Graph Regularized Non-Negative Matrix Factorization for Image Retrieval

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ABSTRACT

This paper presents a novel matrix factorization method for effectively performing the image retrieval in large image databases. Non-negative Matrix Factorization (NMF) provides a parts-based representation of data by finding two non-negative matrices whose product can well approximate the original data matrix. Although it has been applied successfully for several applications, simply using NMF for image retrieval provides low performance results which results from the fact that NMF fails to consider the geometric structure that is contained within the data. To solve the above problem, we encode the geometrical information contained in the data by constructing a nearest neighbor graph and perform matrix factorization based on this graph structure. In this work, 500 images from Corel dataset have been taken into consideration. We compared NMF and GNMF based method in the context of image retrieval. Experimental results demonstrate the effectiveness and robustness of GNMF based approach.

Keywords

Non-negative Matrix Factorization, Local Invariance Assumption, Graph Laplacian

1. INTRODUCTION

Image retrieval has been a very active research area since the 1970. In the recent years there has been a considerable growth in the availability of digital image collections which makes learning from example impossible. We cannot effectively access or make use of the information unless it is organized so as to allow effective searching, browsing and retrieval. One then attempts to make use of the matrix factorization method which finds two or more lower dimensional matrices whose product can well approximate the original data matrix. The technique of matrix factorization can be used to solve problems in computer vision, pattern recognition, information retrieval etc.

Many experimental studies have showed the evidences for parts-based representation[2] in human brain. NMF is a matrix factorization algorithm that factorizes the initial data matrix, which represents the whole database, into two non-negative matrices and consequently produces a parts-based representation of images because it allows only additive, not subtractive, combinations of basis images. Due to its parts-based representation [3] property, NMF can be used for image classification [4], face expression recognition [5], face detection [6], document clustering [7] etc. NMF is optimal for learning the parts of the object. When applied for data representation, a major disadvantage of NMF is that it fails to consider the geometric structure in the data which is essential for data clustering and classification problems. Also NMF needs comparatively more computational costs due to the alternate iterations.

In this paper we present a new NMF algorithm, called Graph-regularized Non-negative Matrix Factorization(GNMF) [1], to overcome the above limitations of NMF and thus performing a more efficient image retrieval. The geometrical information is encoded by constructing a nearest neighbor graph which is used to achieve better performance results. To detect the underlying manifold structure many manifold learning algorithms has been proposed all of which use the locally invariant idea [8], i.e., the nearby points are likely to have similar embeddings.

The rest of the paper is organized as follows: in Section 2, we give a brief review of NMF. Section 3 introduces GNMF algorithm. The GNMF based image retrieval is presented in Section 4. Section 5 gives the experimental results and provides comparative performances of NMF and our GNMF based image retrieval system. Finally, Section 6 presents the conclusions of this paper.

2. A BRIEF REVIEW OF NMF

Assume that the image database is represented as an m×n matrix X, each column of which contains m non-negative pixel values of one of the n images. In order to compress data or reduce the dimensionality, NMF [9] finds two non-negative matrices U and V such that

$$X_{mn} \approx (UV^T)_{mn} \qquad (1)$$

The dimensions of U and V are m×k and n×k respectively. Here the k columns of U are called the bases and the rows of H are the combining coefficients. The dimensionality reduction parameter or the rank, 'k' is chosen such that $(m+n)k < m\times n$ and thereby achieving a highly compressed representation. To find the quality of above approximation the cost function based on the square of the Euclidean distance between two matrices(the square of the Frobenius norm of two matrices difference).

$$O_1 = \|X - UV^T\|^2 \qquad (2)$$

To find lower dimensional approximations U and V of the original data matrix, X the multiplicative update algorithm [10] proposed by Lee and Seung is used. The algorithm minimizing the objective function O_1 in Eq (2) is as follows:

$$u_{ik} \leftarrow u_{ik} \frac{(XV)_{ik}}{(UV^TV)_{ik}} \qquad (3a)$$

$$v_{jk} \leftarrow v_{jk} \frac{(X^TU)_{jk}}{(VU^TU)_{jk}} \qquad (3b)$$

We can view this approximation column by column as

$$x_j \approx \sum_{k=1}^{K} u_k\, v_{jk} \qquad (4)$$

where u_k is the k-th column vector of U. Thus, each data vector x_j is approximated by a linear combination of the columns of U, weighted by the components of V.

Algorithm 1: NMF
Input: m×n matrix X, each column of which denotes the image vector, and rank k.
Output: m×k matrix U and n×k matrix V
Step 1: Initialize U and V with non-negative random values.
Step 2: Repeat steps 3 and 4 while not convergent
Step 3: Update the base matrix U using Eq. (3a)
Step 4: Update the coefficient V using Eq. (3b)

3. GRAPHREGULARIZED-NON NEGATIVE-MATRIX FACTORIZATION

NMF tries to find a set of basis vectors that can well approximate the data. By imposing the non-negative constraints on the lower dimensional factors, NMF can be used to learn the parts-based representation. When applied for real world applications, the problem with NMF is that it fails to consider the geometrical structure that is present within the data space. To solve this problem we encode the geometrical information by constructing an affinity graph and then performs NMF by considering this graphical structure.
To construct the affinity(weight) matrix consider each data point x_j and its 'p' nearest neighbors and put the edges between x_j and its neighbors. There are several approaches to construct the graph structure such as 0-1 weighting, heat kernel weighting and dot product weighting each of which is efficient for different class of data. For this work we use the heat kernel weighting which is efficient for image data applications.

Heat Kernel Weighting: If nodes j and l are connected then,

$$W_{jl} = e^{-\frac{(||x_j - x_l||)^2}{\sigma}}$$

Similar to NMF, the GNMF aims to find the non-negative matrices U and V for a given data matrix X of size m×n. As in the case of NMF we consider the cost function based on Euclidean distance which minimizes the objective function as:

$$O_1 = \| X - UV^T \|^2 + \lambda \mathrm{Tr}(V^T L V) \qquad (5)$$

where Tr(.) denotes the trace of the matrix and D is the diagonal matrix whose elements are the row or column sums of W. The graph laplacian [11], L is obtained as L=D-W. The regularization parameter, $\lambda \geq 0$ controls the smoothness of the reduced representation.
The updating rules to find the lower dimensional representation using GNMF is as follows:

$$u_{ik} \leftarrow u_{ik} \frac{(XV)_{ik}}{(UV^T V)_{ik}} \qquad (6)$$

$$v_{jk} \leftarrow v_{jk} \frac{(X^T U + \lambda WV)_{jk}}{(VU^T U + \lambda DV)_{jk}} \qquad (7)$$

Algorithm 2: GNMF
Input: Matrix X of size m×n, rank k and weight matrix W of size n×n.
Output: m×k matrix U and n×k matrix V whose product can well approximate the original matrix X.
Step 1: Initialize U and V with non-negative random values.
Step 2: Repeat steps 3 and 4 while not convergent
Step 3: Update the base matrix U using Eq. (6)
Step 4: Update the coefficient V using Eq. (7)

4.GNMF-BASED IMAGE RETRIEVAL SYSTEM

In this section, we describe the structure of the proposed GNMF based image retrieval system in detail. Fig. 1 shows the main components of the proposed GNMF based image retrieval system and the control flows among them. The functionality of each component in achieving efficient image retrieval is explained below.

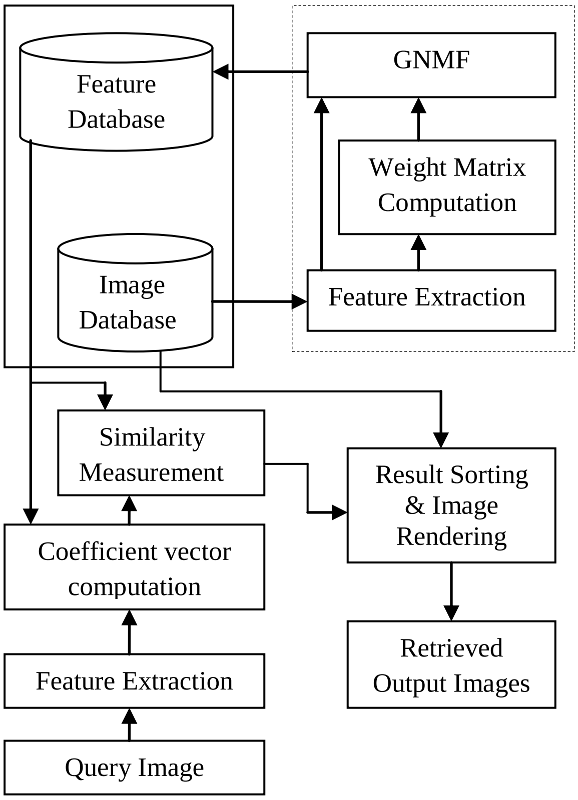

Fig.1 Proposed GNMF-based image retrieval

The images in the image database are given as input to the feature extraction block. Here for each of the images in the database denoising is performed and the features including the color histogram and edge features are extracted to form the feature matrix. This feature matrix is used to construct the affinity graph to encode the geometrical information. This weight matrix together with feature matrix is given as input to the GNMF block that results in feature database which includes two lower dimensional representations called the basis matrix and the co-efficient matrix whose product provides a good approximation to the feature matrix.
When a query image is submitted to retrieve similar images, preprocessing is applied on it and the co-efficient vector of query image is computed by using the basis matrix in the feature database and the feature vector of the query image. The co-efficient vector of the query image thus obtained is compared with each of the co-efficient vectors in the featuredatabase to measure the similarity using the cosine similarity. The result of the similarity measurement is sorted in the ascending order in a manner that returns the indices of

the most similar images in order of similarity which is used for retrieving the images [12].

4.1Functional Blocks:

4.1.1. Preprocessing

This is the initial step where the images from the image database are taken and the features of individual images are extracted. Firstly image denoising is performed to remove the noise in each image using median filter. Feature extraction process use these enhanced images for extracting the features namely color and edge features. In this work, we consider a total of 256 color features and 8 edge features. This block gives a feature matrix, X of size mxn where 'm' is the total number of features of an image (264 in our case) and 'n' is the total number of images in the database. The feature matrix, X is one of the main inputs to the GNMF for creating the lower dimensional feature database.

4.1.2. Weight matrix construction:

The purpose of this step is to create a weight matrix that can model the geometric structure of the data. This weight matrix computation is based on the local invariant assumption about the data points i.e. closer points are likely to have similar embeddings. The input to this step is the feature matrix, X constructed in the previous phase and the output is a weight matrix, W of size nxn where 'n' is the number of images in the image database. The steps involved in this stage are as follows. Initially set W with an nxn zero matrix. Iterate over the feature matrix, X such that for each image x_i we find its closeness with all the other images in the database using the heat kernel weighting equation as

$$W_{ij} = e^{-\frac{(||x_i - x_j||)^2}{\sigma}}$$

The weight matrix, W along with the feature matrix X is used by the GNMF based feature database creation block.

4.1.3. Feature database creation:

This is an important step which involves the creation of feature database. Feature database contains the lower dimensional representations U and V called the basis and co-efficient matrices of size mxr and nxr respectively whose product can well approximate the original data (feature) matrix, X. The steps involved can be summarized as follows. Initially set U and V with non-negative values. U and V are then updated using the multiplicative update rule as shown in Eqns. (6 & 7) respectively. U and V are updated until we get a good approximation i.e. the product of U and V well approximate the original feature matrix, X .Since these update rules consider the geometric information contained in the data space by considering the weight matrix, W which encodes these information this step promises to produce a well approximate lower dimensional representation which in turn can be used for efficient image retrieval.

4.1.4. Similarity measurement:

Similarity measurement computes the similarity of the query image with each of the images in the image database. Pre-processing and feature extraction is performed on the query image to create the feature vector. Then we obtain the co-efficient vector of the query image 'v_q' using the basis vector U present in the feature database and the feature vector of the query image. The output of this step is a row vector of size 1xn whose elements represents the measure of similarity of each image in the test database with the query image. To find this row vector, S we compute the similarity of co-efficient vector 'v_q' and 'v_j's' using the cosine similarity as follows:

$$S[j] = \frac{v_q . v_j}{||v_q|| \, ||v_j||}$$

4.1.5. Result sorting and image retrieval

The row vector, S computed in the previous step is used as input to the final result sorting and image retrieval phase. The steps involved during this stage are: We sort S in such a way that we obtain another row vector, I such that the elements are sorted in the decreasing order of the measure of similarity from S. Then the results are rendered from the database in the order as in I.

5. EXPERIMENTAL RESULTS

In this section, we demonstrate some experimental results to show the performance of the proposed GNMF based image retrieval system. These results are based on experimentation on a test image dataset(subset of Corel dataset) consisting of 500 images including 5 classes and each class has 100 images. Two parameters play vital role in GNMF based image retrieval system: the dimensionality reduction parameter "k" and the number of nearest neighbor "p". The metrics named Acceptance Ratio(AR) and Rejection Rate are used to evaluate the performance of the proposed system. The acceptance ratio and the rejection rate are computed using the following equations referring to [13]:

$$\text{Acceptance ratio} = \frac{1}{N} \sum_{i=1}^{N} (\text{accept ratio})_i$$

$$\text{accept ratio} = \frac{1}{K} \sum_{j=1}^{K} \left(\frac{\text{No.of relevant images retrieved}}{\text{Total no.of relevant images}} \right)_j$$

where N and K are the number of image groups and the number of images in each group respectively.

$$\text{Rejection rate} = \frac{1}{S} \sum_{i=1}^{S} \left(\frac{\text{No. of relevant images rejected}}{\text{Total no. of processed images}} \right)_i$$

where S is the total number of images that is processed.

Table. 1 shows the comparison of acceptance ratio in percentage for different 'k' values of the NMF and GNMF based image retrieval systems. Using this optimum value of 'k=8' the acceptance ratio of for the proposed system for different 'p' values is shown in Table. 2. The comparison of acceptance ratio and rejection rate for 20, 50, 100, 200, 300, 400, 500 images using NMF and GNMF is shown in Table 3. The graphical analysis of the same is as shown in Fig.2 and Fig 3.

Table.1 Acceptance ratiofor different k values

k	Acceptance Ratio NMF	Acceptance Ratio GNMF
3	44.74	58.87
5	56.82	59.15
8	64.33	73.03
10	64.24	72.43
15	60.16	70.29
17	62.77	66.09
19	60.41	64.88

Table.2 Acceptance ratio for different p

p	Acceptance Ratio
1	65.62
2	72.59
3	66.45
4	73.01
5	72.57

Table.3 Comparison of acceptance ratio and rejection rate for varying number of images.

No. of images	Acceptance ratio(%)		Rejection rate	
	NMF	GNMF	NMF	GNMF
20	92.5	96.25	0.015	0.008
50	89.92	94.40	0.02	0.011
100	86.2	92.80	0.028	0.014
200	83.32	88.23	0.033	0.024
300	73.23	86.80	0.054	0.026
400	66.43	80.40	0.067	0.039
500	64.33	73.03	0.071	0.054

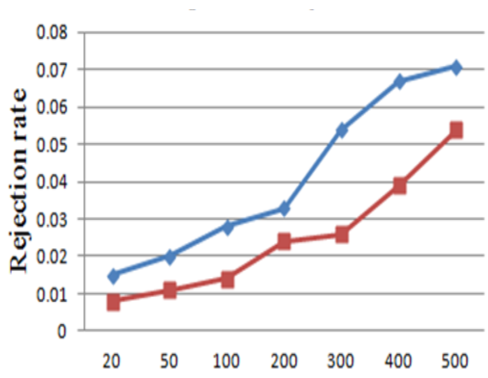

Fig.2 Comparison of Acceptance Ratio

No. of images NMF GNMF

No. of images NMF GNMF

Fig.3 Comparison of Rejection rate

The above experimental results shows that for the image database we considered the performance of the image retrieval continues to increase with increasing value of 'k' up to a certain point beyond which it becomes oscillatory. So for our system we select the k=8 at which the system achieves maximum performance result. By setting the k value as 8 we test the system with different 'p' values and the best choice of p was found to be 4 for the particular database we considered. The sample results of the GNMF based image retrieval system is as shown in Fig.4 and Fig.5, where the first image is the query image, which is also the top match.

Fig.4 The results of GNMF based image retrieval system for the class of flowers.

Fig.5 The results of GNMF based image retrieval system for the class of tribes

6. CONCLUSION

This paper presents a new method called Graph Regularized NMF that can be used for representing huge image databases by lower dimensional representation which provides the way for efficient image retrieval. GNMF consider the geometric structure that is inherent in the data by constructing a p-nearest neighbor (affinity graph) using heat kernel method and performs non-negative matrix factorization which respects this graph structure. GNMF have more discriminating power than ordinary NMF which simply imposes the non-negativity constraint. The performance of GNMF based system is influenced by the values of dimensionality reduction parameter, k(i.e. increases up to a certain point in our case k=8 beyond which it becomes oscillatory) and the nearest neighbor, p. From the experimental results it can be seen that our GNMF based image retrieval system outperforms by providing an acceptance ratio of 87.41%.GNMF can be used in biometric identification applications such as face and iris recognition which can be applied in security measures at air ports, passport verification and criminal list verification in police departments.

7. FUTURE WORK

This paper provides a simple and efficient method for image retrieval which performs retrieval based on the computation and sorting of the values of cosine similarity between the features of query image and the images in the database. This is a one to one mapping where the accuracy of the retrieval is fixed always and cannot be improved. The system performance can be enhanced by incorporating Back Propagation Neural Network (BPNN) consisting of input, hidden and output layers that can be trained using the images in the database to create a knowledge base. This training process can be repeated until an improved accuracy as desired is reached thus providing better image retrieval performance.

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