric.

2) Consider Voltage controlled voltage source (VCVS)

$V_3(s)=V_2(s)/k$ (2)

Substitute this in (1)

$$\begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} Y_{11}(s)Y_{12}(s) + Y_{13}(s)/K \\ Y_{21}(s)\ Y_{22}(s) + Y_{23}(s)/K \\ Y_{31}(s)Y_{32}(s)+Y_{33}(s)/K \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{bmatrix} (3)$$

3) Current I_2(s) is not independent, hence deleting it.

$$\begin{bmatrix} I_1(s) \\ I_3(s) \end{bmatrix} = \begin{bmatrix} Y_{11}(s)Y_{12}(s) + Y_{11}(s)/K \\ Y_{31}(s)Y_{32}(s) + Y_{33}(s)/K \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} (4)$$

Since the VCVS has infinite input impedance

I_3=0 (5)

$Y_{31}(s)V_1(s)+[Y_{32}(s) + \frac{Y_{33}(s)}{K}]V_2(s)$ (6)

Therefore,

$\frac{V_2(s)}{V_1(s)} = \frac{-KY_{31}(s)}{Y_{33}(s)+KY_{32}(s)}$(7)

2.2) Voltage transfer function for a specific network configuration

Consider a specific network configuration. Figure 2.3 shows a specific network configuration which is obtained by replacing the passive network in figure 2.2 by the some passive components.

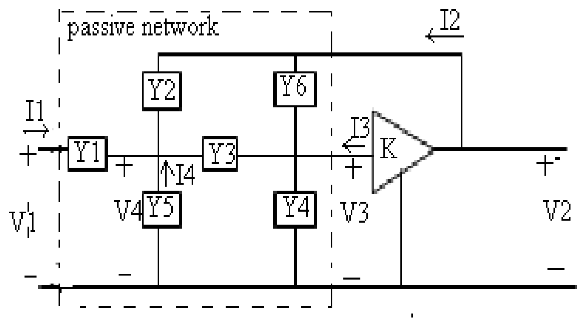


Fig. 2.3 Specific network configuration

The transfer function for this network is

$$\frac{V2(S)}{V1(S)} = \frac{KY_1Y_3}{(Y_1+Y_2+Y_5)(Y_3+Y_4+Y_6)+Y_3(Y_4+Y_6)-K[Y_6(Y_1+Y_2+Y_3+Y_5)+Y_2Y_3]}$$

(8)2.3) General single –Amplifier filter configuration,From equation (7) & properties of passive networks we know that- Y_{31}(s), Y_{33}(s) &Y_{32}(s) cannot be negative valued coefficients. Thus we write-

$Y_{31}(s) = \frac{N_{31}(s)}{D(s)}$(9)

$Y_{32}(s) = \frac{N_{32}(s)}{D(s)}$(10)

$Y_{33}(s) = \frac{N_{33}(s)}{D(s)}$(11)

Thus equation (7) becomes

$$\frac{V_2(s)}{V_1(s)} = \frac{KN_{31}(s)}{N_{33}(s)-KN_{32}(s)} (12)$$

Comparing equation (12) and (8), we get

$N_{31}(\mathrm{s}) = Y_1 Y_3 (13)$

$N_{32}(\mathrm{s}) = Y_6(Y_1 + Y_2 + Y_3 + Y_5) + Y_2 Y_3 (14)$

$N_{33}(\mathrm{s}) = (Y_1 + Y_2 + Y_5)(Y_3 + Y_4 + Y_6) + Y_3(Y_4 + Y_6)$ (15)

2.4)Low pass single Amplifier Filter

The general form of the voltage transfer function for the second order active low pass is -

$$\frac{V_2(s)}{V_1(s)} = \frac{H_0 \times W_n^2}{s^2 + \left(\frac{W_n}{q}\right)s + W_n^2} (16)$$

Where-

H_0- Direct Current Gain

W_N-undamped natural frequencies

Q -Quality factor

From equation 13,

$N_{31}(\mathrm{s}) = Y_1 Y_3$

Comparing with equation (16), $Y_1 = G_1$ & $Y_3 = G_3$

According to equation (16) , N_{33}(s) must be second order equation with negative real zeroes i.e.

$N_{33}(\mathrm{s}) = (\mathrm{s}+\alpha_1)(\mathrm{s}+\alpha_2) (17)$

Therefore

$(\mathrm{s}+\alpha_1)(\mathrm{s}+\alpha_2) = (G_1 + Y_2 + Y_5)(G_3 + Y_4 + Y_6) + G_3(Y_4 + Y_6)$ (18)

Also,

From equation 16,

N_{32}(s) Must be simple zero at origin

i.e. $N_{32}(\mathrm{s}) = \alpha\mathrm{s} (19)$

Therefore,

$\alpha \mathrm{s} = \mathrm{Y}_6(\mathrm{Y}_1 + \mathrm{Y}_2 + \mathrm{Y}_3 + \mathrm{Y}_4 + \mathrm{Y}_5) + \mathrm{Y}_2 \mathrm{G}_3 (20)$

Hence

$Y_2 = \mathrm{s}C_2 (21)$

$Y_6 = 0$ (22)

$(G_1 + sC_2 + Y_5)(G_3 + Y_4 + Y_6) + G_3(Y_4) = (\mathrm{s}+\alpha_1)(\mathrm{s}+\alpha_2)$
(23)

This is possible if –

$Y_4 = sC_4 (24)$

$Y_5 = 0 (25)$

Therefore,

$\mathrm{N}_{31}(\mathrm{s}) = \mathrm{G}_1 * \mathrm{G}_3 (26)$

$\mathrm{N}_{32}(\mathrm{s}) = \mathrm{sC}_2 * \mathrm{G}_3 (27)$

$\mathrm{N}_{33}(\mathrm{s}) = [\mathrm{G}_1 + \mathrm{sC}_2][\mathrm{G}_3 + \mathrm{sC}_4] + \mathrm{G}_3 \mathrm{sC}_4] (28)$

Hence the transfer function for the active low pass Sallen key filter is

$$\frac{\mathrm{V}_2(\mathrm{s})}{\mathrm{V}_1(\mathrm{s})} = \mathrm{K}\frac{\frac{1}{\mathrm{R}_1\mathrm{R}_3\mathrm{C}_2\mathrm{C}_4}}{\mathrm{S}^2 + \mathrm{S}\left[\frac{1}{\mathrm{R}_3\mathrm{C}_4} + \frac{1}{\mathrm{R}_1\mathrm{C}_2} + \frac{1}{\mathrm{R}_3\mathrm{C}_2} - \frac{\mathrm{K}}{\mathrm{R}_3\mathrm{C}_4}\right] + \frac{1}{\mathrm{R}_1\mathrm{R}_3}} (29)$$

Thus a generalized sallen Key filter is Modeled to get a Low pass filter which is shown in figure 2.4

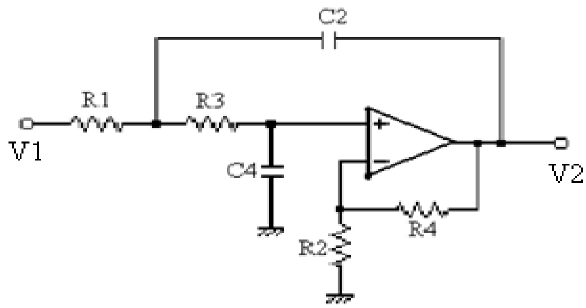


Fig. 2.4 Low pass sallen key filter

Forthis filter almost any Q can be realized, limited mainly by the physical constraints of the power supply and component toleran-ces. Capacitor C2, no longer connected to ground, provides a positive feedback path. Flexibility in gain and frequency adjust-ment, no loading problem, pass band gain, small component size and use of inductor are avoided which are some of the added advantages of this filter.

III.DESIGN CONSIDERATION

There are two methods of designing, equal component and unequal component design.In equal component.fc and Q are independent of one another, and design is greatly simplified although limited. The gain of the circuit now determines Q. RC sets fc .With K=3, Q becomes negative, the poles move into the right half of the s-plane, and the circuit oscillates. The main problem is overshoot that start when K=3, this can be reduced using unequal component design which is being explained later in detail..In the second page, our design is implemented using Spice and the result are shown. It is seen by our design, the amplitude of spike is reduced and bandwidth is increased as compare to equal component result.

Now ,comparethe derived equation with standard form:

$w_n^2 = \frac{1}{R_1R_3C_2C_4}$ (30)

$w_n = \frac{1}{\sqrt{R_1R_3C_2C_4}}$ (31)

$\frac{w_n}{Q} = \frac{1}{R_1C_4} + \frac{1}{R_1C_2} + \frac{1}{R_3C_2} - \frac{K}{R_3C_4}$ (32)

$\frac{1}{Q} = \sqrt{\frac{R_3C_4}{R_1C_2}} + \sqrt{\frac{R_1C_4}{R_3C_2}} + (1 - K)\sqrt{\frac{R_1C_2}{R_3C_4}}$ (33)

$H_O w_n^2 = \frac{K}{R_1R_3C_2C_4}$ (34)

$H_O = K.$ (35)

Designing of equal component:

1)Choose the cut-off frequency f_c.

2)Assume $R_1 = R_3 = R and C_2 = C_4 = C$.

3)Then choose C in the range of 0.001µF -0.1µF

4)Calculate the value of R as $R = \frac{1}{2\pi f_c C}$

5)K=3 $-\frac{1}{Q}$.

6)Assume the value of R_2.

7) $R_4 = (K-1)R_2$.

Design of unequal component:

1) Set Filter Components as Ratios:

1.1) Choose the cut-off frequency f_c.

1.2)Assuming $R_3 = nR_1$, $C_4 = mC_2 and R_1 = R, C_2 = C$.

Therefore, $R_3 = nR$, $C_4 = mC$

1.3) $w_n^2 = \frac{1}{R^2C^2mn}$, $w_n = \frac{1}{RC\sqrt{mn}}$

1.4) $\frac{1}{Q} = (n+1)\sqrt{\frac{m}{n}}$,

1.5)Select the value of m and n as:

m$\leq \frac{1}{Q^2 4}$

n=$(\frac{1}{2mQ^2} - 1) \pm \frac{1}{2mQ^2}\sqrt{1 - 4mQ^2}$

2) Set Resistors as Ratios and Capacitors Equal

2.1) $assume C_2 = C_4 = C$

2.2) $assume R_3 = R$, and also the value of n.

$$R_1 = nR$$

2.3) $f_{c} = \frac{1}{2\pi RC\sqrt{n}}$

2.4) $\frac{1}{Q} = \frac{\sqrt{m}}{1 + 2m - mK}$

Calculation of component values:

1) For equal component:

$$f_c = 5kHz and C_2 = C_4 = 0.1\mu F$$

$$R_1 = R_3 = \frac{1}{2\pi 5 * 10^3 * 0.1 * 10^{-6}} = 318.30\Omega$$

Now by selecting the value of gain and assuming the value of R_2 obtain the value of R_4.

1.1) For K=1, no resistances in feedback path i.e.

$R_4 = R_2$=0

1.2) For K=1.586 and $R_2 = 1k\Omega$.

$R_4 = (1.586 - 1) * 1k\Omega$=0.586$k\Omega$

1.3) For K=1.8 and $R_2 = 1k\Omega$.

$R_4 = (1.8 - 1) * 1k\Omega$=0.8$k\Omega$

1.4) For K=2 and $R_2 = 1k\Omega$.

$R_4 = (2 - 1) * 1k\Omega$=1$k\Omega$

1.5) For K=2.9 and $R_2 = 1k\Omega$.

$R_4 = (2.9 - 1) * 1k\Omega$=1.9$k\Omega$

1.6) For K=1.586 and $R_2 = 1k\Omega$.

$R_4 = (1.586 - 1) * 1k\Omega$=0.586$k\Omega$

1.7) For K=3 and $R_2 = 1k\Omega$.

$R_4 = (3 - 1) * 1k\Omega$=2$k\Omega$

1.8) For K=20 and $R_2 = 1k\Omega$.

$R_4 = (20 - 1) * 1k\Omega$=19$k\Omega$

2) For unequal component design:

Let n=5, $C_2 = C_4 = 0.1\mu F$, $f_c = 5kHz$

R=$\frac{1}{2\pi * 5 * 10^3 * 0.1 * 10^{-6}} = 159.15\Omega$=$R_3$

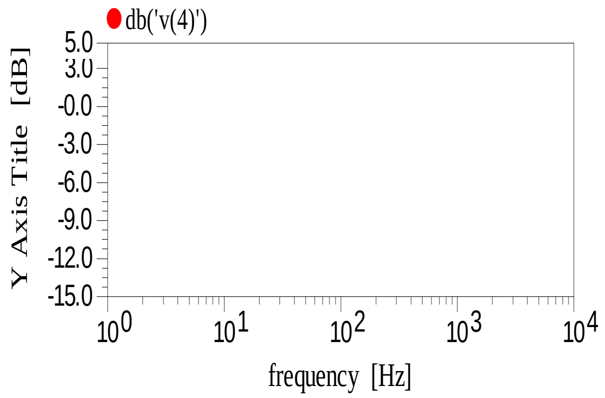
$R_1 = 5 * 159.15 = 636.61\Omega$

Here also by changing the value of K and keeping $R_2 = 1k\Omega$ the value of R_4 is calculated in similar manner as done for equal component.

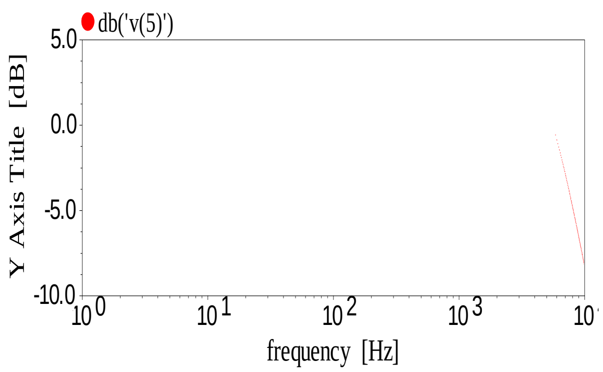
IV.RESULTS AND DISCUSSION

For equal component designing low pass filter for fc=5 kHz by varying gain the changes in fc are noted along with any overshoot if occurring.

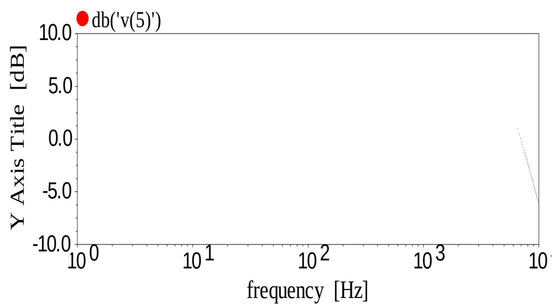
1)k=1 , fc=3.2kHz and at f=5kHz the gain is -6 db down.



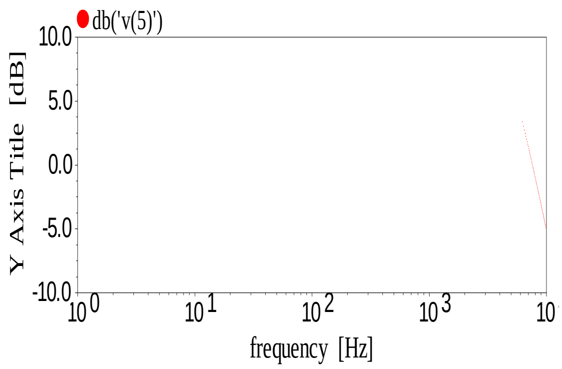
2) k=1.586, fc=5kHz



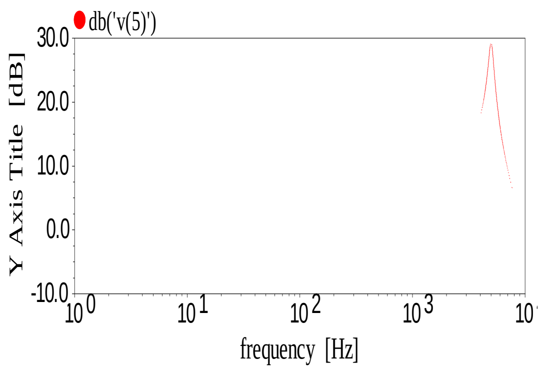
3)k=1.8,fc=6kHz but here overshoot occurs, overshot begins from k=1.85 onward and at f=5kHz, k=4.32 and bandwidth reduces



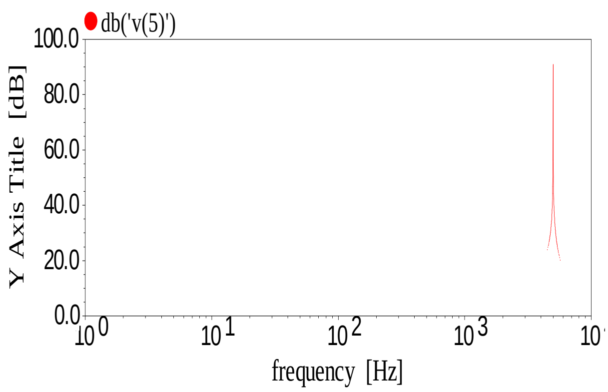
4) k=2,fc=5kHz overshoot further increases and band width further decreases.



5)k=2.9,fc=5.2kHz maximum overshoot occurs for k=2.9 at f=5kHz and k rises upto 30db.



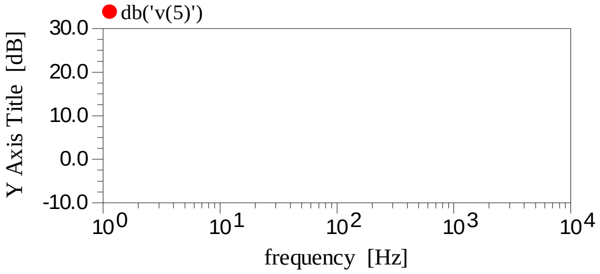
6)k=3, Q becomes infinity and system becomes totally unstable,fc=5.



2kHz and at f=5kHz maximum overshoot occurs and k rises upto 80db.

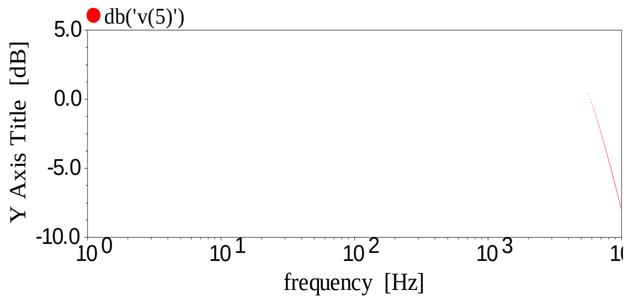
Now from k=3.1 to k =4 the similar pattern as I case of 3, 4,5 is observerd but reduction in bandwidth is more as compare to cases above and if k is further increased spikes doesn't occur but fc shifts towards left which implies further reduction in bandwidth. For understanding this consider one more case for k=20.

7)k=20,fc=280Hz severe reduction in Bandwidth i.e. roll off occurs much before required desired f=5kHz



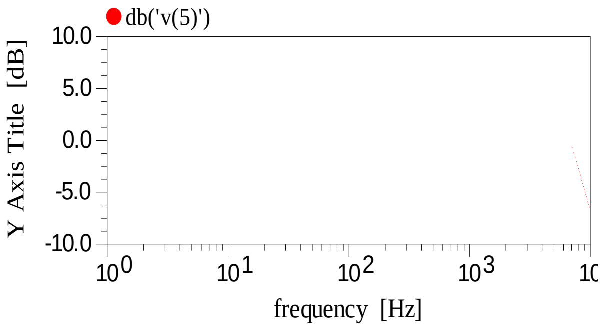
Now in order to reduce the range of over shoot and increase gain and bandwidth we will observe the results of unequal system from the design method mention above we selected 3rd method for designing i.e. keeping capacitor value same and selecting resistance as ratios so that capacitor spread is reduced.

1)k=1.586,fc=5kHz response similar to equal component.

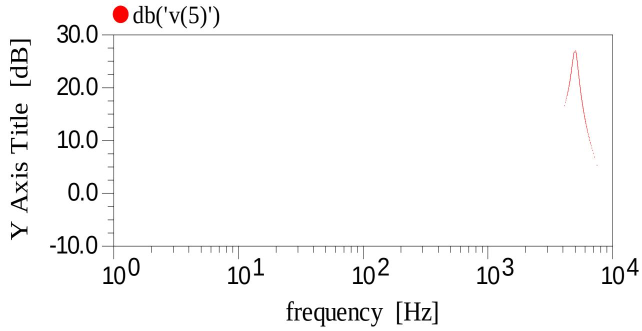


Now when k is increased beyond 1.586 i.e. in the range of 1.7 to 2.3 overshoot appears as shown below.

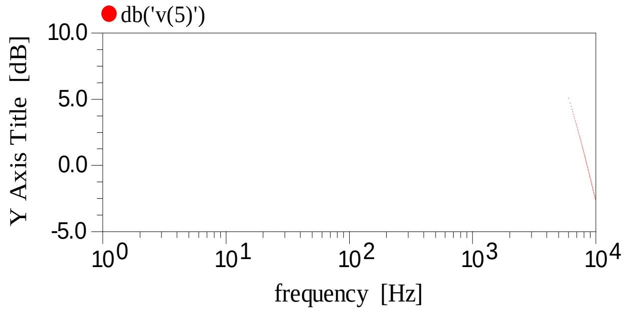
2)k=1.7,fc=8kHz, overshoot begins.



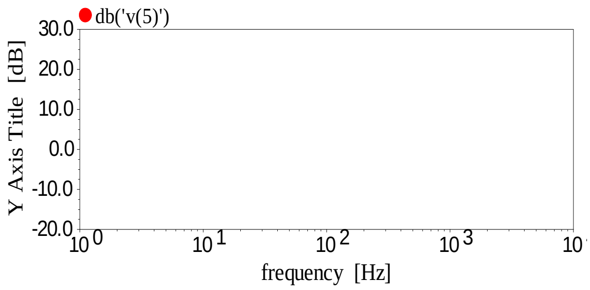
3)k=2.3,fc=5.2kHz and maximum overshoot occurs for k=2.3 and its magnitude is 28.7db atf=5kHz



4)k=2.9,fc=5.6kHz



5) k=20, fc=150Hz response off much before desired frequency.



From the above plots we observe that the range of occurrence of overshoot and reduction in bandwidth is reduced , in unequal component design method the amplitude of overshoot is reduced and bandwidth is increased.

However this design also gives similar plots that has been observed for equal component above k=4 i.e. the plot begins to shift towards left, thus reducing the bandwidth.

We designed our circuit for fc=5kHz and Q=0.707 which gives k=1.586 now keeping fc constant the gain of designed filter is increased and the changes in fc is noted down .

Table I:For equal component:

k (gain) d**B**	**fc (experimental)**	**│Δf│=(5-fc)kHz**
1	3.2kHz	1.8
1.586	5kHz	0
1.8	6kHz	1
2	5kHz	0
2.9	5.2kHz	0.2
3	5.2kHz	0.2
20	280Hz	4.720

The above readings are obtained from the plots of equal component design of Sallen key filter . The filter was designed for K=1.586 and fc=5kHz and then by varying the value of K its effect was observed on fc. As it is seen from the plot the range of overshoot was high and since the bandwidth was reduced , we moved to unequal component design

Table2: For unequal component:

K (gain)dB	**fc(experimental)kHz**	**│Δf│=(5-fc)**
1.586	5	0
1.7	8	3
2.3	5.2	0.2
2.9	5.6	0.6
20	0.15	4.85

The above readings are obtained from the plots of unequal component design and as it is observed from plots and above table that the range of overshoots is reduced and bandwidth is increased.

CONCLUSION

In this paper a sallen key low pass filter is designed and implemented, from the plots and reading of equal component it is seen that range of overshoot is high and bandwidth is less, and this demerit of equal component design was overcome by implementing unequal component design, where the ranges of overshoot is reduced and bandwidth is also increased

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