# Performance Analysis of MIMO Ball and Beam System using Intelligent Controller

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## ABSTRACT

In this paper, to design a PID controller for Ball and Beam system to track the ball to a commanded position by varying the beam angle.The present work deals with the PID controller implementation of highly nonlinear ball and beam system.The mathematical model is developed for this system and it is implement and simulating using MATLAB software. The control algorithm used in the present study was found to be effective in controlling the non-linear unstable system.

## General Terms

Performance, Design, Verification, Measurement, Control, Analysis.

## Keywords

MIMO,PID Controller, Non-linear system,stepper motor, Fuzzy controller,MATLAB.

## 1. INTRODUCTION

The aim is to design a PID controller for Ball and Beam system to track the ball to a commanded position by varying the beam angle.The ball and beam system is also called 'balancing a ball on a beam'. It can usually be found in most university control labs. It is generally linked to real control problems such as horizontally stabilizing an airplane during landing and in turbulent airflow. There are two degrees of freedom in this system. One is the ball rolling up and down the beam, and the other is beam rotating through its central axis. The aim of the system is to control the position of the ball to a desired reference point. The control signal can be derived by feeding back the position information of the ball. The control voltage signal goes to the DC motor via a power amplifier, then the torque generated from the motor drives the beam to rotate to the desired angle. Thus, the ball can be located at the desired position.It is important to point out that

the open loop of the system is unstable and nonlinear.The problem of 'instability' can be overcome by closing the open loop with a feedback controller. The modern state-space method can be employed to stabilize the system. The nonlinear property is not significant when the beam only deflects a small angle from the horizontal position. In this case, it is possible to linearize the system. . However, the non-linarites become significant when the angle of the beam from the horizontal is larger than 30 degrees, or smaller than -30 degrees. In this case, a simple linear approximation is not accurate. Thus a more advanced control technique such as nonlinear control should work better.

## 2. METHODS AND MATERIALS

### 2.1 Ball and Beam System

The ball and beam system is one of the basic examples of nonlinear and unstable control system .This system commonly used for control theory verification or control system design and implementation practice. This system getting popular and become an important laboratory models for teaching control system engineering due to very simple to understand as a system and the control technique that can be studied and it cover many important classical and modern design methods. The system also used as a control training tool in many industrial processes and their application. The ball and beam mechanism consists of a beam and solid ball on it. The ball is rolling free along the beam according to the changing angle of the beam. The beam rotates in the vertical plane driven by a torque usually using servo motor at the side. The required ball position is being control by applying an electrical control signal to the motor amplifier. The position of ball on the beam can measured using special sensor. The control task is difficult

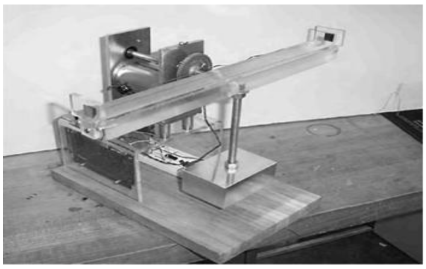

**Fig.1. 'Ball on balancing beam' build by Berkeley Robotics Laboratory.**

because the ball does not stay in one place on the beam but moves with an acceleration that is proportional to the tilt of beam. In control technology the system is open loop unstable because the system output (ball position) increases without limit for a fixed input (beam angle). Feedback control must be used to keep the ball in a desired position on the beam. There are so many method and type in designing the model of ball and beam system hardware [4]. Most the ball and beam system using stainless steel material because it looks clean and it using gears to connect to the motor. Fig.1 Shows the ball on balancing beam build by Berkeley Robotics Laboratory. The ball on beam balancer system is one of the most enduringly popular and important laboratory models for teaching control systems engineering. Control Job: Automatically regulating the position of the ball on the beam by changing the angle of the beam.

The open loop is unstable.

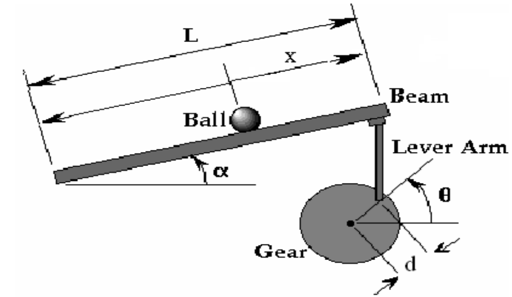

**Fig.2. System identification of Ball and Beam System**

L - Length of the beam
x - Ball position coordinate
α -Beam angle
Θ -gear angle
d - lever radius

Ball is placed on the beam where it is allowed to roll with one degree of freedom along the length of the beam. Ball is positioned by controlling the beam angle using DC stepper motor [8].To control the position of the ball two variables have to be measured: ball position and beam angle.

## 2.2 Fuzzy Controller

Fuzzy controllers are used to control consumer products, such as washing machines, video cameras, and rice cookers, as well as industrial processes, such as cement kilns, underground trains, and robots[7]]. Fuzzy control is a control method based on fuzzy logic. Just as fuzzy logic can be described simply as ’’computingwith words rather than numbers’’, fuzzy control can be described simply as ’’control with sentences rather than equations’’.A fuzzy controller can include empirical rules, and that is especially useful in operatorcontrolled plants

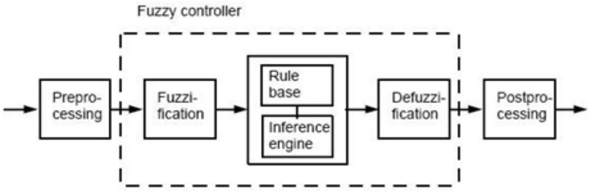

**Fig.3. Block diagramof fuzzy controller.**

Fig.3Shows the block diagram of the fuzzy controller, Fuzzy controllers are very simple conceptually. They consist of an input stage, a processing stage, and an output stage. The input stage maps sensor or other inputs, such as switches, thumbwheels, and so on, to the appropriate membership functions and truth values.The processing stage invokes each appropriate rule and generates a result for each, then combines the results of the rules. Finally, the output stage converts the combined result back into a specific control output value.

## 2.3 PID Controller

PID controllers are realized by using one proportional, integral and differential controller each in parallel. All three controllers receive the same input; their outputs are added to form the compound output of the PID controller. PID control can be and has been implemented in many technical forms throughout its history, from relays to microprocessors [3]. PID controllers are very common; about 90 to 95% of all control problems can solved by this controller. A simple form of control, where the controller response is proportional to the control error. The controller error is defined as the difference between the set point and the process output. The PID controller calculation (algorithm) involves three separate parameters; the proportional, the integral and derivative values. The proportional value determines the reaction to the current error, the integral value determines the reaction based on the sum of recent errors, and the derivative value determines the reaction based on the rate at which the error has been changing. The weighted sum of these three actions is used to adjust the process via a control element suchas the position of a control valve or the power supply of a heating element.

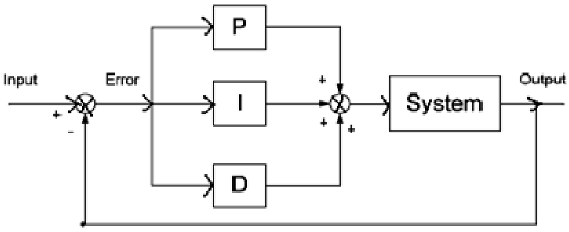

**Fig.4.Block diagram of PID controller**

By tuning the three constants in the PID controller algorithm, the controller can provide control action designed for specific process requirements. The response of the controller can be described in terms of the responsiveness of the controller to an error, the degree to which the controller overshoots the set point and the degree of system oscillation. Use of the PID algorithm for control does not guarantee optimal control of the system or system stability. Some applications may require using only one or two modes to provide the appropriate system control. This is achieved by setting the gain of undesired control outputs to zero. A PID controller will be called a PI, PD, P or I controller in the absence of the respective control actions. PI controllers are particularly common, since derivative action is very sensitive to measurement noise, and the absence of an integral value may prevent the system from reaching its target value due to the control action.

## 2.4 PID Controller Theory

This section describes the parallel or non-interacting form of the PID controller. For other forms the Section "Alternative notation and PID forms". The PID control scheme is named after its three correcting terms, whose sum constitutes the manipulated variable (MV). Hence:

$$MV(t) = P_{out}+I_{out}+D_{out} \quad (1)$$

where Pout, Iout, and Dout are the contributions to the output from the PID controller from each of the three terms, as defined below.

## 2.5 Proportional Term

The proportional term (sometimes called gain) makes a change to the output that is proportional to the current error value. The proportional response can be adjusted by multiplying the error by a constant Kp, called the proportional gain.

The proportional term is given by:

$$P_{out} = K_p + e(t) \quad (2)$$

A high proportional gain results in a large change in the output for a given change in the error. If the proportional gain is too high, the system can become unstable. In contrast, a small gain results in a small output response to a large input error, and a less responsive (or sensitive) controller. If the proportional gain is too low, the control action may be too small when responding to system disturbances. In the absence of disturbances, pure proportional control will not settle at its target value, but will retain a steady state error that is a function of the proportional gain and the process gain. Despite the steady-state offset, both tuning theory and industrial practice indicate that it is the proportional term that should contribute the bulk of the output change.

## 2.6 Integral Term

The contribution from the integral term (sometimes called reset) is proportional to both the magnitude of the error and the duration of the error. Summing the instantaneous error over time (integrating the error) gives the accumulated offset that should have been corrected previously. The accumulated error is then multiplied by the integral gain and added to the controller output. The magnitude of the contribution of the integral term to the overall control action is determined by the integral gain, $K_i$.

The integral term is given by

$$I_{out} = K_i \int_0^t e(T)\,dT \quad (3)$$

The integral term (when added to the proportional term) accelerates the movement of the process towards setpoint and eliminates the residual steady- state error that occurs with a proportional only controller. However, since the integral term is responding to accumulated errors from the past, it can cause the present value to overshoot the setpoint value (cross over the setpoint and then create a deviation in the other direction). For further notes regarding integral gain tuning and controller stability.

## 2.7 Derivative Term

The rate of change of the process error is calculated by determining the slope of the error over time (i.e., its first derivative with respect to time) and multiplying this rate of change by the derivative gain $K_d$. The magnitude of the contribution of the derivative term (sometimes called rate) to the overall control action is termed the derivative gain, $K_d$.

The derivative term is given by:

$$D_{out} = K_d \frac{de}{dt}(t) \quad (4)$$

The derivative term slows the rate of change of the controller output and this effect is most noticeable close to the controller setpoint. Hence, derivative control is used to reduce the magnitude of the overshoot produced by the integral component and improve the combined controller-process stability. However, differentiation of a signal amplifies noise and thus this term in the controller is highly sensitive to noise in the error term, and can cause a process to become unstable if the noise and the derivative gain are sufficiently large.

## 2.8 DC Servo Motor

The position servo consists of a DC servomotor with built in gearbox ratio 70:1. The output of the gear box drives a potentiometer and an independent output shaft to which a load can be attached. The DC motor drives a rotational inertial load. The motor shaft position is measured using a sensor. An optical tachometer is also available to measure motor speed.

**Fig.5. Rotary Servo Plant with Encoder**

Fig.5 Shows the Rotary servo plant with encoder, A high quality DC servo motor is mounted in a solid aluminum frame. The motor drives a built-in Swiss-made 14:1 gearbox whose output drives an external gear. The motor gear drives a gear attached to an independent output shaft that rotates in a precisely machined aluminum ball bearing block. The output shaft is equipped with an encoder. This second gear on the output shaft drives an anti-backlash gear connected to aprecision potentiometer. The potentiometer is used to measure the output angle. The external gear ratio can be changed from 1:1 to 5:1 using various gears. Two inertial loads are suppliedwith thesystem in order to examine the effect of changing inertia on closed loop performance. In the high gear ratio configuration, rotary motion modules attach to the output shaft using two 8-32 thumbscrews. The square frame allows for installations resulting in rotations about a vertical or a horizontal axis.

# 3. MATHEMATICAL MODELS

## 3.1 Mathematical Model of Dc Motor

The mathematical model for DC servomotor is given below,

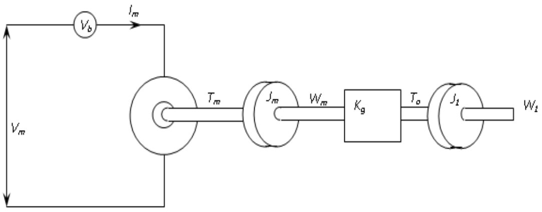

**Fig.6. Diagram of DC motor**

Fig.6. Shows the block diagram of DC motor,the DC servo motor plant is used as an actuator for the ball and beam system.

$$\frac{\theta(s)}{V_m(s)}=\frac{1}{s\left(\frac{sJ_{eq}R_m}{K_gK_m}+K_gK_m\right)} \quad (5)$$

Equation 5 is the mathematical model of DC motor, Where 'R$_m$' - Armature resistance, 'K$_g$'is a motor gear ratio and 'K$_m$' is a Back emf constant.

From rotary servo motor parameters:R$_m$=2.6Ω ; K$_m$=0.00767Nm/amp ;K$_g$=70 ;

$J_{eq}$ = JmKg$^2$ + J1; J$_m$=3.87×10-7 Kgm$^2$; J1=0.00003 Kgm$^2$

Therefore,

$J_{eq}$=1.9263×10-3 Kgm$^2$

K$_m$×K$_g$=0.5369

$$\frac{J_{eq}R_m}{K_mK_g}=9.328\times10^{-3}$$

Therefore,

$$\frac{\theta(s)}{V_m(s)}=\frac{1}{s\left(0.009328s+0.5369\right)} \quad (6)$$

Equation 8 is a final implementation mathematical model of DC motor.

## 3.2 Control System Design of Motor

The open loop position response of the DC motor is unstable due to the pole at the origin.

### *3.2.1 PD Control*

A PD controller can be used for the output to track the desired angle.

$$V_m = K_p\ (\theta_d - \theta) - K_d\theta$$

$$V_m(s) = N_p\theta_d(s) - \left(K_p + sK_d\right)\theta(s) \quad (7)$$

Finally we got the values of K$_p$=18.40 and K$_d$=0.0489.

### *3.2.1 PID Control*

A PID controller can be used for the output to track the desired angle.

$$V_m(s)\ =\ Kp\left(\theta_d(s)\ -\ \theta(s)\right)\ + sK_d\theta(s) + \frac{Ki\left(\theta_d(s)\ -\ \theta(s)\right)}{s} \quad (8)$$

Tp=100ms=0.1s

ζ (z)= 0.707

Since $T_p=\frac{\pi}{W_n\sqrt{1-\varsigma^2}}$; $W_n=44.42$

General third order equation is $s^3+1.75W_ns^2+2.15W_n{}^2s+W_n{}^3$

Substituting the values of Wn and ζ we get

S$^3$ + 77.735 S$^2$ + 4242.24326 S + 87646.7189 (9)

Equation 9 is the third order equation.

Finally we got the values of K$_p$=0.1882 and K$_I$=817.5686 K$_d$=39.5716.

## 3.3 Mathematical Model of the Plant

A ball is placed on a beam, where it is allowed to roll with one degree of freedom along the length of the beam. The position of the ball is changed by changing the angle of the beam. When the angle is changed from the vertical position, gravity changes the ball to roll along the beam. Forces acting on the system are shown below:

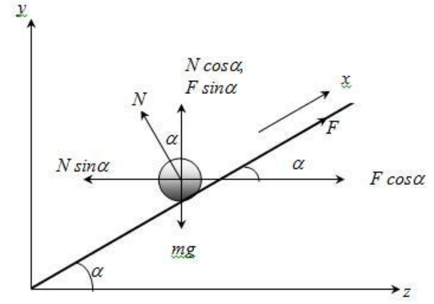

**Fig.7.Forces act on the system**

Fig.7. Shows the forces acting on the system, Where'α' is a Beam angle coordinate, 'g' is a Gravitational Acceleration, 'm' is a Mass of the Ball, 'f' is a Friction al Force, 'n' is a Normal Force, 'r' is a Radius of the Ball'x' is a Ball Position Coordinate, 'j' is a Ball's Moment of Inertia.

The linear mathematical model of the plant is:

$$\ddot{x}\left(\frac{J}{r^2}+m\right)+mg\ \alpha=0 \quad (10)$$

### *3.3.1 Control system Design of the Plant*

*Relation between Beam Angle And Gear Angle*

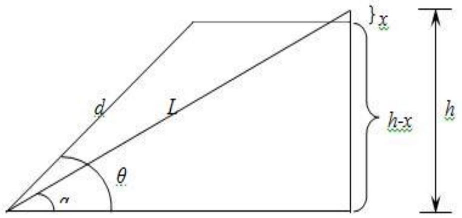

**Fig.8.Diagram of relation between beam angle and gear angle**

$$\frac{X(s)}{\theta(s)}=-\frac{mgd}{L\left(\frac{J}{r^2}+m\right)}\frac{1}{s^2} \quad (11)$$

Equation 8 is a relation between beam angle and gear angle of the plant.

### *3.3.2 PD Control*

The mathematical model of ball and beam is

$$\frac{X(s)}{\theta(s)}=-\frac{mgd}{L\left(\frac{J}{r^2}+m\right)}\frac{1}{s^2} \quad (12)$$

Substituting the parameters: m=0.06Kg, r=0.0027m, L=0.425m, J=9.99x10$^{-6}$Kgm$^2$, d=0.0254m

Finally we got the values $K_{p1}$= -5.272, $K_{d1}$= -5.034

### *3.3.3 PID Control*

$$\frac{X(s)}{X_d(s)} = -\frac{0.416(K_{p_1} + s^2 K_{d_1} + K_{i_1})}{s^3 - 0.416 s^2 K_{d_1} - 0.416 s K_{p_1} - 0.416 K_{i_1}} \qquad (13)$$

Characteristics equation is

$S^3$ –0.416 $K_{d1}S^2$ –0.416 $K_{p1}S$ –0.416 $K_{i1}$
(14)
Constraints are:
Tp=3s
ζ=0.707
Therefore,Wn=1.481
Substituting the values in general third order equation we get

s3 +2.592s2+4.716s +3.248
(15)

Finally we got the values of $K_{p1}$= -11.497and $K_{i1}$=-7.918

$K_{d1}$= -6.3195

## 4. DESIGN AND SIMULATION

The proposed model including Fuzzy controller has been simulated using MATLAB software.

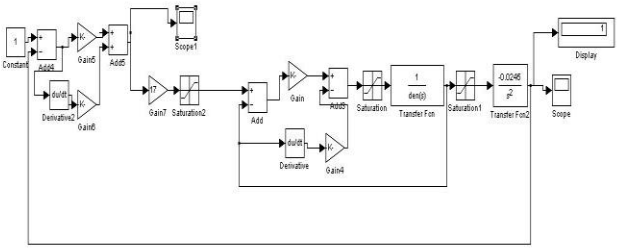

**Fig.9. Simulation model of plant.**

Fig.9 shows the simulation model of plant, the PID mathematical model developed and using a MATLAB software to simulate and controlled the model.

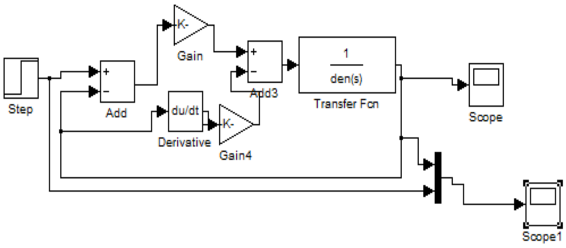

**Fig.10. PD controller for Simulink model of motor**

Fig.10 shows the PD controller Simulink model of motor, the PD control mathematical model developed and using a MATLAB software to simulate and controlled the model.

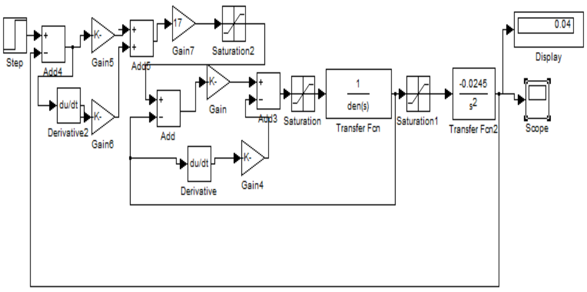

**Fig.11.Simulink model of plant for tracking step input**

Fig.11 shows the Simulink model of plant for tracking step input, the mathematical model developed and using a MATLAB software to simulate and controlled the model.

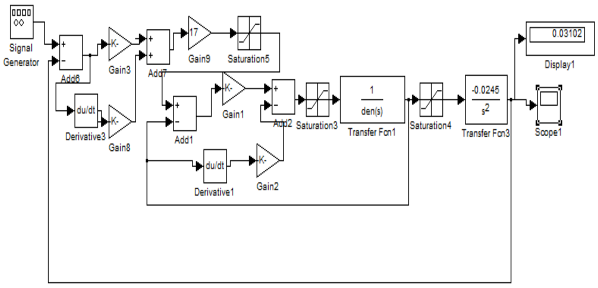

**Fig.12. Simulink model of plant for tracking sine input**

Fig.12 shows the Simulink model of plant for tracking sine input, the mathematical model developed and using MATLAB software to simulate and controlled the model.

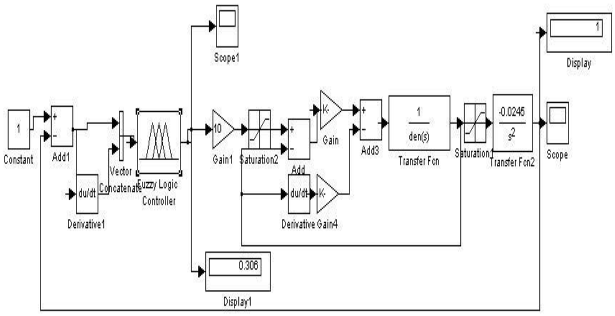

**Fig.13. Simulink model of plant withfuzzy controller**

Fig.13 shows the Simulink model of plant with fuzzy controller, the mathematical model developed and using a MATLAB software to simulate and controlled the model. The above simulation model of the plant, motor and its shown

## 5. RESULTS AND DISCUSSION

The proposed model has been simulated using MATLAB software.

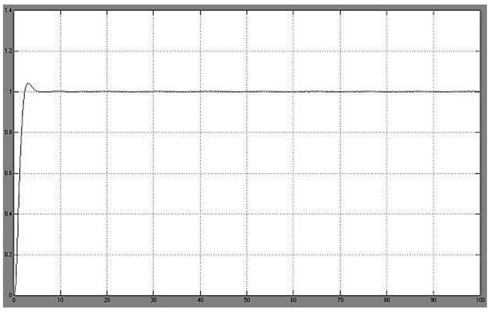

**Fig.14. Response of the plant**

Fig.14 shows, PD control of plant was implemented for a step input of 1 units and step time one, with the designed values of Kp and Kd. It is observed that the response of the system has a peak time of 3sec (4sec-1sec). At steady state the response settles at the desired value of 1 units.

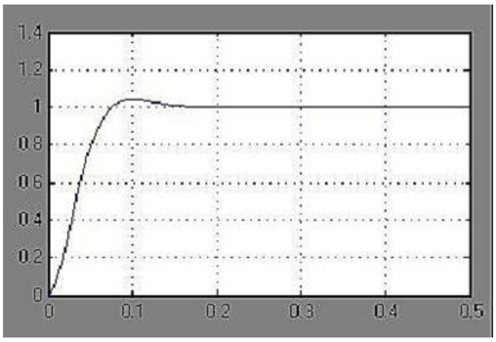

**Fig.15. Response of the motor**

Fig.15 shows, PD motor was implemented for a step input of 1 unit and step time zero, with the designed values of Kp and Kd. It is observed that the response of the system has a peak time of 0.1sec, as per the design. At steady state the response settles at the desired value of 1 unit.

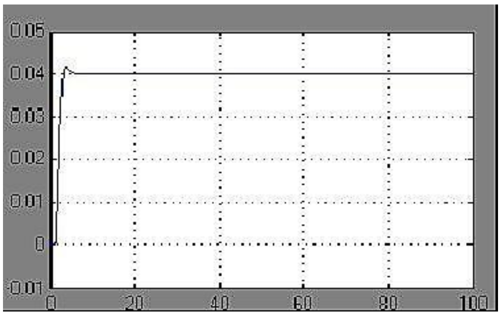

**Fig.16. Response of the plant for tracking step input**

Fig.16 shows, PD control of plant was implemented for a step input of 0.04 units and step time one, with the designed values of Kp and Kd. It is observed that the response of the system has a peak time of 3sec (4sec-1sec). At steady state the response settles at the desired value of 0.04 units.

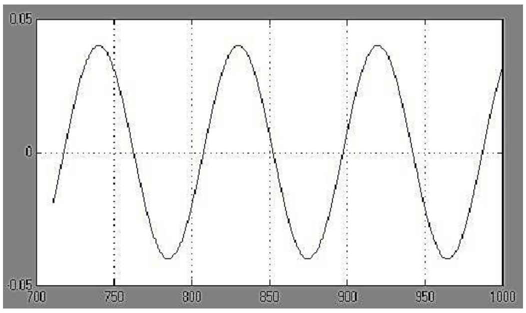

**Fig.17. Response of the plant for tracking sine input**

Fig.17 shows, PD control of plant was implemented for a sine input of 0.04 units and frequency of 0.7 rad/sec, with the designed values of Kp and Kd. At steady state the response settles at the desired value of 0.04 units.

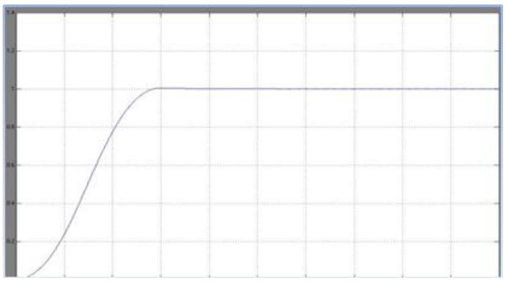

**Fig.18. Response of the plant with fuzzy controller**

Fig.18 shows, the response of the plant with fuzzy controller, the mathematical model developed and using a MATLAB software to simulate and controlled the model.

Current Variable
Name input1
Type input
Range [-3.5 3.5]
Display Range [-3.5 3.5]

Current Membership Function (click on MF to select)
Name mf1
Type trimf
Params [-5.25 -3.5 -1.75]

Selected variable "input1"

**Fig.19. Membership Function Editor**

Fig.19 shows, the membership function editor of fuzzy logic controller.

## 6. CONCLUSIONS

To Conclude, we have shown the mathematical model of the DC motor has been developed and the mathematical model of the plant has been developed. The open loop unstable system will be controlled by the classical and fuzzy controller.The fuzzy controller is to be designed for control the system and the motor position is controlled by the controller. And also we have shown the simulation results for PD controller and Fuzzy controller using LabVIEW software.

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