Analysis of Discrete Time Sliding Mode Control for a Magnetic Levitation System

Ankur Goel
Department Of Electrical Engg.
NIT Kurukshetra
Kurukshetra

A Swarup
Department Of Electrical Engg.
NIT Kurukshetra
Kurukshetra

ABSTRACT
The advent of highly efficient and superior digital devices and fast microprocessors for control application has opened the field of discrete time controller design. This paper presents an exciting application of magnetic levitation system using discrete sliding mode control. There is a limited volume of literature available for discrete sliding mode control as applied to magnetic levitation system. This paper presents an application of magnetic levitation system using discrete sliding mode control. In this work, a discrete first order sliding mode control (1-DSMC) and second order sliding mode control (2-DSMC) is investigated in order to show the difference between the both strategies. A comparative study of both the approaches is presented.

General Terms
Control system, Sliding mode control

Keywords
Sliding Mode control, single order discrete sliding mode, second order discrete sliding mode, MAGLEV.

1. INTRODUCTION
A Magnetic Levitation System (Maglev) is considered as a good benchmark problem for the design and analysis of control systems. It has many engineering applications such as high speed maglev trains, frictionless bearings, and levitation of wind tunnel models. Magnetic levitation systems are usually open loop unstable and highly nonlinear. Therefore, it is necessary and important to design a robust feedback controller for regulating the position of the levitated object.

Several authors have proposed several control strategies to stabilize the position of the levitated object. The PID controller is a simple method for the operation point linearization in nonlinear system. It is suitable in the small region of operating point and sensitive to the parameter variations and external disturbances. Authors [1-3] discussed the feedback linearization controller which utilizes a complete nonlinear description and yields consistent performance which is largely independent of the operating point. But the feedback linearization control does not guarantee robustness when the modeling errors are present. There are several other methods for nonlinear system control like H^{∞} control, H_2 control, Fuzzy control [4-7]. But the sliding mode control (SMC) is one of the popular methods in robust control for nonlinear systems due to its robustness and invariance property. However, SMC has a drawback of high frequency oscillations which is due to discontinuous dynamic system. These oscillations which are known as chattering are very serious phenomenon from practical implementation point of view and it becomes necessary to reduce their magnitude and frequency.

Generally the SMC controllers are designed for a magnetic levitation system [8]-[9] are in continuous time. The advantages of digital control over analog counterpart are high accuracy, cheap, better noise rejection, and higher reliability. Not only these advantages but also the advent of digital computers, samplers and the availability of cheap microprocessors for implementation of controllers with fast sampling rates, motivates for the use of a discrete-time sliding mode controller for the control of a magnetic levitation system.

In this paper first a single order discrete-time sliding mode controller (1-DSMC) for a magnetic levitation system is designed by using Gao's discrete time reaching law [10]. In 1-DSMC [10- 11], the control effort is calculated once in every sampling interval which remain constant during this period. Due to the finite sampling frequency, the system state trajectory moves in a zigzag manner along the surface, which is termed as quasi-sliding motion (QSM). In the eighties a new control technique, called high order sliding mode control, has been proposed. The main idea is to reduce the sliding function, along with its high order derivatives to zero. In the case of the n-order sliding mode control, the discontinuity is applied on the $(n - 1)$ derivative of the control. The effective control is obtained by $(n-1)$ integrations and can, then, be considered as a continuous signal. In other words, the oscillations generated by the discontinuous control are transferred to the higher derivatives of the sliding function [13-14]. This approach shows the promising result by reducing the oscillations amplitude and keeping the robustness of the sliding mode system intact [12]. Here we have used the second order discrete sliding mode control (2-DSMC) to justify this property. This strategy is more practical for a magnetic levitation system in comparison to state feedback based methods, in which separate observers are required to estimate the velocity of the ferromagnetic ball and other state variables. Our simulation results show the effectiveness of the 2-DSMC over 1-DSMC for the control of a magnetic levitation system.

This paper has been organized as follows: Section II describes the mathematical model of the magnetic levitation system. Section III gives the design of a single order discrete time sliding mode controller for the maglev. Section IV describes the second order discrete time sliding mode controller. Section V demonstrate the application of both the methodologies on the Maglev model with the help of MATLAB software and compare the results. Conclusion is given in section VI.

2. SYSTEM MODEL

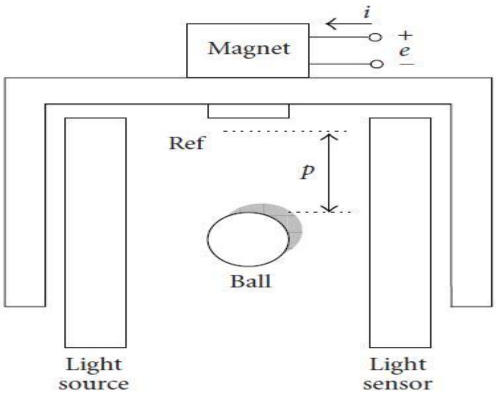

Fig 1. Magnetic Levitation System [9]

The diagram in Fig.(1) shows a popular gravity-based one degree-of freedom magnetic levitation system, in which an electromagnet exerts attractive force to levitate a steel ball (in some references a steel plate is levitated).This attractive force counteract the gravitational force which is needed to keep the ball at a desired height. The dynamic model for the system is as follows:-

$$\begin{aligned} \frac{dp}{dt} &= v \\ Ri + \frac{d}{dt}L(p)i &= e \\ m\frac{dv}{dt} &= mg_c - Q\left(\frac{i}{p}\right)^2 \end{aligned} \tag{1}$$

Where, p = x_1 =position of ball, v = x_2 =velocity of ball, i = x_3 = current in the electromagnet coil, e= applied voltage, R = coil resistance, L = coil inductance, g_c = gravitational constant, Q = magnetic force constant, and m = mass of the levitated ball.

Assuming that the inductance L is a nonlinear function of the position of the ball p, we approximate as follows:

$$L(p) = L_1 + \frac{2Q}{p} \tag{2}$$

Here, L_1 is a parameter of the system which is determined by the electromagnetic coil inductance.

Considering the states of the system and the control input as follows:

x_1 = p, x_2 = v, x_3 = i and voltage e(t) as an control input u(t), the state vector is represented as $X=(x_1, x_2, x_3)^T$.

Now the non linear state space model of the magnetic levitation system can be written as follows:-

$$\begin{aligned} \frac{dx_1}{dt} &= x_2 \\ \frac{dx_2}{dt} &= g_c - \left(\frac{Q}{m}\right)\left(\frac{x_3}{x_1}\right)^2 \\ \frac{dx_3}{dt} &= -\left(\frac{R}{L}\right)x_3 + \left(\frac{2Q}{L}\right)\left(\frac{x_2 x_3}{(x_1)^2}\right) + \left(\frac{u}{L}\right) \end{aligned} \tag{3}$$

The objective of the control scheme is to drive the states x_1, x_2, x_3 to their desired steady state values x_{1d}, x_{2d}, and x_{3d} respectively. The equilibrium point for the system is $x_d=(x_{1d}, 0, x_{3d})^T$, in which x_{3d} satisfies

$$x_{3d} = x_{1d}\sqrt{\frac{(m)(g_c)}{Q}} \tag{4}$$

Now, considering the following non linear change of coordinates –

$$\begin{aligned} z_1 &= (x_1 - x_{1d}) \\ z_2 &= x_2 \\ z_3 &= \left(g_c - \left(\frac{Q}{m}\right)\left(\frac{x_3}{x_1}\right)^2\right) \end{aligned} \tag{5}$$

The objective has been changed to drive the z_1, z_2, z_3 to zero as time $t \to \infty$. Hence the dynamic model of the magnetic levitation system in the new coordinates system can be re-written as-

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= f_1 + g_1 u \end{aligned} \tag{6}$$

Where,

$$\begin{aligned} f_1 &= 2(g_c - z_3)\left(\left(1 - \frac{2Q}{L(z_1 + x_{1d})}\right)\left(\frac{z_2}{z_1 + x_{1d}}\right) + \frac{R}{L}\right) \\ g_1 &= \frac{-2}{L(z_1 + x_{1d})}\sqrt{\frac{Q(g_c - z_3)}{m}} \end{aligned} \tag{7}$$

Let the output of the system be

$$y = z_1 = x_1 - x_d \tag{8}$$

Now using model (5), (6), (7), (8), the design of discrete time SMC system schemes for the magnetic levitation system will be considered in the following sections.

To verify the performance of the 1-DSMC and 2-DSMC based controller, the system was simulated using MATLAB software, with the parameters given in the Table 1. MATLAB has a function Real Time Workshop Target (RTWT) which helps in the real time implementation of the controller.

3. SINGLE ORDER DISCRETE SLIDING MODE (1-DSMC)

A discrete-time sliding mode control is important when the control is implemented by computers with a relatively slow sampling period [10]. The Non linear Maglev model (6) is first converted into the following discrete time system form:-

$$z(k+1) = \phi_\tau z(k) + \Gamma_\tau w(k) \tag{9}$$

where, z(k) is the state vector, w(k) is the control input and ϕ_τ Γ_τ are system and input matrices of appropriate dimensions, respectively.

Using the model of the magnetic levitation system (5)-(8), and the state feedback concept, the control input will be in form:-

$$\text{control input} = u = \frac{-f_1 + w}{g_1} \quad (10)$$

We obtain the transformed state representation as follows:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w$$
$$y = z_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad (11)$$

Here, 'w' represents the control input after the linearization of the system.

This linear model is subsequently discretized with sampling time to get the best system response and minimum chattering. Thus, the continuous-time system as given in (11) is converted to a discrete time system of the form (9) with the sampling time of τ = 0.1 sec.

After linearization and discretization of the system, the system states are restricted to a hyper plane of the state space termed as the switching surface s(k) which is designed as follows:

$$s(k) = c^T z(k) = \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} \quad (12)$$

Here, the coefficients are tuned to have closed loop poles at the desired position in the left hand side of the s-plane.

After designing the switching surface, the next step is to design the discrete sliding mode controller to restrict the states on the desired switching surface, and this controller in our case is based on the Gao's discrete time reaching law [27] which is given as follows.

$$s(k+1) - s(k) = -q\tau s(k) - \varepsilon\tau sign(s(k))$$
$$\varepsilon > 0, q > 0, 1 - q\tau > 0 \quad (13)$$

Where ε, q are non zero constants and τ is the sampling time.

Using this reaching law, the control law obtained for our case is described by-

$$w(k) = -(c^T \Gamma_\tau)^{-1}(c^T \phi_\tau z(k) - c^T z(k) + q\,\tau\, c^T z(k))$$
$$+\varepsilon\,\tau\, sign(c^T z(k)) \quad (14)$$

Where, $(c^T \Gamma_\tau) \neq 0$

4. SECOND ORDER DISCRETE SLIDING MODE (2-DMC)

A sliding mode is said to be "r^{th} order sliding mode" if and

Only if

$$S(t,x) = \dot{S}(t,x) = ... = \dot{S}^{r-1}(t,x) = 0 \quad (15)$$

In high order sliding mode control, the state are forced to move on the switching surface $S(t, x) = 0$ and to keep its $(r - 1)$ successive derivatives null.

There are few contributions in higher order discrete sliding mode domain. Bartolini proposes the approach [16] which is the direct discretization of his proposition in continuous-time [15]. In this approach the calculation of the control law does not require the knowledge of the system's model. The other approach which is based on the equivalent control method for second order sliding mode control requires the use of a system's model and allows an asymptotic convergence of the sliding function to zero. M. Mihoub et al. [17] proposes the second order sliding mode control with asymptotic sliding function convergence which is investigated here for maglev system. Let us consider the system defined as

$$x(k+1) = Ax(k) + Bu(k) \quad (16)$$

$$y(k) = Hx(k)$$

The sliding function designed for this system is in this linear form:

$$s(k) = C^T (x(k) - x_d(k)) \quad (17)$$

Where $x_d(k)$ is a desired state vector.

Now let us consider a new sliding function μ(k) such that

$$\mu(k) = s(k+1) + \beta\, s(k) \quad (18)$$

Here s (k+1) is defined as –

$$s(k+1) = C^T (x(k+1) - x_d(k+1))$$
$$= C^T (A\,x(k) + B\,u(k) - x_d(k+1))$$

In order to ensure the convergence of μ(k), β is chosen in between the interval of [0,1].

Following the equivalent control analogy, the control force needed to force the states to the sliding surface is evaluated from the condition

$$\mu(k+1) = \mu(k) = 0 \quad (19)$$

Hence (17), (18) and (19) will give

$$S(k+1) + \beta S(k) = 0$$

And, $s(k+1) = C^T (A\,x(k) + B\,u_{eq}(k) - x_d(k+1))$

Hence,

$$u_{eq}(k) = (C^T B)^{-1}[-\beta\, S(k) - C^T A\, x(k) + C^T (x_d(k+1)] \quad (20)$$

The condition of robustness is ensured by adding the discontinuous term which depends on the sign of the new sliding function μ (k). As in the continuous-time case, the integral of the discontinuous term which will be approximated by a first order transformation is applied to the system (16).

$$u_{dis}(k) = u_{dis}(k-1) - \tau\, M\, sign(\mu(k)) \quad (21)$$

The control at the instant k is then:

$$u(k) = u_{eq}(k) + u_{dis}(k) \quad (22)$$

The integration of the discontinuous term of the control will provide the benefit to many applications where actuators can be damaged by the discontinuity of the 1-DSMC like actuators, gates etc.

Table 1. SIMULATION PARAMETER FOR MAGLEV SYSTEM

Simulation Parameter	Symbol	Value	Units
Coil Resistance	R	25.6	Ohm
Coil Inductance	Lc	0.68	H
Gravitational Constant	g_c	9.81	m/s2
Magnetic Force Constant	Q	0.000121	-
Levitated Object Mass	m	0.01387	Kg
Plant Initial Condition	x1	0.0255	m
	x2	0	m/s
	x3	1.1	A
Desired Steady State Value	x1d	0.02	m
	x2d	0	m/s
	x3d	0.6707	A

5. SIMULATION RESULTS

The discrete time sliding mode controller schemes discussed previously are applied to the non linear magnetic levitation model discussed in section II. It is a 3rd order system where the position of ball is taken as the output.

After using the state feedback concept, the continuous model of this system is represented by the (11) is given by following equation-

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} w \qquad (23)$$

$$y = z_1 = [1 \quad 0 \quad 0] \times \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

Here, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C = [\,1 \;\; 0 \;\; 0\,]$

This model (23) is converted into the discrete form with the sampling time of τ = 0.1 sec which is represented in the form (9)

$$z(k+1) = \phi_\tau z(k) + \Gamma_\tau w(k)$$

Where,

$$\phi_\tau = e^{A\tau} = \begin{bmatrix} 1 & 0.1 & 0.005 \\ 0 & 1 & 0.1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Gamma_\tau = A^{-1}(\phi_\tau - I) = \begin{bmatrix} 0.0002 \\ 0.0050 \\ 0.1000 \end{bmatrix}$$

The simulation parameter for the 1-DSMC control law are (ϵ, q, C^T) which is chosen as $\epsilon = 0.3$, q= 0.4. The matrix C^T is arbitrarily chosen to obtain the desired poles location. The control action w(k) is computed by (14).These parameters are tuned to get the best performance of the system.

For the 2-DSMC controller, whose control law is computed by (22), the tuning parameters are (β, M). The synthesis parameter β determine the sliding function dynamics, while M ensure the sliding mode existence. The values for the tuning of 2-DSMC is taken as $\beta = 0.9$ and M = 0.1. Fig.2 and Fig.3 show the comparison of the plots for position x_1 , velocity x_2, current x_3, control effort v/s time for a 1-DSMC and 2-DSMC when the desired position of levitated object is 0.02 m and 0.03 m. The response show the effectiveness of the 2-DSMC in terms of the fast convergence and better control action than 1-DSMC. The ferromagnetic ball attains its desired steady state position in lesser time when 2-DSMC is applied.

Now in order to compare the robustness of 1-DSMC and 2-DSMC, both the controllers are applied to the maglev where the desired position is kept at 0.025 and a disturbance is injected at k = 30. The Fig. 4 shows that 2-DSMC demonstrated the better result than 1-DSMC in terms of reduced frequency of the oscillations and robustness. Fig. 5 shows the switching surface v/s time plot. It has been shown that chattering in sliding mode function and the control action can be reduced when 2-DSMC controller is implemented.

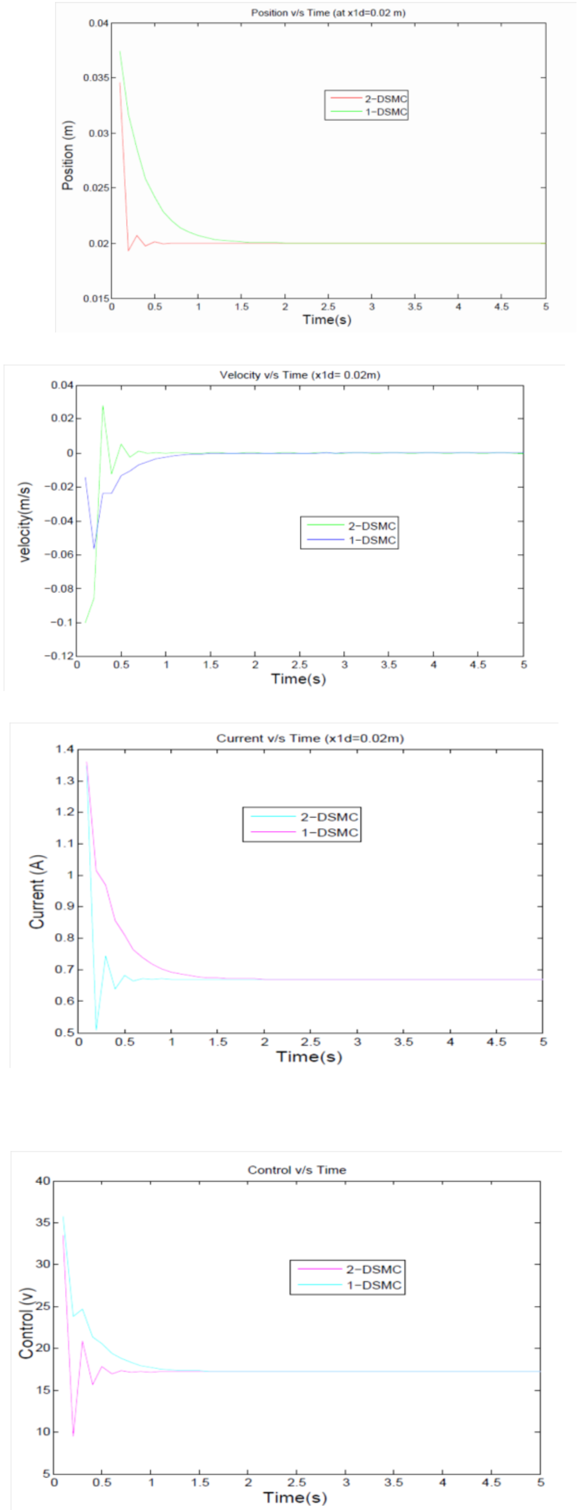

Fig. 2 : State Response for desired position on *x*1*d*=0.02 m.

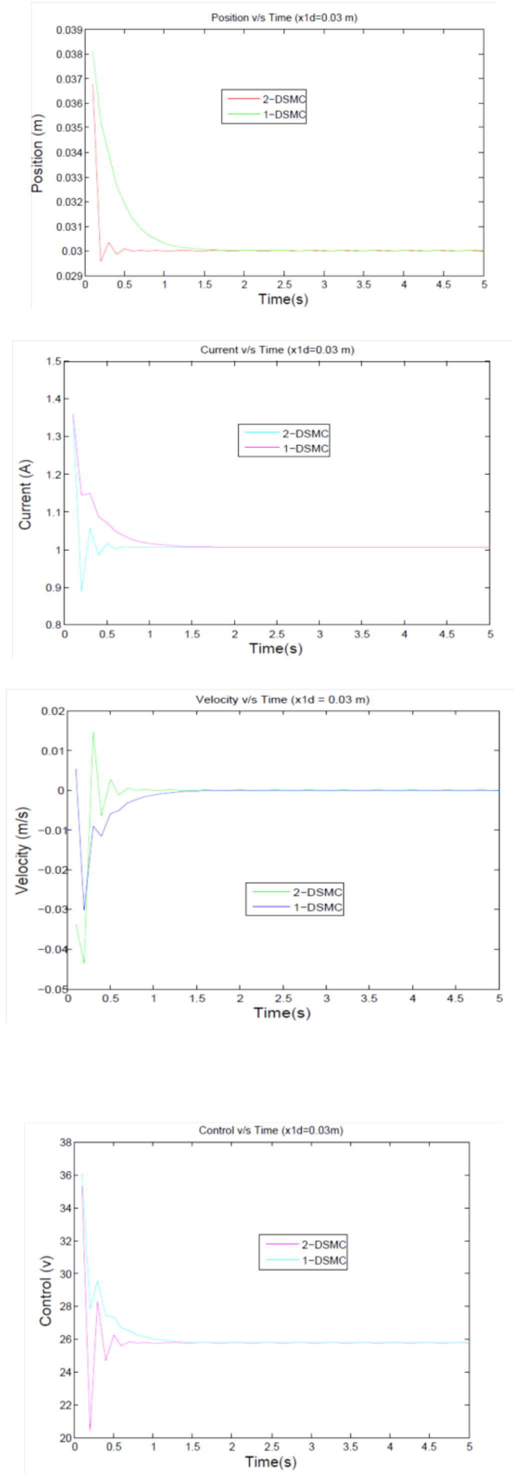

Fig.3 : State Response for desired position on *x*1*d*=0.03 m

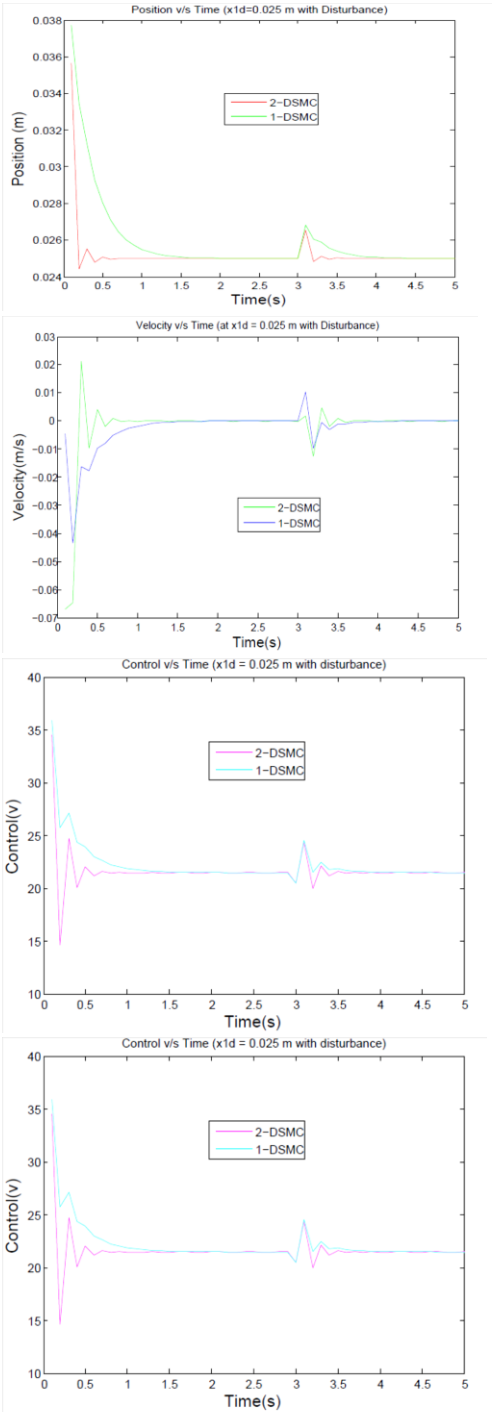

Fig.4: State Response for desired position on *x*1*d*=0.025 m with disturbance at k =30.

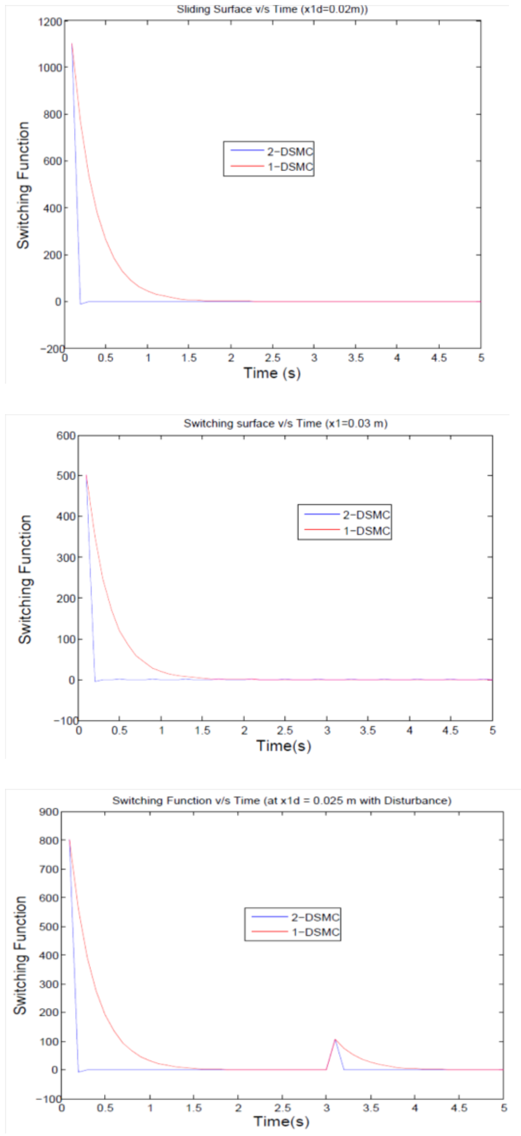

Fig 5 :.Plot for Switching Function v/s time for x1d= 0.02m, 0.03m , 0.025m with disturbance.

6. CONCLUSION

Simulation results of 1-DSMC and 2-DSMC control algorithms for non-linearized magnetic levitation system were presented. Although no experimental result has been given in this paper, the numerical results show that second order discrete time sliding mode control law performs well in the presence of nonlinearity and better method for the reduction of chattering and is an promising method for application to Levitation system.

7. REFERENCES

[1] C.J. Munaro, M.R. Filho, R.M. Borges, S. da Silva Munareto and W.T. da Costa, "Modeling and observer-based nonlinear control of a magnetic levitation system," IEEE International Conference on Control Applications, Vol. 1, pp. 162-167, 2002.

[2] M.R. Filho, and C.J. Munaro, "A design methodology of tracking controllers for magnetic levitation systems," IEEE International Conference on Control Applications, pp. 47-51, 2001.

[3] R.K.H. Galvao, T. Yoneyama, F.M.U. de Araujo, and R.G. Machado, "A simple technique for identifying a linearized model for a didactic magnetic levitation system," *IEEE Transactions on Education*, Vol. 46, No. 1, pp. 22-25, February 2003.

[4] D. Cho, Y. Kato and D. Spilman, "Sliding mode and classical controllers in magnetic levitation systems," *IEEE Control Systems Magazine*, Vol. 13, pp. 42-48, 1993.

[5] J.C. Shen, "H∞ control and sliding mode control of magnetic Levitation system," *Asian Journal of Control*, Vol. 4, No. 3, pp. 333-340, 2002.

[6] Y. Niu, Z. Wang and X. Wang, "Robust sliding mode design for uncertain stochastic systems based on H∞ control method," o*ptimal Control Applications and Methods*, Vol. 31, No. 2, pp. 93-104, Mar./Apr. 2010.

[7] L. Ma, Z.D. Wang and Z. Guo, "Robust H2 sliding mode control for nonlinear discrete-time stochastic systems," *IET Control Theory and Applications*, Vol. 3, No. 11, pp. 1537-1546, Nov. 2009.

[8] M. Jalili-Kharaajoo, M.M. Tousi, H. Bagherzadeh, A.E. Ashari, "Sliding mode control of voltage-controlled magnetic levitation systems**", p***roceedings of 2003 IEEE Conference on Control Applications*, vol.1, pp.83-86, 23-25 June 2003.

[9] N. F. Al-Muthairi and M. Zribi. "Sliding Mode Control of a Magnetic Levitation System". *Mathematical Problems in Engineering*, vol.2004 (2004), Issue 2, pp. 93-107.

[10] W. Gao, Y. Wang, and A. Homaifa, "Discrete-time variable structure control systems," IEEE Transactions on Industrial Electronics, vol. 42, no. 2, pp. 117–122, Apr 1995.

[11] S. Z. Sarpturk, Y. Istefanopulos, and O. Kaynak, "On the stability of discrete-time sliding mode control systems," *IEEE Transactions on Automatic Control*, 1987, vol. 32, no. 10, pp. 930-932.

[12] A. Levant. "sliding order and sliding accurcy in sliding mode control". *International Journal of Control*, 58(6):1247–1263, 1993.

[13] Levant,A. "Sliding order and sliding accuracy in sliding mode control", International JournalofControl, 58(6), 1247–1263, 1993

[14] Nollet, F.,Floquet,T.,&Perruquetti,W. "Observer- based second order sliding mode control laws for stepper motors". Control Engineering Practice, 16(4), 429–443, 2008.

[15] G. Bartolini, A. Ferrara, and E. Usai. Chattering avoidance by second order sliding mode control. *IEEE Transactions on Automatic Control*, 43(2):241–246, February 1998.

[16] G. Bartolini, A. Pisano, and E. Usai. "a Digital 2- sliding control algorithm for a class of uncertain non-linear plants". In *CC99. European Control Conference 1999*, 1999.

[17] Mohamed Mihoub , Ahmed Said Nouri, Ridha Ben Abdennour, "Real-time application of discrete second order sliding mode control to a chemical reactor", *Control Engineering Practice* ,vol.17, pp 1089-1095, 2009.