Understanding Grids and Effectiveness of Hexagonal Grid in Spatial Domain

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ABSTRACT
A grid is a pattern of geometrical shapes used to segment a surface or region and which can be defined using a simple mathematical function or formula. Dividing a space in grids helps in better understanding of the space and location of objects. The grids are of great use in translating the spatial information stored in the maps. The shape, size and the type of grid varies depending on the need of the study and the data to be plotted. Most common shapes used are square and triangle. Hexagons are rapidly emerging as the most efficient shape. The selection of the most suitable grid depends on two major criteria; the reality check and the ease of use.

Hexagon is the highest order polygon which is capable of covering an area completely by itself. Hexagonal grids are not frequently used due to the referencing problem. Hexagon maintains linearity and directionality. This paper focuses on the selection criteria of the most suitable grid and the efficiency comparison of the squares and hexagons. The results prove that the hexagonal grids are more efficient and accurate.

Index Terms
Grids, Hexagon, linear distance, angularity, spatial coverage

1. INTRODUCTION
A grid is a pattern of geometrical shapes used to segment a surface or region which can be defined using a simple mathematical function or formula. The grid can be imaginary or be actually marked on the space to be studied. The grid can be regular or irregular but there always exists a basic unit which is self-repeating and creates a pattern. The grid consists of three components, namely the vertices or nodes (zero dimensional), edge (one dimensional) and the face or surface (two dimensional). Representation of any spatial parameter like the distance, area or volume is made simpler by the use of grids. The basic utility of the grid system is to measure the spatial dimensions namely the one dimensional (length, breadth, height, depth and perimeter), two dimensional (area) and three dimensional (volume). Grids can easily be transformed into a graph by converting the faces into nodes. [1]

Dividing a space in grids helps in better understanding of the space and location of objects. The grids are of great use in translating the spatial information stored in the maps. As Josef Müller-Brockmann has quoted "The grid system is an aid, not a guarantee. It permits a number of possible uses and each designer can look for a solution appropriate to his personal style. But one must learn how to use the grid; it is an art that requires practice." [2]

2. SELECTION OF A GRID
The shape, size and the type of grid varies depending on the need of the study and the data to be plotted. Most common shapes used are square and triangle. The hexagonal grid is gaining attention owing to the fact that it approximates the human vision better than a square [3]. In the human visual system, the irregularly and non-uniformly placed sensors sample the scenes. This visual system needs to perform image reconstruction from these irregularly placed samples. This image reconstruction is better approximated by the hexagonal system [4]. In nature the most compact and efficient shape found is circular or spherical, and a hexagon is found to be a better approximation than a square [3].

Square, triangle and hexagon, all have their own pros and cons. The selection of the most suitable grid depends on two major criteria - the resemblance to reality and the ease of use. The resemblance to reality corresponds to three important factors namely the similarity of the mapped and actual distance, the areal coverage and maintenance of the angularity and linearity of the distance.

In recent times, GIS (Geographic Information System) has gained major importance and relevance for handling spatial data. Mostly the spatial data handled are satellite images, toposheets or maps. Earth's surface is one of the most common but important spatial domain being worked on in the current times. Data is geo-referenced and a projection is set with reference to a predefined datum. This projection is based on some established grid system. The gridded data structures help to generate efficient algorithms for such spatial dataset. Images get slightly distorted due to the curvature of the earth which is taken into account while deciding on the projections. Efforts are being made to create data structures which can retain the earth's surface topology [5]. The grid can be azimuthal, conformal, equiareal, or equidistant; depending on whether the user wants to preserve direction, shape, area, or distance respectively for the region of interest.

3. HEXAGONS AND SQUARES
Hexagon, square and triangle are the few shapes that are capable of creating a regular grid by tessellation. Other polygons can also be used to create a grid but they require the help of some other shape to cover the surface, like the pentagon cannot tessellate to create a regular grid on its own. [6]

Square is the most commonly used shape, so this paper compares hexagons with squares for their pros and cons. The grids are compared mainly with their ability to have comparable Euclidean and grid distances. This resemblance to reality corresponds to three important factors namely the similarity of the mapped and actual distance, the areal coverage and maintenance of the angularity and linearity of the distance.

In a grid, distance is measured from center to center of each cell in the grid. When a square grid of side 'x' is considered the straight distance between two adjacent cells is 'x' and that between two diagonal cells is '2x' if only primary neighborhood or von Neumann neighborhood is considered. The primary neighborhood constraint causes the distance to be '2x' because the distance travelled is 'x' to the primary cell and then 'x' to the primary cell of this cell, instead of going directly to the diagonal cell (secondary neighbor). In the Figure 1(a) shown below, the blue cells indicate the primary neighborhood of the central black cell, whereas the red cells in Figure 1 (b) indicate the secondary neighborhood of the central black cell.

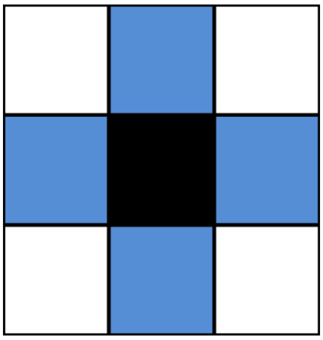

Fig. 1(a) Primary neighborhood for a square grid

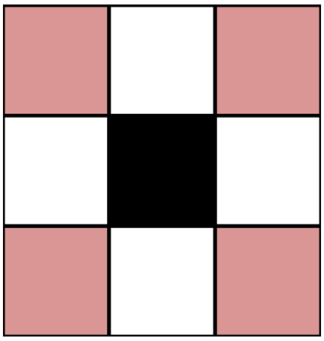

Fig. 1(b) Secondary neighborhood for a square grid

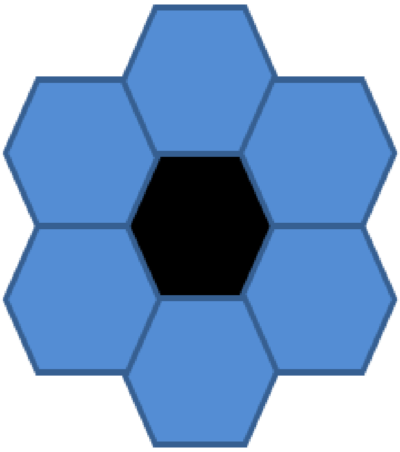

Fig. 1 (c) Primary neighborhood for a hexagonal grid

If secondary neighbors or Moore neighborhood are also taken into consideration the distance between two diagonal cells is '√2 x'. Hence, in a square grid having primary neighbors, the mapped distance is higher than the actual distance and also the angularity is lost in most cases where the actual distance is not linear [7]. Hexagons are simpler and less confusing when the concern is for connectivity and neighborhood [8].

In a three dimensional grid, squares have six primary neighbors, four in the same plane and one each above and below the cell (Fig. 2 (a) and 2 (b)). For a three dimensional hexagonal grid, for every central hexagon there are eight primary neighbors – six in the same plane and one each above and below the cell (Fig. 3 (a) and 3 (b)). This arrangement has been shown in the figure given below. The black cell indicates the central cell and blue cells are its primary neighbors.

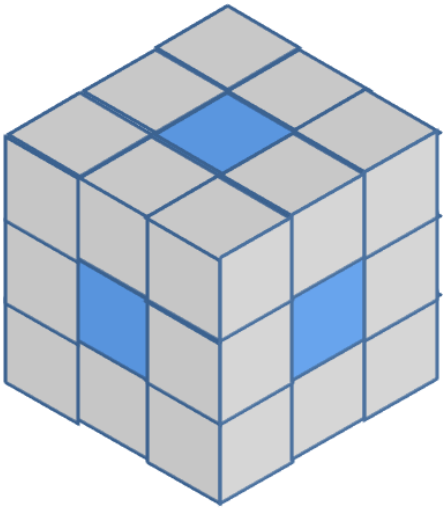

Fig. 2(a) Primary neighborhood for a 3D square grid

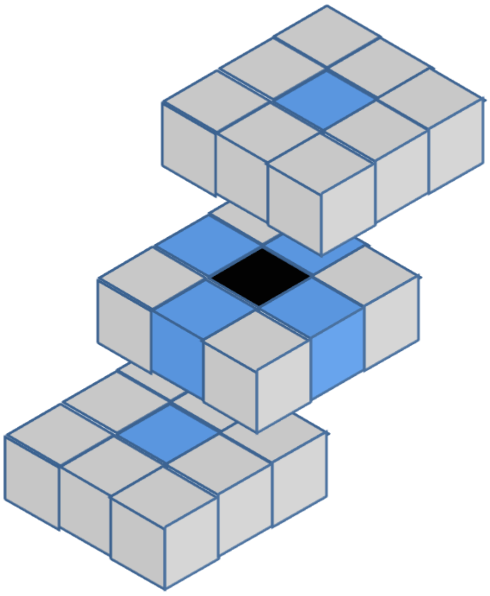

Fig. 2(b) Layer-wise view of primary neighborhood for a 3D square grid

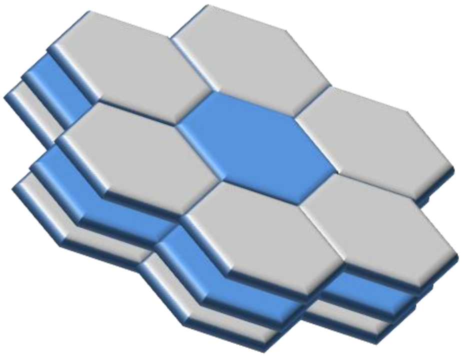

Fig 3(a) Primary neighborhood for a 3D hexagonal grid

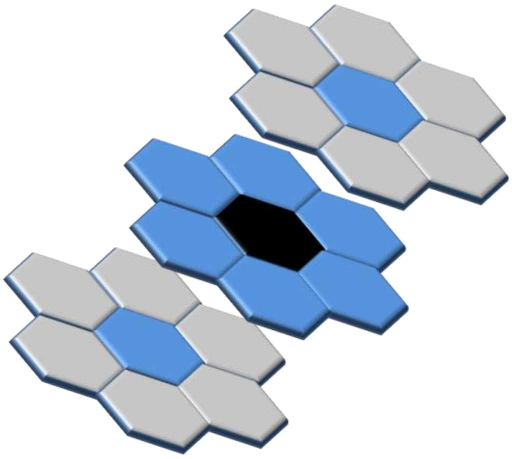

Fig 3(b) Layer-wise view of primary neighborhood for a 3D hexagonal grid

The areal coverage is an important feature necessary for mapping and studying the data. The areal coverage is attained by taking one shape only or taking two or more complementary shapes. For example square, hexagon and triangle are self-sufficient i.e. they do not need any other shape to cover the given area. In the case of triangle though, the orientation needs to be changed so as to cover the area and this complicates the calculation. If a circular cell is considered, there needs to be a combination of circle and triangle so as to ensure areal coverage. The hexagonal shape is the highest order polygon which is capable of covering the area all by itself.

Along with the areal coverage, the earth's curvature also needs to be kept in mind. The latitude and longitude system currently used is not adequate to take care of the curvature. Hexagons are closer to circle than square, and are thus more appropriate for spatial datasets. If a square and hexagon of equal area are considered, then it is found that the hexagon has a shorter perimeter than square which in turn helps in reducing the bias due to edge effect. For example, a hexagon with unit area has a perimeter of 3.722 units and a square with unit area has a perimeter of 4 units [8].

Handling of large datasets and performing advanced and complicated spatial operations are necessary in many fields and they require high representational and algorithmic efficiency. As advances are being made in the geospatial field, their performance is being greatly enhanced even by the slightest improvement in the efficiency and/or accuracy of the datasets. This could have been commonly feasible if the commercial availability of software and display systems which support hexagons existed [9]. For any operation that needs both a regular grid and the efficient areal coverage, a hexagonal tessellation is the logical selection. Hexagonal grids are now commonly being used in various fields like cellular automata, statistical sampling, board games, video games etc. [10]

One of the major challenges faced while tackling a hexagonal grid is assigning a coordinate system to the grid. The coordinate systems which can be followed for a hexagonal grid are using a staggered y-axis, assigning row and column numbers or tilting the y-axis to coincide with the direction of hexagons [11]. After deciding on the coordinate system, the hexagons can be easily incorporated in the study. Such coordinate systems are new to most of the users and hence many users are reluctant to shift to this system. This reluctance needs to be overcome so that hexagons can have a widespread acceptance.

Summary and Discussion

The studies indicate that the higher efficiency and better accuracy of hexagons is making them the preferred grid over square or triangular grids. The major hindrance in the hexagons being more widely used is the lack of infrastructure and commercially available software and display support. The square grid is the grid that has been prevalently used since ages hence all supporting infrastructure and display systems follow the square grid. Hexagons have been proved to be a better choice in most of the cases and are hence gaining popularity.

4. ACKNOWLEDGMENT

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