Evaluating Students' Performance using Fuzzy Logic

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ABSTRACT
Fair results give motivation and encouragement to the students. So reforms in education are must not only in curriculum development but also in students' performance assessment. Proposed method is useful when questions in the examination are of subjective or objective type and total time duration is given to attempt all questions rather than individual question. This method considers importance and complexity of question into account. It makes use of fuzzy inference system (FIS) and fuzzy logic.

General Terms
Fuzzy Logic Controller (FLC), Fuzzy Inference System(FIS), MFs(Membership Functions)

Keywords
Student Evaluation, Fuzzification, Defuzzification, Computational Intelligent (CI), Soft Computing.

1. INTRODUCTION
Many universities and institutions conduct examination in subjective or objective type. In such examinations, based on the correctness or accuracy of answers written by students, marks are awarded. The complete question paper has to be answered within fixed time duration. Since time per question is not specified, the overall time management is done by students. Besides the accuracy/correctness of answer this paper gives considerations for two more factors complexity and importance of the questions. The values of complexity and importance are being taken from domain expert(s).

In recent years, many researchers applied fuzzy logic, fuzzy sets, fuzzy logic controller (FLC) in educational grading systems. Biswas [2] highlighted the importance of education system - "The chief aim of education institutions should be to provide students with the evaluation reports regarding their test/examination as sufficient as possible with unavoidable error as small as possible so as to make evaluation system more transparent and fairer to students". He used fuzzy set theory in student evaluation and is potentially finer than awarding grades or numbers when evaluating answerscipts. The methods presented by him are fem and generalized fem. Chen and Lee[3] presented methods which removes drawbacks of Biswas's methods. Their methods performs calculation in much faster manner and don't require to use complicated matching operations. Later on [9] proposed method for evaluating student answerscripts using fuzzy numbers associated with degree of confidence. They have considered degree of confidence of evaluator when awarding satisfaction level to questions of student answerscripts. Bai and Chen [15] proposed a method for automatically constructing grade membership functions of fuzzy rules for students' evaluation. [1] Proposed a method for automatically generating the weights for several attributes with fuzzy reasoning capability.

2. PROPOSED METHOD FOR STUDENTS' EVALUATION

2.1 Basics of Fuzzy Logic
Fuzzy Logic was proposed by Prof. Lotif Zadeh in 1965 as a means of representing or manipulating data that is not precise but rather fuzzy. Fuzzy set theory is use to solve problems involving the absence of sharply defined criteria. Because fuzziness and vagueness are common characteristics in many decision-making problems, good decision-making models should be able to tolerate vagueness or ambiguity. A Fuzzy set has a membership function that allows various degrees of membership for the elements of a given set.

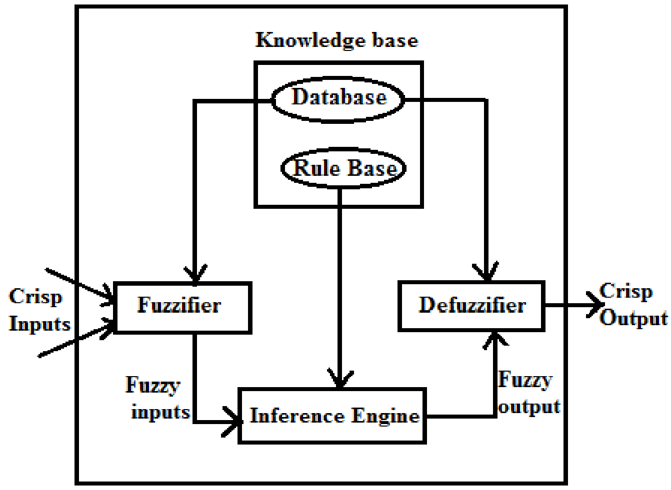

Fig 1: Basic structure of a fuzzy inference system

Fuzzy Controller: A fuzzy controller works similar to a conventional system: it accepts an input values, performs some calculations and generate an output value. Figure 1 shows the basic structure of Fuzzy System .It includes four main components. **A Fuzzifier**: It translates crisp (real valued) inputs into fuzzy values. **An Inference Engine**: That applies a fuzzy reasoning mechanism to obtain a fuzzy output. **A Defuzzifier**: Which translates this latter output into a crisp values. **A Knowledge Base:** It contains both an ensemble of fuzzy rules known as the rule base, and an ensemble of membership functions, known as the database.

2.2 Two-Node Evaluation Method

In this paper we have modified the method proposed by [4]. Saleh and Kim used three node structures, they have taken accuracy rate and time rate to get the difficulty. But as discussed earlier in many universities and institution time management is done by the students as total time allocated to attempt complete question paper is fixed. So how much time is taken by every student to attempt each question we don't know. But complexity and importance of questions are important issues that we have considered. Difficulty of question we did not consider. Bai and Chen (2008b) [15] pointed out that the difficulty factor is a very subjective parameter and may cause an argument regarding fairness in evaluation.

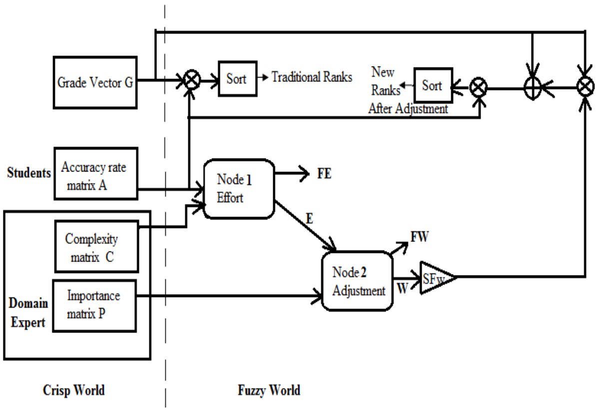

Fig 2: Block Diagram of the two node fuzzy evaluation systems.

Block diagram of two node fuzzy Evaluation system is shown in above Figure 2. The system consists of two nodes: the effort node, and the adjustment node. Each node of the system behaves like a fuzzy logic controller (FLC) with two scalable inputs and one output, as shown in Figure 3. It maps a two-to-one fuzzy relation by inference through a given rule base. The inputs to the system are given either by examination results or domain expert(s). The inputs are fuzzified based on the defined levels (fuzzy sets) in Figure 4. In the first node, one input is given by examination results and other by domain expert. Whereas in the second node, one input is the output of its previous node and the other is given by a domain expert(s). The output of each node can be in the form of a crisp value (defuzzified) or in the form of linguistic variables (MFs).

Figure 3 shows representation of node structure of Fuzzy Logic Controller (FLC). Each node has two scale factors (SFs). This paper considers both scaling factors have the same value of unity assuming the equal influence of each input on the output.

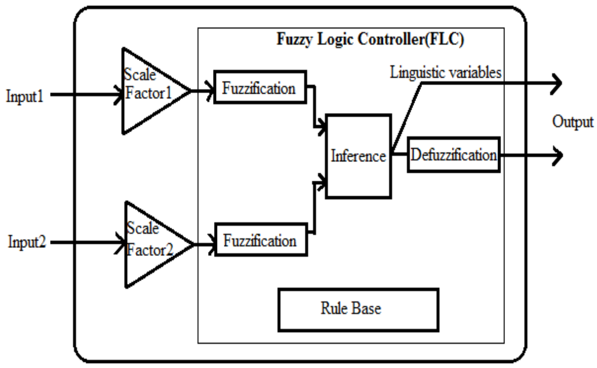

Fig 3: Representation of node process as a Fuzzy Logic Controller (FLC)

Let us assume there are 'n' students to answer 'm' questions. Accuracy rate of student answerscript means students score in each question divided by the maximum score assigned to this question and is represented by

$$A = [a_{ij}], m \times n,$$

Where $a_{ij} \in [0, 1]$ denotes the accuracy rate of student *j* for question *i*.

A Grade vector means maximum score assigned to each question i, is represented by

$$G = [g_i], \ m \times 1,$$

Where g ϵ [1,100], satisfying the following constraints:

$$\sum_{i=1}^{m} g_i = 100$$

Original total score vector of dimension $n \times 1$ can be obtained by using the accuracy rate matrix A and the grade vector G, following formula can be use,

$$S = A^T G = [s_j], \ n \times 1, \quad \ldots\ldots\ldots\ldots\ldots (1)$$

Where $s_j \in [0,100]$ is the total score of student j. To get the 'Traditional-Classical' ranks of students, sort the values of S in descending order.

In this paper importance and complexity of questions are taken into consideration and values are determined by Domain expert or group of domain experts. If we are considering group of domain expert the average is taken for the values of complexity and importance. We have l levels of Importance to describe the degree of Importance of each question in the fuzzy domain. The importance of the questions is an important factor to be considered. Following is the Importance matrix of dimension m × l,

$$P = [p_{ik}], \quad m \times l,$$

Where $p_{ik} \in [0, 1]$ denotes the membership value (degree of the membership) of question i belonging to the importance level k. In this paper, five levels (fuzzy sets) of importance (l = 5) are used.

k = 1 use for linguistic term ''Low'',

k = 2 use for ''More or Less Low'',

k = 3 use for ''Medium'',

k = 4 use for ''More or Less High'', and

k = 5 use for ''High''.

Figure 4 shows their membership functions.

The complexity of questions which indicates the ability of students to give correct answers and is given below as matrix of dimension m × l,

C = $[c_{ik}]$, m × l,

Where c_{ik} ϵ [0, 1] denotes the membership value of question i belonging to the complexity level k.

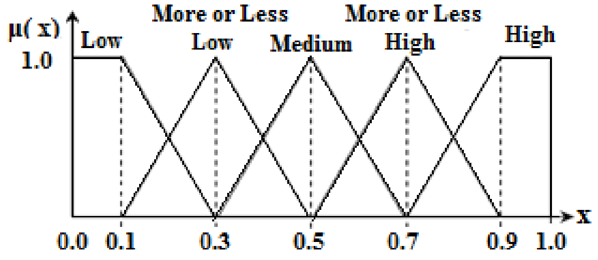

Fig 4: Fuzzy membership functions of the five levels

Following are three steps to evaluate students' answerscripts. First step is Fuzzification, second is Inference and third is Defuzzification.

Step 1 (Fuzzification): In the first step, inputs are converted into membership values of the fuzzy sets as shown in Figure 4. Triangular MF is commonly used because of its simplicity and easy computation.

Calculate the average accuracy rate vector, based on accuracy rate (A)

$\bar{A} = [a_i \cdot]$, *m × n,*

Where $a_i \cdot$ denotes the average accuracy rate of question *i* which is obtained by

$$a_i \cdot = \sum_{j=1}^{n} a_{ij} / n \qquad \text{.................... (2)}$$

Then, obtain the fuzzy accuracy rate matrix of dimension *m × l,* by fuzzification

FA = $[fa_{ik}]$, *m × l,*

Where fa_{ik} ϵ [0, 1] denotes the membership value of the average accuracy rate of question *i* belonging to level *k* matrix of dimension *m × l,*

We have assumed that the fuzzy values of complexity and importance are taken from domain expert(s).

Step 2 (Inference): In the second step, inference is performed based on the given rule base, in the form of IF-THEN rules. Mamdani's max–min inference mechanism is used to produce fuzzy sets for defuzzification.

Based on the fuzzy accuracy rate matrix, FA, the fuzzy Complexity matrix, C, and the fuzzy rules, $\Re_E$ given in the form of IF-THEN rules, obtain the fuzzy Effort matrix(answer-cost)of dimension *m × l,*

$E = [e_{ik}]$, *m × l,*

Where e_{ik} ϵ [0, 1] denotes the membership value of the effort of question *i* belonging to level *k*. When the level of accuracy, l_A and the level of complexity, l_C are given, the level of Effort, l_E is determined by the given fuzzy rule base shown in Table 1(a),

$$l_E = \Re_E(l_A, l_C)$$

Table 1. Fuzzy rule bases to infer Effort and Adjustment

(a)Effort-Cost						(b)Adjustment					
	Complexity						Importance				
Accuracy	1	2	3	4	5	Efforts	1	2	3	4	5
1	1	1	2	2	3	1	1	1	2	2	3
2	1	2	2	3	4	2	1	2	2	3	4
3	2	2	3	4	4	3	2	2	3	4	4
4	2	3	4	4	5	4	2	3	4	4	5
5	3	4	4	5	5	5	3	4	4	5	5

1-"Low", 2-"More or Less Low", 3-"Medium", 4-"More or Less High", 5-"High"

In Mamdani's max–min mechanism, implication is modeled by means of the minimum operator, and the resulting output MFs are combined using the maximum operator. The inference mechanism can be written into the form

$$e_{ik} = \max_{\{(l_A,l_C)|\Re_E(l_A,l_C)=k\}} \{\min(fa_{i,l_A}, C_{i,l_C})\} \qquad \text{............ (3)}$$

Next, based on the fuzzy effort matrix, E, and fuzzy importance matrix, P, given the fuzzy rule base in Table 1 (b) ($\Re_W$), obtain the adjustment matrix of dimension *m × l*

$W = [w_{ik}]$, *m × l,*

Where w_{ik} ϵ [0, 1] denotes the membership value of the adjustment of question *i* belonging to level *k*.

Then use the following formula to obtain the adjustment vector,

$\bar{W} = [w_{i\bullet}]$, *m × l,*

Where $w_{i\bullet}$ ϵ [0, 1] denotes the final adjustment value required by question *i* obtained by

$$w_{i\bullet} = \frac{0.1 * w_{i1} + 0.3 * w_{i2} + 0.5 * w_{i3} + 0.7 * w_{i4} + 0.9 *}{0.1 + 0.3 + 0.5 + 0.7 + 0.9} \qquad \text{............(4)}$$

Where 0.1, 0.3, 0.5, 0.7 and 0.9 are the centers of the fuzzy MFs shown in Figure 4.

Step 3 (Defuzzification): In the third step, fuzzy output values are converted into a single crisp value or final decision.

The Center Of Gravity (COG) method is applied. The crisp value of question *i* is obtained by

$$Y_i = \int x.\mu(x)dx \,/\, \int \mu(x)dx \qquad \text{……..(5)}$$

The original ranks of students are adjusted in this step. Where integrals are taken over the entire range of the output and μ(x) is obtained from step 2. By taking the COG, a reasonable crisp value can be obtained.

The adjustment vector, W, is then used to obtain the adjusted grade vector of dimension $m \times 1$,

$\hat{G}$= [$\bar{g}_i$], $m \times 1$

Where $\bar{g}_i$ is the adjusted grade of question *i*,

$$\bar{g}_i = \bar{g}_i \cdot (1 + w_{i.\bullet}), \qquad \text{…………… (6)}$$

and $w_{i.\bullet}$ is the average adjustment of question *i*. Then, the value is scaled to its total grade (i.e., 100) by using the formula

$$\bar{g}_i = \bar{g}_i \cdot \sum_{j=1}^{m} \bar{g}_j \,/\, \sum_{j=1}^{m} \bar{g}_j \qquad \text{……………….(7)}$$

Finally, obtain the adjusted total scores of students by

$$\hat{S} = A^T \hat{G} \qquad \text{………………………………. (8)}$$

After sorting the element values of $\hat{S}$ in descending order, NEW ranks of students are then obtained.

3. EXPRIMENTAL RESULT

In this section, we have used example shown in [4] so that comparison can be done easily. Consider that there are five question Q1, Q2... and Q5 and ten students S1, S2 ….and S10 to answer these questions. Assume that Accuracy Matrix (A), the Score Matrix (G), Importance Matrix (P) and complexity matrix (C) are shown below:

$$A = \begin{bmatrix} 0.59 & 0.35 & 1 & 0.66 & 0.11 & 0.08 & 0.84 & 0.23 & 0.04 & 0.24 \\ 0.01 & 0.27 & 0.14 & 0.04 & 0.88 & 0.16 & 0.04 & 0.22 & 0.81 & 0.53 \\ 0.77 & 0.69 & 0.97 & 0.71 & 0.17 & 0.86 & 0.87 & 0.42 & 0.91 & 0.74 \\ 0.73 & 0.72 & 0.18 & 0.16 & 0.5 & 0.02 & 0.32 & 0.92 & 0.9 & 0.25 \\ 0.93 & 0.49 & 0.08 & 0.81 & 0.65 & 0.93 & 0.39 & 0.51 & 0.97 & 0.61 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0.8500 & 0.1500 & 0 & 0 \\ 0 & 0 & 0.3300 & 0.6700 & 0 \\ 0 & 0 & 0 & 0.6900 & 0.3100 \\ 0.5600 & 0.4400 & 0 & 0 & 0 \\ 0 & 0 & 0.7000 & 0.3000 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0.33 & 0.67 & 0 & 0 \\ 0 & 0 & 0 & 0.15 & 0.85 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0.07 & 0.93 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 10 \\ 15 \\ 20 \\ 25 \\ 30 \end{bmatrix}$$

Eq. (2) gives value of average accuracy and its transpose is shown

$\bar{A}$ = [0.4500 0.3100 0.7110 0.4700 0.6370] `

Fuzzified values of Accuracy Matrix, Effort Matrix and Adjustment are shown below:

$$FA = \begin{bmatrix} 0 & 0.2500 & 0.7500 & 0 & 0 \\ 0 & 0.9500 & 0.0500 & 0 & 0 \\ 0 & 0 & 0 & 0.9450 & 0.0550 \\ 0 & 0.1500 & 0.8500 & 0 & 0 \\ 0 & 0 & 0.3150 & 0.6850 & 0 \end{bmatrix}$$

$$FE = \begin{bmatrix} 0 & 0.8050 & 0.1950 & 0 & 0 \\ 0 & 0.2900 & 0.7100 & 0 & 0 \\ 0 & 0 & 0 & 0.7750 & 0.2250 \\ 0.1250 & 0.8750 & 0 & 0 & 0 \\ 0 & 0 & 0.3500 & 0.6500 & 0 \end{bmatrix}$$

$$FW = \begin{bmatrix} 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0.3600 & 0.6400 & 0 & 0 \\ 0 & 0 & 0 & 0.2650 & 0.7350 \\ 0.9500 & 0.0500 & 0 & 0 & 0 \\ 0 & 0 & 0.4900 & 0.5100 & 0 \end{bmatrix}$$

Table 2. Original Score and New score by proposed method

Students	Original Score	New Score
1	67.6000	68.5594
2	54.0500	53.4176
3	38.4000	42.7799
4	49.7000	54.0271
5	49.7000	47.7035
6	48.8000	54.1155
7	46.1000	49.5455
8	52.3000	49.0691
9	85.9500	85.5717
10	49.7000	52.2738

As per Eq. (1) and based on accuracy matrix (A) and score matrix (G), obtained original score of each student as shown in Table 2. New scores and ranks are obtained by inserting the values in the Eq. (2) to Eq. (8). Table 2 also shows marks obtained by classical method and the proposed method. Comparison of proposed and classical method is shown in Table 3. Surface view and rule viewer for effort are shown in Figure 5 and Figure 6 respectively.

Different values of results can be obtained depending on the values provided by Domain Expert(s) for complexity matrix and importance matrix.

Table 3. Comparison between Proposed fuzzy method and Classical method

	Ranks obtained by both the methods									
Method	1	2	3	4	5	6	7	8	9	10
Classical	9	1	2	8	4=	5=	10=	6	7	3
Proposed fuzzy	9	1	6	4	2	10	7	8	5	3

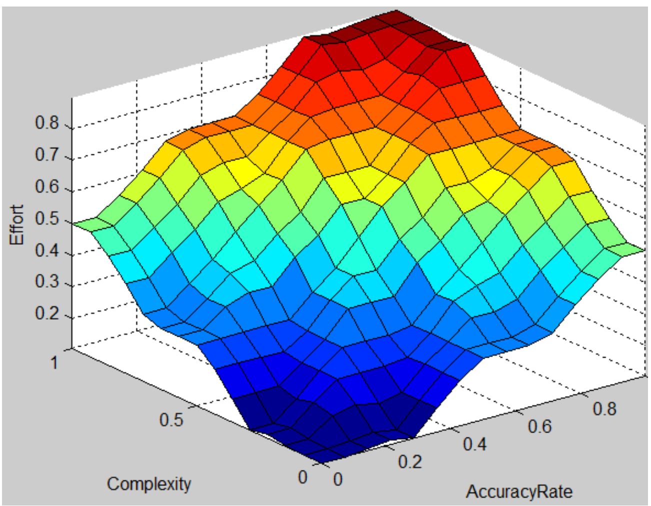

Fig 5: Surface view of the Effort and Adjustment

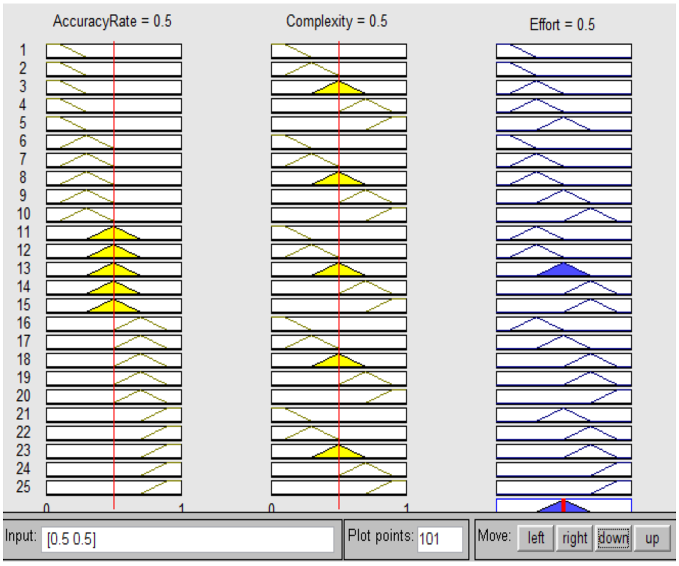

Fig 6: Experimental view of rule viewer for effort

4. CONCLCUSION

Student Evaluation if not done properly, it will effect him/her adversely. This will create negative impression on their present and future teachers, peers and employers. To overcome the said problem the proposed system makes uses of two node fuzzy logic controllers (FLC). System adjusts the original scores of students based on complexity and importance of questions based on fuzzy inference mechanism. Result of the proposed method provides fairer result and which is beneficial to all the students.

5. ACKNOWLEDGMENTS

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